LMU GUT Course

Lecture VI

20/11/2020

LMU

Fall

Monopoles- summary

$$SU(2) \longrightarrow U(1) \qquad \boxed{g=e}$$

$$\mathcal{L} = \mathrm{Tr}(D_\mu \Sigma)(D^\mu \Sigma) - V(\Sigma)$$
$$- \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} \qquad (1)$$
$$a = 1, 2, 3$$

$$\Sigma = T_a \phi_a$$

$$V(\Sigma) = \frac{\lambda}{4}\left(2\,\mathrm{Tr}\,\Sigma^2 - v^2\right)^2 \qquad (2)$$

$$\Downarrow$$

$$\cancel{\Box \phi = -\frac{\partial V}{\partial \phi}}$$

$$\Downarrow$$

- $$(D^\mu D_\mu \phi)^a = -\frac{\partial V}{\partial \phi^a} \qquad (3)$$

$$= \lambda \phi^a (\vec{\phi}^2 - v^2)^2$$

$$\vec{\phi}^2 = \phi^a \phi^a$$

$$\boxed{g = e}$$

- $$D^\mu F_{\mu\nu}^a = j_\nu^a$$

$$j_\nu^a = \epsilon_{abc} \phi^b D_\nu \phi^c e$$

(4)

- $$D^\mu \tilde{F}_{\mu\nu}^a = 0 \qquad \text{(Bianchi)}$$

$$\tilde{F}_{\mu\nu}^a \equiv \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}^a \qquad (5)$$

$$\tilde{F}_{\mu\nu} \equiv F_{\mu\nu}^d$$

ϕ^a , $\quad \Sigma = T_a \phi^a$

$\Sigma \to U \Sigma U^\dagger$ $\quad \boxed{Q = \frac{\Sigma}{|\phi|}}$ (6)

$\hat{Q}\,\Sigma = [Q, \Sigma] = 0$

$$\boxed{F_{\mu\nu} = \left(F_{\mu\nu}{}^a - \frac{\epsilon_{abc}}{e} \frac{(D_\mu\phi)^b (D_\nu\phi)^c}{|\phi|^2}\right) \frac{\phi^a}{|\phi|}} \quad (7)$$

$U(1)$ = Maxwell

(– YM int.)

fixed frm $\Sigma_0 = v T_3$

$\Rightarrow F_{\mu\nu} = (F_{\mu\nu}^{3} - \qquad)$

$$\boxed{A_\mu = A_\mu^3} \quad = \underbrace{\partial_\mu A_\nu^3 - \partial_\nu A_\mu^3}$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu$$

· magnetic current $= k_\mu$

$$k_0 = \nabla \cdot \vec{B} = \partial_i B_i$$

$$= \frac{1}{2} \partial_i \epsilon_{ijk} F_{jk}$$

Static, finite energy configuration

but $\phi_a^a \phi_a^a = v^2$ at ∞

$$D_i \phi^a = 0 \Rightarrow$$

$$\boxed{A_i^a = -\frac{1}{e v^2} \epsilon_{abc} \phi^b \partial_i \phi^c}$$

check

(8)

$$\Downarrow$$

$$k_0 = \frac{1}{2}\, \epsilon_{ijk}\, \epsilon_{abc}\, \partial_i \phi_a\, \partial_j \phi_b\, \partial_k \phi_c$$

$$\Downarrow$$

$$\boxed{\int k_0 = \oint \vec{B}\, d\vec{S} = g_m = \frac{4\pi}{e}}$$

(monopole)

$$\phi_a^\infty = v\, x^a / r$$

(n) $\phi^3 = v \cos\theta$

$\phi^1 = v \sin\theta \cos n\varphi$

$\phi^2 = v \sin\theta \sin n\varphi$ (9)

$$\Downarrow$$

$$\boxed{g_m = \frac{4\pi}{e} n} \qquad (10)$$

$$E = \int d^3x \left[V + \frac{1}{2} |D_i \phi^a|^2 + \frac{1}{2} (B^i_a)^2 \right]$$

$$\text{|||}$$

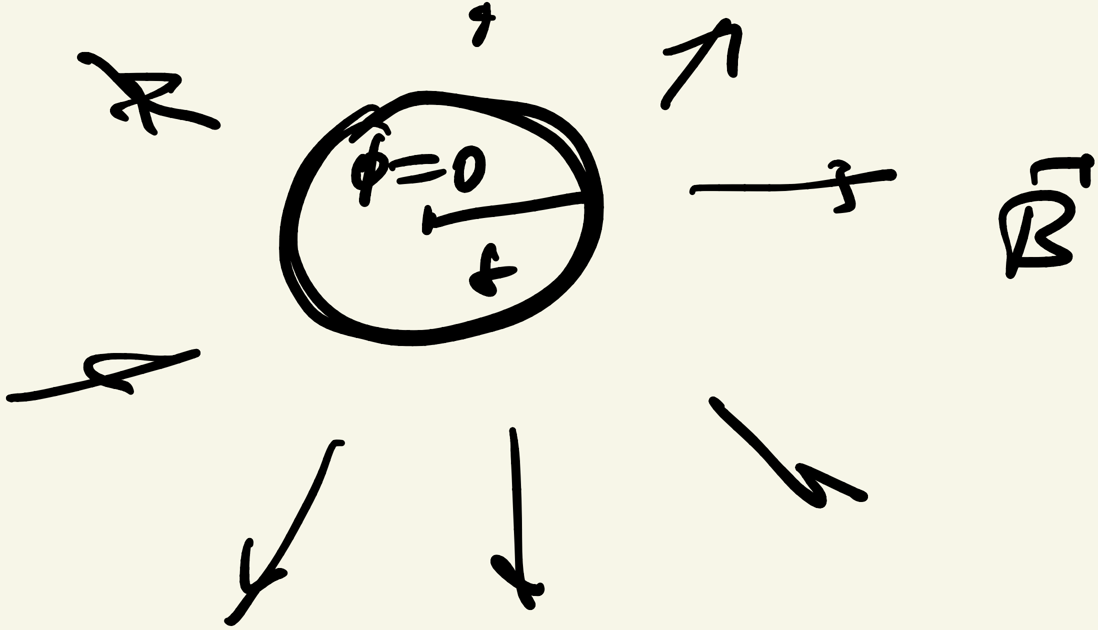
$$M_m$$

$$(11)$$

$$B^i_a = \frac{1}{e} \cdot \epsilon_{iaj} \frac{x^j}{r^3}$$

$$\left(\frac{1}{r^2}\right)^2 \to \frac{1}{r^4} \frac{1}{e^2}$$

$$E_{(\vec{B})} = \int_\delta^\infty \frac{1}{r^4} \frac{1}{e^2} d^3r \propto \frac{1}{e^2 \delta}$$



$$E_v = \lambda v^4 \int_0^\delta d^3r = \lambda v^4 \delta^3$$

$$E_{\text{monopole}} \simeq (\lambda + \dots) v^4 \delta^3 + \frac{1}{e^2\delta}$$

$$\lambda = g^2 = e^2$$

$$e^2 v^4 \delta^2 \simeq \frac{1}{e^2 \delta^2}$$

$$\Downarrow$$

$$\boxed{\delta \simeq 1/ev}$$

(only scale)

$\boxed{\text{Precise}}$

$$M_m = \int d^3x \left[V + \frac{1}{2} (D_i \phi^a)^2 + (B_i^a)^2 \tfrac{1}{2} \right]$$

$$= \int d^3x \left[V + \frac{1}{2} (D_i \phi^a - B_i^a)^2 + D_i \phi^a \cdot B_i^a \right] \quad (12)$$

$$\int d^3x \, (D_i \phi^a) B_i^a =$$

$$= \int d^3x \, (\partial_i \phi^a + e \, \epsilon_{abc} A_i^b \phi^c) B_i^a$$

$$= \int d^3x \, \partial_i \phi^a B_i^a \; +$$

$$+ \int d^3x \, \underbrace{\epsilon_{cba}}_{-\,\epsilon_{acb}} A_i^b \phi^a B_i^c$$

$$= \int d^3x \Big[\partial_i (\phi^a B_i^a) - \int d^3x \, \phi^a \partial_i B^a$$

$$- \phi^a \epsilon_{abc} A_i^b B_i^c$$

$$\int dS_i \, \phi^a B_i^a - \int d^3x \, \phi^a \underbrace{D_i B_i^a}_{=\,0}$$

$$= \int dS_i \, \frac{\phi^a}{v} v B_i^a \leftarrow \text{monopole}$$

$$= v \int dS_i \, B_i = v g_m$$

$$\left(B_i = \phi^a/v \, B_i^a \right)$$

$$\Downarrow$$

$$\boxed{M_m = \int d^3x \left[V + \frac{1}{2}\left(D_i\Phi^a - B_i^a\right)^2 \right] + g_m v}$$

$$\boxed{M_m \gtrsim g_m v} \qquad g_m = \frac{4\pi}{e}$$

$$\boxed{M_m \geq \frac{4\pi}{e} v}$$

- $\delta \simeq \dfrac{1}{ev}$ $\quad (R_m)$

monopole radius

$$\boxed{M_W = g_m v}$$ when?

$$\Rightarrow \boxed{\lambda = 0, \quad D_i \phi^a = B_i^a}$$

Богомольный

$$\bullet \quad \phi^a = v \frac{x^a}{r} H(evr) \qquad (**)$$

$$A_i^a = -\epsilon_{aij} \frac{x^j}{e r^2} K(evr)$$

$$\Rightarrow \quad H, K \xrightarrow[r \to \infty]{} 1$$

$$H, K \xrightarrow[r \to 0]{} 0$$

$$\Downarrow$$

$$\boxed{\begin{aligned} H(e\vartheta r) &= (e\vartheta r \coth e\vartheta r - 1)\frac{1}{e\vartheta r} \\ K(e\vartheta r) &= \frac{e\vartheta r}{\sinh e\vartheta r} - 1 \end{aligned}}$$

$$\boxed{R_m = \frac{1}{e\vartheta}}$$

exponential damping

$$M_m = \frac{4\pi}{e}\vartheta = \frac{4\pi}{e^2}(e\vartheta)$$

$$\boxed{M_m = \frac{1}{\alpha} R_m^{-1}}$$

$$\boxed{\cdot\ h_0(r) \Leftarrow \text{inside}}$$

$$\boxed{\begin{aligned} R_c(\text{monopole}) &\simeq \frac{1}{M_m} = \\ &= \alpha R_m \leq 1/100\, R_m \\ \text{monopole} &= \text{classical} \end{aligned}}$$

Topological defects

(i) domain walls

(ii) strings (cosmic strings)

(iii) magnetic monopoles

↳ finite energy

$$\mathcal{M}_\infty \xrightarrow{\text{maps}} \mathcal{M}_0$$

$\uparrow$ infinity $\qquad\qquad$ $\uparrow$ vacuum

$$\mathcal{M}_\infty = S_2 \quad \Leftarrow \quad \mathcal{M}_0 = S_2$$

(1) $\boxed{\text{Domain wall}}$ $\phi \in \mathbb{R}$

$\mathbb{Z}_2$ symmetry

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi)$$

$$V = \frac{\lambda}{4}(\phi^2 - v^2)$$

$D: \quad \phi \longrightarrow -\phi$

$\boxed{\cancel{D} \text{ spont.}}$

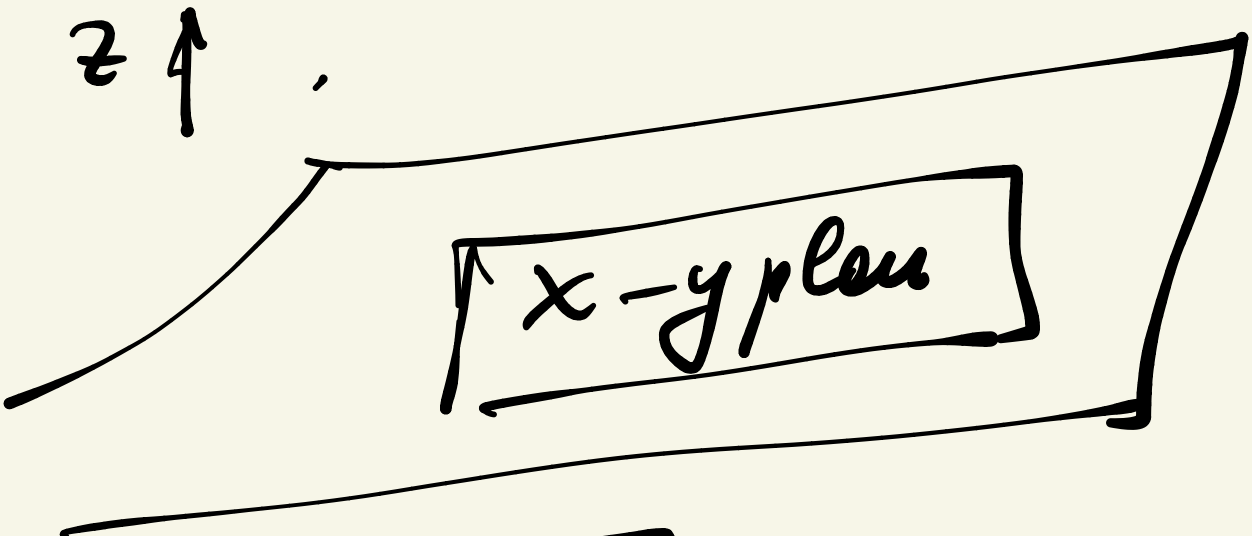
$$\mathcal{M}_0 = \{\phi_0^2 = v^2\} \quad (V=0)$$

$$= \{\pm v\}$$

$$\mathcal{M}_\infty = \{z = \pm\infty\}$$

$$\boxed{\phi_{cl}(\pm\infty) \in \mathcal{M}_0}$$

· finite energy (density)



$$\boxed{\phi_{cl}(z)}$$

· $\phi_{cl} = v = \phi_0$

found $E(\phi_0) = 0$

· $\phi_{cl}(z)$ $\left| \begin{array}{l} \phi_{cl}(+\infty) = +v \\ \phi_{cl}(-\infty) = -v \end{array} \right|$

$$\boxed{\phi_{cl}(z) = v \tanh \sqrt{\lambda}\, v z}$$

G = simple YM group

$\hookrightarrow$ SM

$$\underbrace{SO(10)}_{r=5} \xrightarrow{M_{GUT}} SU(2)_L \times SU(2)_R \times U(1) \times SU(3)_c \qquad g_L = g_R$$

$$\downarrow M_R$$

$$SU(2)_L \times U(1)_Y \times SU(3)_c$$

$$\boxed{M_{GUT} \gg M_R}$$

$$SO(10) \to \underbrace{SU(2)_L \times SU(2)_R}_{r=2\ \ SO(4)} \times \underbrace{\overset{\boxed{r=3}}{SU(4)_C}}_{15=4^2-1}$$

$$\| \quad SO(6)$$

$$6\cdot 5/2 = 15$$

$SU(2)$ $\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

$SU(3)$ $\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$

$SU(4)$ $\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -2 & \\ & & & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$

$$\to SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$$

monopole

$$SU(5) \longrightarrow SU(2) \times U(1) \times SU(3)_c$$

monopole at $U(1)_Y$

$$Q = T_{3_L} + Y/2$$

$$Q = T_{3_L} + T_{3_R} + \frac{B-L}{2}$$