LMU GUT Course

Lecture VIII

27/11/2020

Fall 2020
LMU

$$SU(3) \supseteq SU(2) \times U(1)$$

- quarks

$p = uud \qquad n = udd$

$q_u = 2/3 \qquad q_d = -1/3$

$$\boxed{W \longleftrightarrow \text{LH fermions}}$$

$$F = \begin{pmatrix} u \\ d \\ \hline D \end{pmatrix}_L \leftarrow \qquad F = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

$$\boxed{SU(3) \times \text{Lorentz}}$$

$$F \to U_3 F$$

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \to U_2 \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

$L \qquad R$

U_3 acts on LH fermions

on RH —"—

separate

$$Q_{em} = \sum_{\text{Cartan}} c_a T_a$$

(T_3, T_8)

$$\boxed{Tr\, Q_{em} = 0}$$

$$3_L = F_L = \begin{pmatrix} u \\ d \\ \hdashline D \end{pmatrix}_L \quad \begin{pmatrix} U \\ D \\ \hdashline d \end{pmatrix}_R$$

W

$\boxed{d_R,\ u_R}$? $\longrightarrow$ SU(2) singlets

D_R

$$\psi_R \to (\psi^c)_L = C\,\bar{e}_R{}^T$$

Dirac C

$$C^T = -C$$

$$C\gamma_\mu C^T = -\gamma_\mu{}^T$$

$$C^T C = 1$$

$$\left(\begin{array}{ll} u & \leftarrow 2/3 \\ d & \leftarrow -1/3 \\ \hdashline D & \;\; -1/3 \end{array}\right.$$

charges

$$\Rightarrow \left(\begin{array}{ll} \psi_1 & \to 2/3 \\ \psi_2 & \to -1/3 \\ \hline \psi_3 & - \;\; -1/3 \end{array}\right.$$

u_R must be ψ_3 <u>because</u>

u_R is $SU(2)$ singlet

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \qquad \psi_3 \longrightarrow \text{only } \psi_3 = \text{singlet}$$

$SU(2) \times \boxed{U(1)}$ $\qquad$ $\boxed{\text{Glashow '61}}$

easy

(p, n) $\qquad$ (ν, e)

NO quarks

$\boxed{V-A}$ $\qquad$ Marshak, Sudarshan '57

$\Longleftrightarrow$ p_L, n_L have weak int

$$\begin{pmatrix} p \\ n \\ \hdashline P^c \end{pmatrix}_L \Longleftarrow \begin{pmatrix} n^c \\ p^c \end{pmatrix}_R$$

$$(n^c)_R = C\,\bar{n}_L^{\,T}$$

$$(p^c)_R = C\,\bar{p}_L^{\,T}$$

$$Q_P = Q_p = 1$$

$$\boxed{n_R \sim (n^c)_L = \text{singlet}}$$

$$P^c \stackrel{?}{=} p^c$$

$$Q = \sum_{\text{Cartan}} c_a T_a$$

$$\boxed{Q_{\text{singlet}} = 0}$$

$$\boxed{T_a \text{ singlet} = 0}$$

Green : $(\psi^c)_L = (\psi^c)_L$

$$(\psi^c)_L = C \, \overline{\psi_R}^{\,T}$$

SM weak int. singlet

$$(\psi^c)_L = C \gamma_0 \psi_R^* \quad (C = i\gamma_2\gamma_0)$$

$$= i\gamma_2 \psi_R^*$$

$$= \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ u_R^* \end{pmatrix}$$

$$= \begin{pmatrix} i\sigma_2 u_R^* \\ 0 \end{pmatrix} \leftarrow LH$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$L(R) = \frac{1 \pm \gamma_5}{2}$$

$$L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

$$(\psi^c) = C\gamma_0 \psi_R^*$$

$$= C \gamma_0 \frac{1-\gamma_5}{2} \psi^*$$

$$R(\psi^c) = \frac{1-\gamma_5}{2} C \gamma_0 \frac{1-\gamma_5}{2} \psi^*$$

$$= C \frac{1-\gamma_5}{2} \gamma_0 \frac{1-\gamma_5}{2} \psi^*$$

$$= C \gamma_0 \frac{1+\gamma_5}{2} \frac{1-\gamma_5}{2} \psi^*$$

$$= 0$$

$$L(\psi^c) = C \gamma_0 \frac{1-\gamma_5}{2} \frac{1-\gamma_5}{2} \psi^*$$

$$\boxed{\psi^c(\psi_R) = LH}$$

$(\psi^c)_L \leftrightarrow$ no weak int.

$$\boxed{(\psi^c)_R \equiv \underbrace{C\bar{\psi}_L{}^T}_{\text{weak int.}}}$$

$$F = \begin{pmatrix} p \\ n \\ \hline p^c \\ ? \end{pmatrix}_L \quad \Bigg\| \quad \begin{matrix} (n^c)_L \\ \Updownarrow \\ n_R = \text{singlet} \end{matrix}$$

$$\bar{f} f = \bar{f}_L f_R + \bar{f}_R f_L$$

$\cancel{\bar{F}_L n_R}$ by SU(3)

SM: $H = D \equiv$ doublet

$$\Phi_{(3)} = \begin{pmatrix} D \\ \hline ? \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \\ \hline ? \end{pmatrix}$$

$\langle \phi^0 \rangle = vev \Longleftrightarrow$ SM situation

$$L_y = \overline{F_L} Y_u \Phi u_R + h.c.$$

$$\rightarrow \overline{F_L} U_3^+ Y_u U_3 \Phi u_R \ldots$$

$$= \overline{F_L} y_u \underbrace{U_3^+ U_3}_{1} \Phi u_R \ldots$$

<u>SU(2)</u> $\quad D \to U D$

$$D^T i\sigma_2 D = D^T \epsilon D = \text{singlet}$$

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\epsilon_{ij} D_i D_j = \text{invariant}$$

<u>SU(3)</u> $\quad T \; (T_i, \; i = 1, 2, 3)$

$$T \to U T \quad \therefore \quad T_i \to U_{ij} T_j$$

$$\epsilon_{ijk} T_i T_j T_k \longrightarrow$$

$$\epsilon_{ijk} \; U_{ii'} U_{jj'} U_{kk'} T_{i'} T_{j'} T_{k'}$$

$$= \varepsilon_{1jk}\, U_{1i'} U_{jj'} U_{kk'} T_{i'} T_{j'} T_{k'}$$

$$+ \varepsilon_{ijk}$$

$$= \varepsilon_{123}\, U_{1i'} \left(U_{2j'} U_{3k'} - U_{3j'} U_{2k'} \right)$$

$$T_{i'} T_{j'} T_{k'}$$

$$= \epsilon_{i'j'k'} \underbrace{(\det U)}_{1} T_{i'} T_{j'} T_{k'}$$

$$\boxed{\text{l'invariant}}$$

$$\varepsilon_{ijk}\, F_{Li}^{T}\, G\, F_{Lj}\, \Phi_k\, Y_p$$

$$F_L = \begin{pmatrix} p \\ n \\ p^c \end{pmatrix}_L$$

Proton Yukawa

$$\Phi_0 = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}$$

$$F_{L1}^T C F_{L3} \Phi_{02} y_p$$

$$= p_L^T C (p_c)_L v y_p$$

$$= p_L^T C C \bar{p}_R^T v y_p \qquad \boxed{C^2 = -1}$$

$$C C^T = 1$$

$$= -p_L^T \bar{p}_R v y_p \qquad C = -C^T$$

$$= \overline{p_R} p_L v\, y_p \;!!$$

$C = i\gamma_2\gamma_0$

σ_2

Eureka

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \qquad K^+ = u\bar{s} \qquad K^0 = d\bar{s}$$

$u\bar{d} = \pi^+$

$\frac{1}{2} \times \frac{1}{2}$

$$\boxed{D_\mu S' = (\partial_\mu - ig\, T_a A_\mu^a) S'}$$

$$F = \begin{pmatrix} p \\ n \\ \hline p^c \end{pmatrix}_L \Big\} SU(2)$$

n_R

$[(n^c)_L]$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \\ \hline s^- \end{pmatrix}$$

$$\left.\begin{array}{l} \bullet\ \ \bar{F}_L \, Y_u \, u_R \, \bar{\Phi} \quad + h.c. \\ \\ \bullet\ \ \bar{F}_{L_i}^{\,T} C \, Y_p \, F_{L_j} \, \Phi_k \, \epsilon_{ijk} + h.c. \end{array}\right\} \mathcal{L}_Y$$

$$m_p = m_u \Rightarrow \underbrace{y_p = y_u}_{\text{miracle}} \,!$$

$\boxed{SM}$ $\quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R,\ d_R$

$$\mathcal{L}_y = (\bar{u}\,\bar{d})_L \underbrace{\Phi}_{\text{doublet}} \ y_d \, d_R \ +$$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

$$+ (\bar{u}\ \bar{d})_L \, i\sigma_2 \, \Phi^* \, y_u \, u_R + h.c.$$

$$m_u = y_u v \qquad \langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$m_d = y_d v$$

$$y_{u,d} \ll 1$$

$$m_{u,d} \simeq 0$$

u u
d

p

u d
d

n

$$y_t = 1, \qquad y_b = 1/20$$

$\boxed{\text{Nambu '60}}$

- $m_{u,d} = 0$

$$\underbrace{\langle \bar{q}_L q_R \rangle \neq 0}_{QCD} \quad \boxed{\text{condensate}}$$

- $q_L \to e^{i\alpha} q_L, \quad q_R \to q_R$

chiral symmetry

$$\mathcal{L}_{kin} = i\bar{q}_L \gamma^\mu \partial_\mu q + i\bar{q}_R \gamma^\mu \partial_\mu q_R$$

$$\partial_\mu \to D_\mu \qquad \partial_\mu \to D_\mu$$

$\Rightarrow$ chiral symmetry $\leftrightarrow$ S S B

$\Rightarrow$ Nambu- Goldstone
'1961'

- $\langle\phi\rangle \neq 0$

$$\phi = v + h$$

$$\Downarrow$$

$$m_h = \sqrt{\lambda}\, v$$

π = Nambu - Goldstone bosons

$$m_{u,d} \neq 0$$

$$\Rightarrow \boxed{m_\pi^4 \simeq m_q \Lambda_{QCD}^3}$$

Leptons

$$\begin{pmatrix} e^c \\ \nu^c \\ \hline e \end{pmatrix}_R$$

$(e^c)_R \equiv C\bar{e}_L^T$

has same charge as proton

$$\boxed{\text{SU(3) theory of } p, n + \nu, e}$$

$\nu = 2$ SU(3) breaking

$\downarrow$

$r=2 \quad SU(2)\times U(1)$

adjoint $\Sigma \to U\Sigma U^+$

$\Sigma_0 \to U\Sigma_0 U^+ = $ diagonal

vacuum

$\Sigma^+ = \Sigma$

$\Downarrow$

rank = preserved

$$\underbrace{[\Sigma_0, \text{ Cartan}]}_{\text{diagonal}} = \underbrace{[\Sigma_0, T_\alpha]}_{\alpha\in\text{Cartan}} = 0$$

diagonal

$$\Sigma_0 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} v_{new}$$

$$F = \begin{pmatrix} p \\ n \\ \hline p^c \end{pmatrix}_L^{W,\,X} \times \begin{pmatrix} e^c \\ \nu^c \\ \hline e \end{pmatrix}_R^{W,\,X}$$

$$\boxed{m_X \simeq f^{\nu}_{\text{new}} \gg m_W}$$

$$b = \begin{pmatrix} p \\ n \\ p^c \end{pmatrix}_L \overset{\text{mirror}}{\longleftrightarrow} \ell = \begin{pmatrix} e^c \\ \nu^c \\ e \end{pmatrix}_R$$

$$\boxed{\bar{b}\,\ell = \text{invariant}}$$

$$\cancel{\boxed{\bar{b}_L\, M\, \ell_R = \text{invariant}}}$$

$\overline{p_L}\, e^c_R$

$$\Rightarrow \boxed{e^c = p}$$

- $M \to 0 \Rightarrow p = \text{proton}$

  $e^c = \text{positron}$

- $\overline{b_L}\, y_{\text{new}}\, \Sigma\, e_R$ allowed

$$\to \overline{b_L}\, y_{\text{new}}\, U^\dagger U \Sigma U^\dagger U\, e_R$$

$$= \overline{b_L}\, y_{\text{new}}\, \Sigma\, e_R \quad \underline{\text{invariant}}$$

$$\Rightarrow \boxed{y_{\text{new}} \to 0}$$

$$\boxed{\begin{aligned} y_{\text{new}} &\ll g \\ M &\ll g v \end{aligned}}$$

particles = SM particles

$$\underbrace{(q,\ l,\ W,\ Z,\ h)}_{\boxed{\text{mass from Higgs}}}$$

Model of leptons

$\Downarrow$

$$\underbrace{\begin{pmatrix} e^c \\ \nu^c \\ e \end{pmatrix}_R \Longleftrightarrow \begin{pmatrix} \nu \\ e \\ \hdashline e^c \end{pmatrix}_L}$$

$\Phi \Rightarrow$ it works!?

$\boxed{\text{Anomalies}}$

kill anomalies?

Gauge theory = anomaly free

$\boxed{\text{baryons}} \longrightarrow b_L = \begin{pmatrix} p \\ n \\ \hdashline p^c \end{pmatrix}_L$

LH – weak int.

leptons $\boxed{Q = \sum c_a T_a}$

$$l = \begin{pmatrix} e^c \\ \nu^c \\ --- \\ e \end{pmatrix}_R$$

same as b

$$\boxed{e^c_R \equiv C \bar{e}_L^T}$$

SM $Q_{em} = T_3 + Y/2$

q_u
q_d = arbitrary, up to

$$q_u = q_d + 1$$

$$q_u - q_d = 1/2 - (-1/2) = 1$$

$$\overline{b_L}\ M\ \ell_R$$

$$\boxed{M \ll yv}$$

$\ell_R \to -\ell_R$

$b_L \to b_L$

$M \to 0 \Rightarrow$ more symmetry

$$m_p = m_n \Leftrightarrow y_p = y_n$$

$$y_p = y_n \Rightarrow SO(2)$$

or

aha ! $SO(2) \Longrightarrow y_p = y_n$

$$G \longrightarrow H \quad \text{(global)}$$

n_G $\qquad\qquad$ n_H

\# of generators

$$\Longrightarrow \boxed{\begin{aligned} &\#\text{ of Nambu-Goldstone} \\ &= n_G - n_H \\ &= \#\text{ of broken generators} \end{aligned}}$$

$\boxed{m_q = 0}$ $\qquad$ $\langle \bar{q}_L \, q_R \rangle \neq 0$

$\frac{3}{2} + \frac{3}{2} = 3$ $\qquad$ $= \Lambda_{QCD}^3$

$$\Downarrow$$

global $SU(3)_L \times SU(3)_R$

$\downarrow$ 3 broken gen.

$SU(2)_{L+R}$

$\Rightarrow$ 3 NG bosons = pions

$$\boxed{m_\pi^2 f_\pi^2 = m_q \Lambda_{QCD}^3}$$

$m_q \to 0 \Rightarrow m_\pi \to 0$

pions = pseudo-NG bosons

$m_q \ll \Lambda_{QCD}$

$$\boxed{\begin{pmatrix} \nu^c \\ e^c \end{pmatrix}_R = SM}$$

$$\|$$

$$\boxed{\begin{pmatrix} \nu \\ e \end{pmatrix}_L \leftarrow \text{def. of left}}$$

$$\psi_R \longleftrightarrow (\psi^c)_L \equiv C\bar{\psi}_R^T$$

$$\boxed{\begin{array}{cccc} e_L & + & e_R & \\ 2 & + & 2 & = 4 \end{array}}$$

or

$$\boxed{\begin{array}{cccc} (e)_L & + & (e^c)_L & = 4 \\ 2 & + & 2 & \end{array}}$$

GUT – work with L

$$\boxed{\begin{pmatrix} q \\ \ell \end{pmatrix}_L} \leftarrow \psi^c$$

$$\boxed{SU(3) \times U(1)}$$

Lee, Weinberg '77 (PRL)

$$X \begin{pmatrix} p \\ n \\ \hline p^c \end{pmatrix}_L$$

$$\boxed{X_\mu \bar{p}_L \gamma^\mu (p^c)_L}$$

$B(X_\mu) = +2$

$\Rightarrow B$ is conserved

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c$$

$$[T_a, T_b] = i f_{abc} T_c$$

$$F_{\mu\nu}^a F^{\mu\nu a} \longrightarrow (\partial_\mu A_\nu^a) f_{abc} A_\mu^b A_\nu^c$$

$$\boxed{g \quad X_- \quad W^+ W^+ \quad ?}$$

break B

$B(W) = 0$

$\bar{p} \, \sigma^{\mu} u \, W_\mu$

B = bonjour monsieur