19/10/2021

Neutrino Mass

and

Grand Unification

$\boxed{\text{Lecture I}}$

LMU

2021-2022

• SM $\Rightarrow m_\nu = 0$

$\Downarrow$ $m_\nu \neq 0$ (oscillations)

neutrino mass = door to

new physics

• Grand Unification =

= unification of SM forces

$\Downarrow$

(a) proton decay

(b) $\exists$ magnetic monopoles !

Standard Model (SM)

$$= \underbrace{\text{em} + \text{weak} + \text{strong}}_{}$$

messengers of forces =

gauge bosons

$$\Updownarrow$$

SM = gauge theory

em = $U(1)$

weak = $SU(2)_L$ flavor

strong = $SU(3)$ color

$\Downarrow$

$\cancel{SU(4)}$

gauge principle

$\Downarrow$

(i) $\exists$ messengers

(ii) renormalizable =

= all physical amplitudes are finite in pert. theory

symmetry : $SU(N)$

$\boxed{F \to U\, F}$ (fund. repr.)

$$\left.\begin{array}{l} UU^+ = U^+U = 1 \\ \det U = 1 \end{array}\right\} \Rightarrow U = e^{iH}$$

$$\Downarrow$$

$$H = H^+ , \quad \mathrm{Tr}\, H = 0$$

$\Rightarrow$ $N^2 - 1$ elements

$$\Rightarrow \quad H = \theta_a T_a \, ; \qquad a = 1, \dots, N^2 - 1$$

$$[T_a, T_b] = i f_{abc} T_c$$

gauge: $\quad \theta_a = \theta_a(x)$

$$\partial_\mu F \rightarrow D_\mu = \partial_\mu - i g T_a A_\mu^a$$

$$A_\mu \equiv T_a A_\mu^a$$

$$D_\mu F \rightarrow U D_\mu F$$ (covariant derivative)

$$U(\partial_\mu - i g A_\mu) F =$$

$$= \left(\partial_\mu - ig\, A_\mu'\right) U F$$

$$\Downarrow$$

$$\boxed{A_\mu' = U A_\mu U^+ + \frac{i}{g}(\partial_\mu U) U^+}$$

(1)

$U(1)$: Q = generator

$$A_\mu = A_\mu Q$$

$$U = e^{i\theta(x) Q} \qquad \Longrightarrow$$

$$Q A_\mu' = Q A_\mu + \frac{1}{g} i (\partial_\mu \theta) Q$$

$\Downarrow$

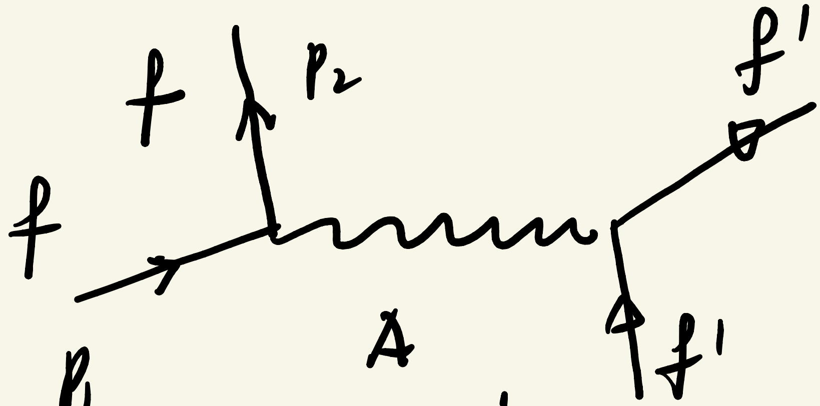
$$\boxed{A_\mu' = A_\mu - \partial_\mu \theta(x) \frac{1}{g}}$$

$\boxed{U(1)}$ $\boxed{Q = \mathrm{diag}(q_1, q_2, \cdots, q_n)}$

$\Downarrow$

fermions would give

$$i \bar{f} \gamma^\mu D_\mu f \to g \bar{f} \gamma^\mu q_f A_\mu f$$



$q = p_2 - p_1$

$\downarrow$

$$q_f q_{f'} \frac{g^2}{q^2} \bar{f} \gamma^\mu f \; \bar{f}' \gamma_\mu f'$$

q = momentum exchange

em: $\quad g = e \Rightarrow \boxed{\alpha_{em} = \frac{e^2}{4\pi} \simeq 1/100}$

$\boxed{SU(2)}$

$\Downarrow$

$$F = \begin{pmatrix} u \\ d \end{pmatrix} \circlearrowleft T_1, T_2$$

$$T_a \equiv \frac{\sigma_a}{2}$$

$\Downarrow$

non-Abelian

Yang, Mills '54

Shaw '54

$$\boxed{[T_a, T_b] = i \varepsilon_{abc} T_c}$$

- $\frac{g}{\sqrt{2}} W_\mu^+ \; \bar{u} \gamma^\mu d$

$$W_\mu^+ = \frac{(A_1 - i A_2)_\mu}{\sqrt{2}}$$

$$A_\mu = T_i A_\mu^i = T_1 A_\mu^1 + T_2 A_\mu^2 + T_3 A_\mu^3$$

$$T_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} 1/2 \, , \quad T_2 = \cdots$$

- $\boxed{Q = T_3}$ $\quad (Q = c T_3)$

$$D_\mu = \partial_\mu - i g T_3 A_\mu^3$$

$$= \partial_\mu - i g \frac{Q}{c} A_\mu^3$$

$$g' = q/c$$

$$\boxed{\Rightarrow \ c \neq \text{physical}}$$

$$\boxed{\Rightarrow \quad q = n\,\frac{1}{2} \qquad n = \text{integer} \qquad \text{quantization of charge}}$$

$$\boxed{\text{em charge} = \text{quantized}} \quad \left(1/10^{20} \right)$$

$$q_e = 3\, q_d, \quad q_u = -2\, q_d$$

$$q_\nu = 0$$

$SU(2) \longrightarrow$ fails!

$$\begin{matrix} 2/3 \\ -1/3 \end{matrix} \begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{pmatrix} \nu \\ e \end{pmatrix} \begin{matrix} 0 \\ -1 \end{matrix}$$

quarks leptons

$$n = ddu$$

$$p = duu$$

$$\Downarrow$$

Glashow '1961

$$G_{ew} = SU(2)_L \times U_Y(1)$$

$$[Y, T_a] = 0$$

$$\Rightarrow \quad Y_\nu = Y_e$$

$$Y_u = Y_d$$

$$\Rightarrow \quad Q = aT_3 + \frac{y'}{2}b$$

$$= a\left(T_3 + \frac{y'b}{a}\right)$$

$$\Rightarrow \quad a = 1 \quad \text{(normalization)}$$

$$y = \frac{b}{a}y'$$

$$\Rightarrow \quad \boxed{Q = T_3 + Y/2}$$

$$\Downarrow$$

$$\boxed{y = 2(Q - T_3)}$$

$\boxed{1957}$

$V - A \quad \Leftarrow$ Marshak, Sudarshan

$\Updownarrow$

$$\mathcal{H}^{W}_{eff} = \frac{G_F}{\sqrt{2}} J^{W}_{\mu} \overline{J^{\mu}_{W}}$$

$$J^{W}_{\mu} = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu d_L$$

"V-A was the key"

Weinberg

em + weak

$\parallel$

$\boxed{\text{ew theory}}$

'57 Schwinger

$\boxed{SU(2)_L}$

$\Downarrow$

$Q_{em} = T_3$ fails

$\Downarrow$ extend

$$\overset{g}{SU(2)_L} \times \underset{Y}{\overset{g'}{U(1)}}$$

does not work

$$Q_{em} = T_3 + \frac{Y}{2} \text{ (arbitrary)}$$

does not work

of $SU(3)$

$$\Rightarrow \quad A_\mu = \sin\theta_W A_\mu^3 + \cos\theta_W B_\mu$$

$\tan\theta_W = g'/g \quad \Downarrow \quad \boxed{e = g \sin\theta_W}$ of $U(1)$

$$\boxed{Z_\mu = \cos\theta_W A_\mu^3 - \sin\theta_W B_\mu}$$

$$\Leftrightarrow \quad J_\mu^{em} = \bar{f}\gamma_\mu Q f$$

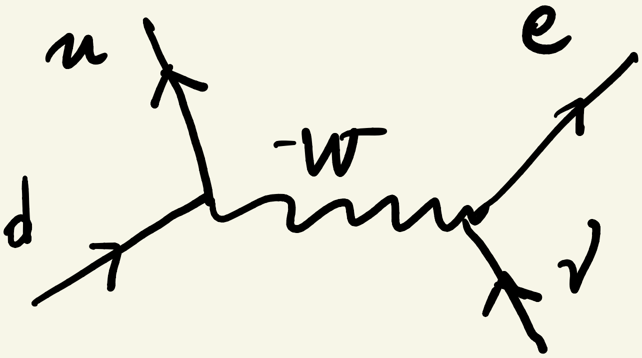
$$\boxed{J_\mu^Z = \bar{f}\gamma^\mu (T_3 - Q\sin^2\theta_W) f}$$

$$\alpha_{em} = \frac{e^2}{4\pi} \simeq 1/100$$

$$\alpha_{weak} = \frac{g^2}{4\pi} \simeq 1/10$$

$\boxed{\text{why weak?}}$

$\boxed{q < \text{GeV}}$



$$\frac{1}{\cancel{q^2} - M_W^2}$$

β decay: $d \longrightarrow u + e + \bar{\nu}_e$

$$M_W \gg q \Rightarrow \underset{\text{weak}}{\frac{1}{M_W^2}} < \underset{\text{em}}{\frac{1}{q^2}}$$

$$\alpha_{\text{eff}}^{\text{weak}} \simeq \alpha^{\text{weak}} \left(\frac{E^2}{M_W^2} \right)$$

$$\alpha_{\text{gr}}^{\text{eff}} = \frac{E^2}{M_p^2}$$

$$G_N = \frac{1}{M_p^2}$$

$$M_p \simeq 10^{19}\ \text{GeV}$$

$$(\hbar = c = 1)$$

$$G_F \simeq \frac{1}{M_W^2}$$

$\Downarrow$

$$\alpha_{gr}^{static} = \frac{M_1 M_2}{M_p^2}$$

$$M_\odot \simeq 10^{60}\,\text{GeV}$$

$$M_{earth} \simeq 10^{50}\,\text{GeV}$$

Strong force = $SU(3)_c$ gauge

$\alpha_s \simeq O(1)$ $\qquad\qquad$ $\alpha_s = \dfrac{g_s^2}{4\pi}$