Neutrino Mass and Grand Unification

Lecture II

22/10/2021

LMU

Fall 2021

Unif. of ew + strong int

= Grand Unification

• $U(1)_{em}$

Maxwell = massless photon

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e\, j_\mu^{em} A^\mu \qquad (1)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\Rightarrow \quad A_\mu \to A_\mu + \frac{1}{e} \partial_\mu \alpha(x)$$

$$\boxed{\partial_\mu j^\mu_{em} = 0}$$

$$e j_\mu^{em} A_\mu \to e j_\mu^{em} A_\mu + j_\mu^{em} \partial^\mu \alpha$$

$$= -''- + \partial^\mu (j_\mu^{em} \alpha) - \alpha \, \partial^\mu j_\mu^{em}$$

$$= -''-$$

$$\Downarrow$$

$$(\Box g^{\mu\nu} - \partial^\mu \partial^\nu) A_\nu = e \, j_\nu^{em} \quad (2)$$

A_μ = photon ($= \gamma$)

2 d. o. f. (degrees of freedom)

(helicity) $h = \bar{S} \cdot \hat{p}$

gauge fixing $\boxed{\partial^\mu A_\mu = 0}$

$$\Rightarrow \boxed{\Box A_\mu = e\, j_\mu^{em} \qquad (3)}$$

$$A_\mu = \frac{e}{\Box} j_\mu^{em}$$

$$\hookrightarrow \frac{e}{q^2} \quad \text{(momentum)}$$

$$\Downarrow$$

$$e A_\mu j^\mu_{em} = e^2 j_\mu^{em} \frac{g^{\mu\nu}}{q^2} j_\nu^{em}$$

propagator of $m_A = 0$

$$\left(\Delta_{\mu\nu}(A^{\mu}) = -\frac{i}{q^2} g_{\mu\nu} \right)$$

- Proca theory of a
  U(1) <u>massive</u> gauge boson

$$\mathcal{L}_P = \mathcal{L}_M + \frac{1}{2} m_A^2 A_\mu A^\mu$$

(a) $\partial^\mu j_\mu = 0$

$$\Downarrow$$

$$\left[(\Box + m_A^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu = g\, j^\mu$$

$\hookrightarrow$ derivative ∂_μ

(g = generalized e)

$$\Rightarrow \left(\cancel{(\Box + m_A^2)} - \cancel{\Box} \right) \partial_\mu A^\mu = g\, \partial_\mu j^\mu$$

conserved current $\partial^\mu j_\mu = 0$

$$\Rightarrow \boxed{\partial_\mu A^\mu = 0}$$

$\Downarrow$

ensures 3 d.o.f. (played)

$$\Rightarrow (\Box + m_A^2) A_\mu = g\, j_\mu$$

$$\boxed{A_\mu = \frac{g}{\Box + m^2} j_\mu \qquad (4)}$$

$$\hookrightarrow \frac{g}{-q^2 + m_A^2}$$

Feynman $(m_A \to 0)$

if $m_A = 0$

(b) $\partial_\mu j^\mu \neq 0$

$$\boxed{\text{propagator} = \text{inverse of the quadratic form}}$$

$\Downarrow$

$$\Delta_{\mu\nu}(A^\rho) = \frac{-i}{q^2 - m_A^2}\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{m_A^2}\right]$$

effective int. $\Downarrow$

$$\boxed{j^\mu \, \Delta_{\mu\nu}(A^\rho)\, j^\nu g^2}$$

$$\downarrow$$

$$\underbrace{g^2 \, j^\mu \frac{g_{\mu\nu}}{q^2 - m_A^2} j^\nu}_{\text{OK} = \text{smooth}} \; \underbrace{- g^2 \frac{1}{q^2 - m_A^2} j^\mu \frac{q_\mu q_\nu}{m_A^2} j^\nu}_{\text{BAD}}$$

the source of all evil

$$\left(\begin{array}{l} \text{iff } \partial^\mu j_\mu = 0 \iff q^\mu j_\mu = 0 \\ \qquad \Rightarrow \text{BAD} \to 0 \end{array} \right)$$

However

$$\boxed{\begin{array}{l}\text{if } m_A \gg q \\ \Rightarrow \text{BAD} \to 0\end{array}}$$

$$\Downarrow$$

$$\frac{g^2}{-\cancel{q^2}+m_A^2}\, j^\mu j_\mu = \mathcal{H}_{eff}$$

$\Longleftrightarrow$ Fermi theory

$$\mathcal{H}_{eff}^F(\text{weak}) = \frac{4G_F}{\sqrt{2}} J^\mu_w J_\mu^w$$

$$\frac{G_F}{\sqrt{2}} = \frac{1}{M_F^2} = \frac{\cdots g^2}{4 M_W^2}$$

<u>Non-Abelian</u> (Yang-Mills)

$\boxed{\text{Show}}$

$$D_\mu F_a^{\mu\nu} = j_a^\nu$$

$$\Rightarrow \partial_\mu F_a^{\mu\nu} = j_a^\nu + \cdots$$

$$\underbrace{\partial_\nu \partial_\mu F_a^{\mu\nu}}_{= 0} = \partial_\nu j_a^\nu + \underbrace{\cdots\cdots}_{\neq 0}$$

$$\Downarrow$$

$$\boxed{\partial_\nu j_a^\nu \neq 0}$$

weak current.

$$J^\mu_W = \underbrace{\bar{u}_L \gamma^\mu d_L} = \bar{u} \gamma^\mu L d$$

$$L(R) = \frac{1 \pm \gamma_5}{2}$$

$$\boxed{J^\mu_W = \frac{1}{2} \bar{u} \gamma^\mu d + \frac{1}{2} \bar{u} \gamma^\mu \gamma^5 d \qquad \partial^\mu J^W_\mu \neq 0}$$

[accept weak current as above]

$$\Downarrow$$

$$\boxed{\text{non-Abelian}}$$

$\Longleftrightarrow$ off-diagonal generators

$$\Big\Downarrow \ \boxed{\text{minimality}}$$

$$\boxed{SU(2)}$$

(gauge theory)

1957

Schwinger

D (doublet) $\to$ U D

$$\boxed{\begin{array}{l} UU^{+} = U^{+}U = 1 \\ \det U = 1 \end{array}}$$

$$U = e^{iH} \qquad H^{+} = H$$

unitary

$$\det U = 1 \Rightarrow \mathrm{Tr}\, H = 0$$

$$\Downarrow$$

$$H = \theta_a T_a \leftarrow \text{generators}$$

$$a = 1, 2, 3$$

$$T_a = \frac{\sigma_a}{2}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (r = 1)$$

rank

$$\boxed{[T_a, T_b] = i\,\varepsilon_{abc} T_c}$$

$$\mathrm{Tr}\, T_a T_b = \frac{1}{2}\delta_{ab} \quad \text{(doublet)}$$

$$T_\pm = T_1 \pm i T_2$$

$$[T_3, T_\pm] = \pm T_\pm$$

$$[T_+, T_-] = 2 T_3$$

$$\Downarrow$$

$$D = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$t_{3u} = t_{3d} + 1$$

$$\boxed{t_{3u} = -t_{3d} = 1/2}$$

$$\Rightarrow \boxed{t_3 \text{ (any repr.)} = n\, 1/2}$$

$\boxed{D = \text{fermion}}$ (f)

$$\mathcal{L}_D = i\bar{f}\gamma^\mu D_\mu f - m_f \bar{f} f$$

$$f = \begin{pmatrix} u \\ d \end{pmatrix} \quad \left(+ \text{gauge} \atop \text{kin} \right)$$

$$\Downarrow \quad D_\mu = \partial_\mu - ig\, T_a A_\mu^a$$

$$\mathcal{L}_{kin} = i\bar{f}\gamma^\mu\partial_\mu f + \underbrace{g\, A_\mu^a \bar{f}\gamma^\mu T_a f}_{\boxed{\mathcal{L}_{int}}}$$

$$\Downarrow$$

$$\mathcal{L}_{int} = \frac{g}{2} \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix} \gamma^\mu \begin{pmatrix} 0 & A_1 - iA_2 \\ A_1 + iA_2 & 0 \end{pmatrix}_\mu \begin{pmatrix} u \\ d \end{pmatrix}$$

$$+ g/_2 \; \bar{f} \gamma^\mu \sigma_3 A_\mu^3 f$$

$$= \frac{g}{\sqrt{2}} \bar{u} \gamma^\mu \left(\frac{A_1 - iA_2}{\sqrt{2}} \right)_\mu d + h.c$$

$$\boxed{+ g \bar{f} \gamma^\mu T_3 A_\mu^3 f}$$

$$W_\mu^+ = \frac{(A_1 - iA_2)_\mu}{\sqrt{2}}$$

$$W_\mu^- = \frac{(A_1 + iA_2)_\mu}{\sqrt{2}}$$

$$\Downarrow$$

$$\mathcal{L}_{weak} = \frac{g}{\sqrt{2}} \bar{u} \gamma^\mu d \, W_\mu^+ + h.c.$$

SU(2) gauge $\Rightarrow$ $\exists$ "weak" int.

$$\boxed{A_\mu^3 = ?} \iff \overset{\text{weak}}{\overset{\downarrow}{g}} = e \leftarrow \text{em}$$

$$\boxed{Q_{em} = T_3}$$

charge is quantized

$$T_3 = \sigma_3/2 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

$\boxed{\text{Fails}}$? ?

NOT ?

(x)

A_μ^3 = new gauge boson

$$G_{ew} = \underbrace{SU(2)_W \times U(1)_{em}}_{}$$

$$U(1)_{em} \updownarrow Q_{em}$$

$$\Updownarrow$$

$$[T_a, Q_{em}] = 0$$

$$\Updownarrow$$

photon = flavor blind

$\uparrow$

$\left[\begin{array}{l} u = \text{up flavor} \\ d = \text{down} \text{ —||— } \end{array} \right]$

$$\Downarrow$$

$$\left[\begin{array}{c} Q_u^{em} \equiv q_u = q_d = Q_d^{em} \\ + \\ q_\nu = q_e \end{array}\right]$$

Photon cannot live outside SU(2) (weak)

$$\Downarrow$$

electro-weak theory

"unified"

$$\Downarrow$$

$$G_{SM} = SU(2) \times \underset{Y}{U(1)}$$

arbitrary

$$\Rightarrow \quad Q = a\,T_3 + b\,\frac{Y}{2}$$

$$\Downarrow$$

$$\boxed{Q_{em} = T_3 + \frac{Y}{2}} \qquad (a = b = 1)$$

normalization

$$D_\mu = \partial_\mu - ieQA_\mu + \cdots$$

$$= \partial_\mu - ie'Q'A_\mu + \cdots$$