

Neutrino Mass
and

Grand Unification

Lecture II

22/10/2021

LMU
Fall 2021



Unif. of ew + strong int

= Grand Unification

• $U(1)_{\text{em}}$

Maxwell = massless
photon

$$\mathcal{L}_H = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e j_\mu^{\text{em}} A^\mu \quad (1)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\Rightarrow A_\mu \rightarrow A_\mu + \frac{1}{e} \partial_\mu \alpha(x)$$

$$\partial_\mu j_\mu^{\text{em}} = 0$$

$$e j_\mu^{em} A_\mu \rightarrow e j_\mu^{em} A_\mu + j_\mu^{em} \partial^\nu d$$

$$= -\square - + \partial^\mu (j_\mu^{em} d)$$

$$- d \partial^\mu j_\mu^{em}$$

$$= -\square -$$



$$(\square g^{\mu\nu} - \partial^\mu \gamma^\nu) A_\nu = e j_\nu^{em}(z)$$

A_μ = photon ($= \gamma$)

2 d. o. f. (degrees of freedom)

(Helicity) $h = \vec{s} \cdot \hat{\vec{p}}$

gauge fixing, $\boxed{\partial^\mu A_\mu = 0}$

$$\Rightarrow \boxed{\square A_\mu = e j_\mu^{\text{em}} \quad (3)}$$

$$A_\mu = \frac{e}{\square} j_\mu^{\text{em}}$$

$$\hookrightarrow \frac{e}{q^2} \text{ (momentum)}$$



$$e A_\mu j_\nu^{\text{em}} = e^2 j_\mu^{\text{em}} \frac{g^{\mu\nu}}{q^2} j_\nu^{\text{em}}$$



propagator of $w_A = 0$

$$\left(\Delta_{\mu\nu} (A) = -\frac{i}{q^2} g_{\mu\nu} \right)$$

- Prove theory of a V_1) massive gauge boson

$$\mathcal{L}_p = \mathcal{L}_M + \frac{1}{2} m_A^2 A_\mu A^\mu$$

$$(a) \quad \partial^\mu j_\mu = 0$$



$$\boxed{\left[(\square + m_A^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu = g j^\mu}$$

derivative ∂_μ ($g = \text{generalized } e$)

$$\Rightarrow (\cancel{\square} + m_A^2) - \cancel{g} \partial_\mu A^\mu = g \partial_\mu j^\mu$$

conserved current $\partial^\mu j_\mu = 0$

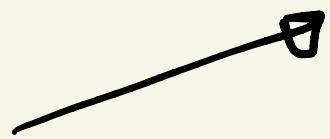
$$\Rightarrow \boxed{\partial_\mu A^\mu = 0}$$

ensures 3 d.o.f. (physical)

$$\Rightarrow (\square + m_A^2) A_\mu = g j_\mu$$

$$\boxed{A_\mu = \frac{g}{\square + m_A^2} j^\mu \quad (4)}$$

$$\hookrightarrow \frac{g}{-q^2 + m_A^2}$$



Feynman ($\frac{m}{A} \rightarrow 0$)

if $m_A = 0$

$$(b) j_\mu j^\mu \neq 0$$

propagator = inverse of
the quadratic form



$$A_{\mu\nu}(A^P) = -\frac{i}{q^2 - m_A^2} \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{m_A^2} \right]$$

effective int.

$$\boxed{\int^\mu \Delta_{\mu\nu} (A^0) j^\nu g^2}$$



$$g^2 j^\mu \frac{g_{\mu\nu}}{g^2 - m_F^2} j^\nu - g^2 \frac{1}{g^2 - m_F^2} \int^\mu \frac{g_\mu g_\nu}{m_F^2} j^\nu$$

$\underbrace{\hspace{10em}}$

$\overbrace{\hspace{10em}}$

OH = smooth

BAD

the source of all evil

$$\left(\text{iff } \partial^\mu j_\mu = 0 \Leftrightarrow g^\mu j_\mu = 0 \right)$$

$$\Rightarrow \text{BAD} \rightarrow 0$$

However

If $m_A \gg \varrho$
 $\Rightarrow B A D \rightarrow 0$



$$\cancel{\frac{g^2}{\cancel{e} + m_A^2}} j^\mu j_\mu = h_{\text{eff}}$$

\Leftrightarrow Fermi theory

$$h_{\text{eff}}^F (\text{weak}) = 4 \frac{G_F}{\sqrt{2}} J_w^\mu J_\mu^\nu$$

$$\frac{G_F}{F^2} = \frac{1}{M_F^2} = \frac{-g^2}{4 M_W^2}$$

Non-Abelian

(Yang-Mills)

Show

$$D_\mu F_a^{\mu\nu} = j_a^\nu$$

$$\Rightarrow D_\mu F_a^{\mu\nu} = j_a^\nu + \dots$$

$$\underbrace{\partial_\nu \partial_\mu F_a^{\mu\nu}}_{\text{---}} = \partial_\nu j_a^\nu + \underbrace{\dots}_{\text{---}}$$

↓

$\partial_\nu j_a^\nu \neq 0$

weak current.

$$J_\nu^\mu = \bar{u}_L \gamma^\mu d_L = \bar{u} \gamma^\mu L d$$



$$L(R) = \frac{1 \pm \gamma_5}{2}$$

$$J_\nu^\mu = \frac{1}{2} \bar{u} \gamma^\mu d + \frac{1}{2} \bar{u} \gamma^\mu \gamma^5 d$$

$$\gamma^\mu J_\mu^\nu \neq 0$$

accept weak current as
above

1L



um-Abeliden

\Leftrightarrow off-diagonal gluons

\downarrow minimality

$SU(2)$

1957

(gauge theory)

Schwinger

D (doublet) \rightarrow U D

$UV^+ = V^+U = I$
 $\det U = 1$

$$U = e^{iH} \quad H^+ = H$$



unitary

$$\det U = 1 \Rightarrow T_U H = 0$$



$$H = \theta_a T_a + \text{generators}$$

$$a = 1, 2, 3$$

$$T_a = \frac{\sigma_a}{2}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (r=L)$$

reich

$$[T_a, T_b] = i \epsilon_{abc} T_c$$

$$T_r T_a T_b = \frac{1}{2} \delta_{ab} \text{ (doublet)}$$

$$T_{\pm} = T_1 \pm i T_2$$

$$[T_3, T_{\pm}] = \pm T_{\mp}$$

$$[T_+, T_-] = 2 T_3$$

$$D = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$t_{3u} = t_{3d} + 1$$

$$t_{3u} = -t_{3d} = 1/2$$

$$\Rightarrow t_3 \text{ (any repr.)} = \underline{u^{1/2}}$$

$D = \text{fermion}$ (f)

$$\mathcal{L}_D = i\bar{f} \gamma^\mu D_\mu f - m_f \bar{f} f$$

$$f = \begin{pmatrix} u \\ d \end{pmatrix} \quad (+ \text{ gage } \underbrace{\text{ term }}_{\text{kin}})$$

$$\Downarrow \quad D_\mu = \partial_\mu - ig T_a A_\mu^a$$

$$\mathcal{L}_{\text{kin}} = i\bar{f} \gamma^\mu \partial_\mu f + g A_\mu^a \bar{f} \gamma^\mu T_a f$$

$\boxed{\mathcal{L}_{\text{int}}}$

\Downarrow

$$\mathcal{L}_{\text{int}} = \frac{g}{2} (\bar{u} \bar{d}) \gamma^\mu \begin{pmatrix} 0 & A_1 - i A_2 \\ A_1 + i A_2 & 0 \end{pmatrix}_\mu \begin{pmatrix} u \\ d \end{pmatrix}$$

$$+ g/2 \bar{f} \gamma^\mu \sigma_3 A_\mu^3 f$$

$$= \frac{g}{\sqrt{2}} \bar{u} \gamma^\mu \left(\frac{A_1 - i A_2}{\sqrt{2}} \right)_\mu d + h.c.$$

$$+ g \bar{f} \gamma^\mu \sigma_3 A_\mu^3 f$$

$$W_\mu^+ = \frac{(A_1 - i A_2)_\mu}{\sqrt{2}}$$

$$W_\mu^- = \frac{(A_1 + i A_2)_\mu}{\sqrt{2}}$$

$$\mathcal{L}_{\text{weak}} = \frac{g}{\sqrt{2}} \bar{n} \gamma^\mu d W_\mu^+ + h.c.$$

$SU(2)$ gauge $\Rightarrow \exists$ "weak" int.

$$\boxed{A_\mu^3 = ?} \Leftrightarrow g = e \leftarrow \text{ew} \quad \downarrow^{\text{weak}}$$

$$\boxed{Q_{\text{ew}} = T_3}$$

charge is quantized

$$T_3 = \sigma_{3/2} = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

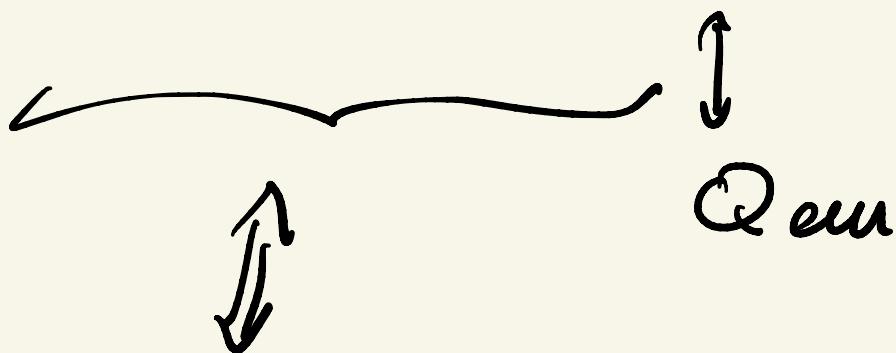
Fails

???

NOT?

$A_\mu^S = \text{new gauge boson}$ (x)

$$G_{EW} = SU(2)_W \times U(1)_Y$$



$$[T_a, Q_{EM}] = 0$$



photon = flavor blind



$u = \text{up flavor}$ }
 $d = \text{down flavor}$ }



$$\left. \begin{aligned} Q_u^{\text{em}} &= e_u = e_d = Q_d^{\text{em}} \\ &+ \\ q_v &= e_e \end{aligned} \right\}$$

Photons cannot live outside
 $SU(2)$ (weak)



electro - weak theory



"unified"

$$\Downarrow$$

$$G_{SM} = SU(2) \times U(1)$$

γ
 \uparrow

orb. theory

$$\Rightarrow Q = a T_3 + b \frac{\gamma}{2}$$

$$\Downarrow$$

$$Q_{em} = T_3 + \frac{\gamma}{2}$$

$$(a = b = 1)$$

$$\gamma$$

normalization

$$D_\mu = \partial_\mu - i e Q A_\mu + \dots$$

$$= \partial_\mu - i e 'Q' A_\mu + \dots$$

