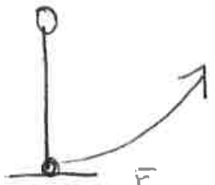
$\mapsto x(t) = A \cdot \sin(\omega t)$



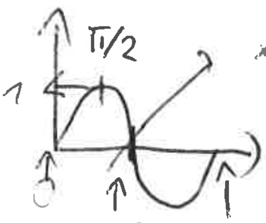
$E_{Pot} \Rightarrow$ minimal

$F = m \cdot a = m \cdot \ddot{x}$; $F = -kx$ (Hooke)

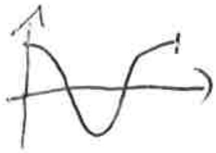
DGL: $\ddot{x} + \omega_0^2 x = 0$ $\qquad \omega_0 = \sqrt{\frac{k}{m}}$

Lösungen: $x(t) = A \cdot \sin(\omega t + \varphi)$

$\hookrightarrow \varphi = \frac{\pi}{2}$



$\Rightarrow x(t) = A \cdot \sin(\omega t) + B \cdot \cos(\omega t)$



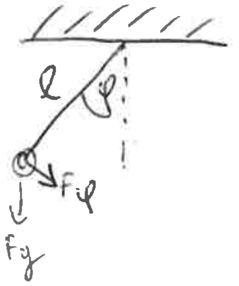
Komplexer Ansatz: $x(t) = c \cdot e^{\lambda t}$

$\ddot{x}(t) = c \cdot \lambda^2 e^{\lambda t}$

$\Rightarrow c \cdot e^{\lambda t} \cdot \omega_0^2 + c \cdot \lambda^2 e^{\lambda t} = 0 \qquad \lambda^2 = -\omega_0^2 \Rightarrow \boxed{\lambda = \pm i \omega_0}$

$x(t) = C_1 \cdot e^{i\omega_0 t} + c_2 \cdot e^{-i\omega_0 t}$

$\sin(\varphi) = \varphi$? Kleinwinkelnäherung



$$F_\varphi = -m g \sin\varphi$$

$$\Rightarrow \boxed{\ddot{\varphi} + \frac{g}{l} \cdot \sin\varphi = 0}$$

Taylor:

$$\sin\varphi = \sum_{k=0}^{\infty} (-1)^k \frac{\varphi^{2k+1}}{(2k+1)!} = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \dots$$

$$\sin\varphi = \varphi$$

$\varphi \ll 2\pi$ [$\varphi < 23^\circ$ $\rightarrow$ Fehler $\approx 1\%$]

Gedämpfte Schwingung:

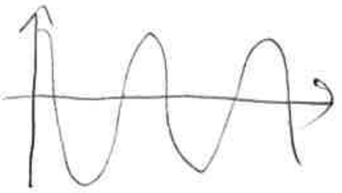
$F = m \cdot a = m \cdot \ddot{x}$; $F = -k x$; $F = -b \dot{x}$ (Stokes)

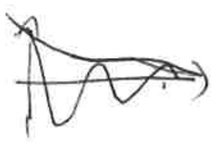
↳ $6\pi\eta r$ für Kugel

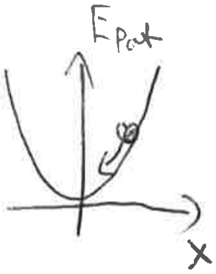
DGL: $\ddot{x} + \frac{b}{m} \cdot \dot{x} + \frac{k}{m} \cdot x = 0$

Lösung: $x(t) = \underbrace{A \cdot e^{\delta t}}_{\text{Einhüllende}} \sin(\omega' t + \varphi)$

$$\boxed{\delta = \frac{b}{2m}} \qquad \boxed{\omega'^2 = \omega^2 - \delta^2}$$







$E_{Pot} = \frac{1}{2} D x^2$

$F = -Dx$

$\omega_0 = \sqrt{\frac{D}{m}}$

$D = k$

Quadratisches Potential

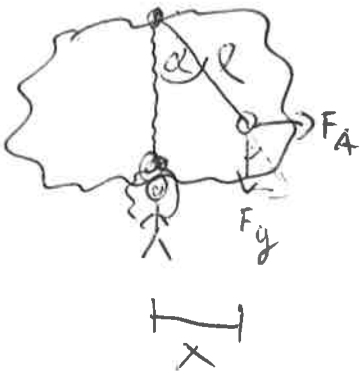
$\Updownarrow$

lineare Kraft

$\Updownarrow$

harmonische Schwingung

Höhenbestimmung per Pendel



① Statische Messung

$|\vec{F}_A|, \ |\vec{F}_G|, \ x$

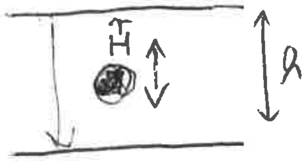
$\sin\alpha = \frac{x}{l} \qquad \tan\alpha = \frac{F_A}{F_g}$

KWN: $\sin\alpha \approx \tan\alpha \approx \alpha \qquad \frac{x}{l} \approx \frac{F_A}{F_G} \qquad \boxed{l = x \cdot \frac{F_G}{F_A}}$

② Dynamisch über T

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \boxed{l = \frac{g \cdot T^2}{4\pi^2}}$$

Prallender Ball



$\boxed{\text{Ohne Energieverlust}}$

1) Gesucht: t_{gesamt} ; $v(h) = 0$; k ; Radius $= r$

3 Teilbewegungen:

(1) Freier Fall $\quad s = h - 2r$

(2) Harmon. Schwingung

(3) Wurf

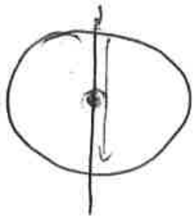
t_1: $\quad s = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2s}{g}}$ $\quad$ mit $s = h - 2r$

$\Rightarrow t_1 = \sqrt{\frac{2(h-2r)}{g}}$

t_2: $\quad 2\pi \cdot \sqrt{\frac{m}{k}}$

t_3: $\quad t_3 = t_1 \quad \Rightarrow \quad t_{gesamt} = 2 \cdot t_1 + t_2$

•)



$\boxed{t=0}$

$v_S = 333\,\frac{m}{s}$; $R = 6{,}37\cdot 10^3\,km$

$m = 55\,kg$ $\quad M = 5{,}98\cdot 10^{24}\,kg$

•) Harmonische Schwingung

„Bahnkurve"

$F_g \sim r \Rightarrow r(t) = R\cdot\cos(\omega t)$

Bestimme D („Federkonstante")

$F = -D r$ $\qquad F_g(r) = -G\,\frac{m M}{R^3}\cdot r$

$$\Rightarrow \boxed{D = G\cdot\frac{m M}{R^3}}$$

$\omega = \sqrt{\frac{D}{m}}$

$$\Rightarrow D = m\omega^2 \Rightarrow \omega = \sqrt{\frac{G M}{R^3}}$$

$$\Rightarrow \omega = 1{,}24\cdot 10^{-3}\,/s \qquad \Rightarrow T = \frac{2\pi}{\omega} = 84\,min$$

$$\Rightarrow v_{max} = R\cdot\omega = 7{,}9\,km/s$$