LMU GUT Course

Fall 2020

Lecture II

6/11/2020

SU(2) gauge theory

$\boxed{\begin{array}{l}\text{Schwinger '57} \\ \text{Glashow '60}\end{array}}$

QED

$\updownarrow$

massless photon ($m_A = 0$)

$\rightarrow$ U(1) gauge

$$\mathcal{L}_{QED} = i\bar{f}\gamma^\mu D_\mu f - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m_f \bar{f} f$$

$$D_\mu = \partial_\mu - ieQA_\mu$$

$$\alpha = e^2/4\pi \simeq 1/137 \quad \begin{array}{c} E \to 0 \\ (\sim \text{MeV}) \end{array}$$

$$f = \psi \quad - \quad 4 \text{ component}$$

$$\bar{f} = f^{+} \gamma^{0}$$

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma_{\mu}^{+} \\ \sigma_{\mu}^{-} & 0 \end{pmatrix} \quad (1)$$

$$\sigma_{\mu}^{\pm} = (I, \pm \sigma_i) \quad (2)$$

$$\Sigma_{\mu\nu} = \frac{1}{4i}[\gamma_{\mu}, \gamma_{\nu}] \quad \text{- Lorentz}$$

$$\{\gamma_5, \gamma_{\mu}\} = 0 \qquad \gamma_5^2 = 1$$

$$[\gamma_5, \Sigma_{\mu\nu}] = 0 \quad (3)$$

$$\psi_{L,R} = L(R)\,\psi = \frac{1 \pm \gamma_5}{2}\psi \quad (4)$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad (5)$$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}, \quad \psi_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix} \qquad (6)$$

$$u_{L,R} \longrightarrow e^{i\vec{\sigma}/2 \cdot (\vec{\theta} \pm i \vec{\varphi})} \qquad (7)$$

ROT   BOOST

- $\bar{\psi} \gamma^\mu \psi$ = vector

$$\bar{\psi} \gamma^\mu \psi \equiv V_\mu \quad \therefore \quad V_i \xrightarrow{P} -V_i, \quad V_0 \xrightarrow{P} V_0$$

$$\Longleftrightarrow \quad \psi \xrightarrow{P} \gamma_0 \psi$$

$$\bar{\psi} \gamma^\mu \psi \longrightarrow \bar{\psi} \gamma^0 \gamma^\mu \gamma^0 \psi \longrightarrow \begin{cases} -\bar{\psi} \gamma_i \psi \\ \bar{\psi} \gamma_0 \psi \end{cases}$$

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\begin{pmatrix} u_L \\ u_R \end{pmatrix} \xrightarrow{P} \gamma_0 \begin{pmatrix} u_L \\ u_R \end{pmatrix} = \begin{pmatrix} u_R \\ u_L \end{pmatrix} \quad (8)$$

$$\boxed{u_L \overset{P}{\longleftrightarrow} u_R}$$

$$\bar\psi \gamma^\mu \psi = \bar\psi_L \gamma_\mu \psi_L + \bar\psi_R \gamma^\mu \psi_R$$

P conserving

P conserving

$$\bar\psi_L \gamma^\mu \psi_L + a\, \bar\psi_R \gamma^\mu \psi_R \Leftrightarrow P \text{ broken}$$

$$a = 0 \Rightarrow \not{P} \text{ maximally}$$

$\boxed{\text{weak int} = \text{gauge theory}}$

$n \to p + e + \bar{\nu}_e$

$\mu \to e + \nu_\mu + \bar{\nu}_e$

messenger is charged $W^\pm$

$\boxed{SU(2)}$ $i = 1, 2, 3$

$$\boxed{[T_i, T_j] = i\,\epsilon_{ijk} T_k} \quad (9)$$

- F = fundamental

$T_i^\dagger = T_i$

$\mathrm{Tr}\, T^i = 0$

$$T_i = \sigma_i/2$$

$$U = e^{i\theta_i T_i} \quad (10)$$

$$\boxed{\begin{gathered} U^+ U = UU^+ = 1 \\ \det U = 1 \end{gathered}}$$

$$T_3 = \frac{\sigma_3}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

"charge"

$$\boxed{[T_\pm, T_3] = \pm T_3}$$

$\boxed{SU(N)}$

$$T_\pm = T_1 \pm i T_2$$

<u>algebra</u>

$$[T_a, T_b] = i f_{abc} T_c$$

ε_{abc} = anti-symmetric

$\boxed{\text{Cartan sub-algebra}}$ $\{T_\alpha\}$

$$\{[T_\alpha, T_\beta] = 0\}$$

$SU(2) \qquad C = \{T_3\} \qquad (f = 4)$

- $m\,\bar{\psi}\psi = m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$

$$\bar{\psi} = \psi^\dagger \gamma^0 \quad \Big| \quad = u_L^\dagger u_R + u_R^\dagger u_L$$

$$\psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

$\boxed{m = 0}$ $\quad \boxed{u_L,\ u_R}$ — move independently

irreducible repr. of Lorentz

$$D = \begin{pmatrix} u \\ d \end{pmatrix} = \text{Fundamental of } SU(2)$$

$$\boxed{\begin{aligned} D &\longrightarrow U D \\ \tilde{D} &= \epsilon D^* \end{aligned}}$$

$$= i\sigma_2 D^*$$

$$\epsilon_{12} = -\epsilon_{21} = 1$$

$$\epsilon_{11} = \epsilon_{22} = 0$$

$$\underline{\tilde{D}} \rightarrow i\sigma_2 U^* D^* = U i\sigma_2 D^*$$
$$= U \epsilon D^* = \underline{U}\,\tilde{D}$$

$$\boxed{D = \text{doublet} \Rightarrow \tilde{D} = \text{doublet}}$$

$$U(1) \longrightarrow SU(2)$$

$$\Rightarrow D_\mu(SU(2)) = \partial_\mu - i g\, T_a A_\mu^a$$

acting on F = fundamental

$$\boxed{T_a \longleftrightarrow F}$$

$\bar{T}_a$ = generic generator on any repr. - R

- $\bar{T}_a \equiv T_a \equiv \sigma_a/2$ (F)

- $\bar{T}_3 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}$ "spin" = 1

- $\bar{T}_3 = \begin{pmatrix} 3/2 & & & \\ & +1/2 & & \\ & & -1/2 & \\ & & & -3/2 \end{pmatrix}$ "spin" = 3/2

$$\mathcal{L}_{SU(2)} = i\bar{f}\gamma^\mu D_\mu f - m\bar{f}f$$

$$- \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} \quad (11)$$

$a = 1, 2, 3$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\,\epsilon_{abc} A_\mu^b A_\nu^c \quad (12)$$

$$D_\mu = \partial_\mu - ig\,T_a A_\mu^a \quad (13)$$

$$f = \begin{pmatrix} u \\ d \end{pmatrix}$$

f_{abc}

$\boxed{SU(N)}$

$\downarrow$

$$i\bar{f}\gamma^\mu D_\mu f = i\bar{f}\gamma^\mu \partial_\mu f$$

$$+\frac{g}{2}(\bar{u}\ \bar{d})\gamma^\mu \begin{bmatrix} A_3 & A_1 - iA_2 \\ A_1 + iA_2 & -A_3 \end{bmatrix}_\mu \begin{pmatrix} u \\ d \end{pmatrix}$$

$$= \frac{g}{\sqrt{2}}\bar{u}\gamma_\mu (A_1 - iA_2)^\mu d + h.c.$$

$$+ \boxed{g\bar{f}\gamma^\mu A_\mu^3 T_3 f}$$

$$W_+ = \frac{A_1 - iA_2}{\sqrt{2}} \qquad W^- = \frac{A_1 + iA_2}{\sqrt{2}}$$

$$\boxed{\frac{g}{\sqrt{2}}\,\bar{u}\,\gamma_\mu W_+^\mu\, d + h.c.}$$

$$\Rightarrow \boxed{A_\mu^3 = \text{photon}}$$

$$\Downarrow$$

$$\boxed{\underset{\text{weak}}{g} = \underset{\text{em}}{e}} \Rightarrow \boxed{Q = T_3} \boxed{= \pm 1/2}$$

$$\Rightarrow \begin{array}{l} Q_u = 1/2 \\ Q_d = -1/2 \end{array} \Rightarrow \begin{array}{l} Q_{up} = Q_{\text{"}\nu\text{"}} \\ Q_d = Q_e \end{array}$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}^\alpha \quad \ell = \begin{pmatrix} \text{"}\nu\text{"} \\ e \end{pmatrix}$$

$$\alpha = r, y, b$$

$$n = ddu$$

$$p = uud$$

$k \ll M_W$

$$\frac{g}{\sqrt{2}}$$

$$\Delta_{\mu\nu} = -i\,\frac{g_{\mu\nu} - \cancel{\frac{k_\mu k_\nu}{M_W^2}}}{k^2 - M_W^2}$$

$$\Downarrow$$

$$\frac{G_F}{\sqrt{2}} J_\mu^{\nu} J^\mu_{\nu} \qquad J_\mu = \bar{u}\gamma_\mu d$$

$$= \frac{e^2}{2M_W^2} \text{ —"— }$$

$$\Downarrow$$

$$4\pi\alpha = \frac{e^2}{2M_W^2} = \frac{G_F}{\sqrt{2}} = 10^{-5}\,\mathrm{GeV}^{-2}$$

$$\Rightarrow \boxed{M_W = 80\ \mathrm{GeV}}$$

$$\boxed{\text{gauge theory + spont. symmetry breaking}}$$

$$\boxed{SU(2)}$$

$\begin{pmatrix} u \\ d \end{pmatrix}_L$

$T_a = \frac{\sigma_a}{2}$

- u_R, d_R singlets
  $T_a = 0$
- $\begin{pmatrix} u \\ d \end{pmatrix}_R$ $T_a = \frac{\sigma_a}{2}$

- singlet

fermions are massive

$\bar{u}_L u_R, \quad \bar{d}_L d_R$

$\cancel{(\bar{u}\,\bar{d})_L \, u_R}$

$\boxed{\text{forbidden}}$

$$y_u \, (\bar{u}\,\bar{d})_L \, \bar{\Phi} \, u_R \qquad (14)$$

$$\langle \Phi \rangle \neq 0 \qquad \text{(vacuum)}$$

$$= \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Downarrow$$

$$\frac{1}{2}\left|D_\mu \Phi\right|^2 = \qquad (15)$$

$$= \frac{1}{2}\left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right)$$

$$\to \frac{1}{2}\left(D_\mu \langle\Phi\rangle\right)^\dagger \left(D^\mu \langle\Phi\rangle\right)$$

$$= \frac{1}{2}\left(\frac{g}{2}\right)^2 \langle\Phi\rangle^\dagger \sigma_b A^\mu_b \sigma_a A_\mu^a \langle\Phi\rangle$$

$$= \frac{1}{2}\frac{g^2}{4} \langle\Phi^\dagger\rangle \left(\delta_{ab} + \cancel{i\,\epsilon_{abc}\sigma_c}\right)\langle\Phi\rangle \, A_\mu^a A^\mu_b$$

$$= \frac{1}{2}\frac{g^2}{4} A_\mu^a A^{\mu a} \left|\langle\Phi\rangle\right|^2$$

$$\Downarrow$$

$$\boxed{m_{A_a} = \frac{g}{2} v} \qquad a = 1,2,3$$

No photon

$$SU(2) \xrightarrow[\langle \Phi \rangle]{} \mathbb{1}$$

doublet

$$\Downarrow$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$(\bar{u}\,\bar{d})_L \, M \begin{pmatrix} u \\ d \end{pmatrix}_R \quad (15)$$

$$M = m\,\mathbb{1}$$

$$\Rightarrow \boxed{m_u = m_d}$$

wrong

$$\boxed{m_{A_i} = 0}$$

<u>Adjoint</u> $\quad \Sigma \to U \Sigma U^{+}$ (16)

2x2 matrix

$\mathrm{Tr}\,\Sigma \to \mathrm{Tr}\,\Sigma \Rightarrow \mathrm{Tr}\,\Sigma = 0$

$\Sigma^{+} = \Sigma \Rightarrow$ remains

$$\Rightarrow \boxed{\begin{array}{c} \Sigma = \Sigma^{+} \\ \mathrm{Tr}\,\Sigma = 0 \end{array}} \text{ (def.)} \quad (17)$$

$\Rightarrow \quad \Sigma = T_i \varphi_i \qquad i = 1, 2, 3$

$\updownarrow$ vector

$U = e^{i\theta_i T_i}$

$\Rightarrow \quad \varphi_i \to \varphi_i \pm \varepsilon_{ijk} \theta_j \varphi_k$

(18)

Assume $\Downarrow$

$\langle \Sigma \rangle \neq 0$

$\hookrightarrow$ diagonalize by

$$\langle \Sigma \rangle \to U \langle \Sigma \rangle U^\dagger$$

$$\langle \Sigma \rangle = v \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow [\langle \Sigma \rangle, T_3] = 0$$

$$\boxed{U_3 \langle \Sigma \rangle U_3^\dagger = \langle \Sigma \rangle}$$

$$SU(2) \to U(1) = SO(2)$$

generated by T_3

$$\boxed{A_\mu = A_\mu^3} \qquad m_A = 0$$

$$\boxed{m_{A_1} = m_{A_2} \stackrel{?}{=} e v}$$

$$\boxed{W_\pm = \frac{A_1 \mp i A_2}{\sqrt{2}}}$$

$$\mathcal{L}_Y = (\bar u\ \bar d)_L \left(M + y\,\Sigma\right) \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$M = m\,\mathbb{1}$

$$U \Sigma U^+$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \to U \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$$

$$\langle \Sigma \rangle = v \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} m_u &= m + y v \\ m_d &= m - y v \end{aligned}}$$

$m_u \neq m_d$

$\boxed{\text{Summarize}}$

- $\begin{pmatrix} u \\ d \end{pmatrix}_L \xleftrightarrow{\;\;} \begin{pmatrix} u \\ d \end{pmatrix}_R$

  $\underline{P}$ ~ good

- Higgs = $\overline{\Sigma}$ (adjoint)

- $M_W = 80$ GeV

- $Q_{em} = \pm n \, 1/2$ = quantized

Failures

(i) Q_{em} = different

$Q_e = 3 q_d, \quad q_u = -2 q_d$

$$(\quad Q_\nu = 0 \quad)$$

(iii) $\not{P}$ = neutral cu weak int.

$$\boxed{\underset{g}{SU(2)_L} \times \underset{Y}{\overset{g'}{U(1)}}}$$

'1961' Glashow

$$[T_a, Y] = 0 \qquad (19)$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R,\ d_R$$

Φ

'1967' Weinberg

$$\boxed{\begin{aligned} &\tan\theta_W = g'/g \\ (20) \quad &e = g\sin\theta_W \end{aligned}}$$

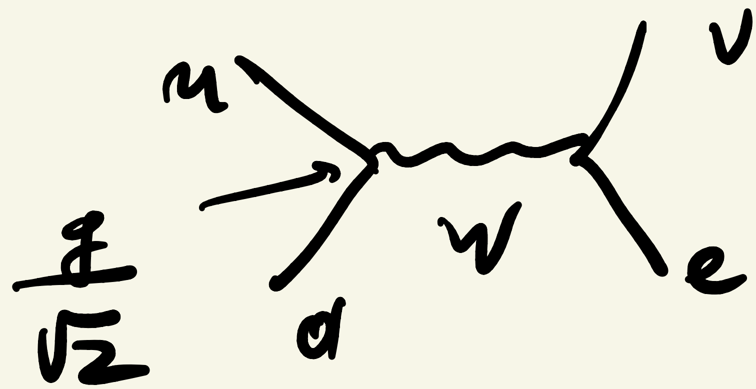
$(e < g)$

"weak" $\Leftrightarrow M_W \gg m_p$

$$J_\mu^W = \bar{u}\,\gamma_\mu \frac{1+\gamma_5}{2}\, d$$

$$= \bar{u}_L\, \gamma_\mu\, d_L$$

$$\Rightarrow \boxed{\frac{4G_F}{\sqrt{2}} = \frac{g^2}{2M_W^2}} \qquad (21)$$



$$\frac{4G_F}{\sqrt{2}}\; \bar{u}\,\gamma^\mu \frac{(1+\gamma_5)}{2}\, d \;\cdots\cdots\; \frac{1}{2}$$

SU(2) $\quad \frac{G_F}{\sqrt{2}} = \frac{e^2}{2M_W^2}$

SU(2) × U(1) $\quad \frac{4G_F}{\sqrt{2}} = \frac{g^2}{2M_W^2}$

$e = g\sin\theta_W \simeq \frac{1}{2}g$

measure

$\boxed{\sin\theta_W M_W = 40\,\text{GeV}}$

$\boxed{M_W \simeq 80\,\text{GeV}}$

Z boson

$$J_\mu^Z = \frac{g}{\cos\theta_W}\left[T_3 - Q\sin^2\theta_W\right]$$

$\textcircled{\Theta_W}$

neutral current (Z)

$$Q = T_3 + \frac{Y}{2} \quad (22)$$

$$\boxed{Y = 2\left[Q - T_3\right]}$$

fixed

arbitrary

$$\boxed{q_R = q_L \Leftarrow Y \text{ arbitrary}}$$

$$\boxed{\text{anomaly} \Rightarrow q_e = 3 q_d}$$

$\Downarrow$ add to SM

$$\underbrace{Q_L, \quad Q_R \ (\text{singlet})}_{\text{arbitrary}}$$

$$\boxed{\begin{array}{c} SU(2) \quad (G = SU(5) \ldots) \\ \Downarrow \\ \text{magnetic monopoles} \end{array}}$$

"Is there a monopole problem?"

Dvali, Melfo, G.S.

"Is there a domain wall problem?"
Dvali, G. S.

Symmetries at high T

Chen

Quarks and Leptons

Halzen, Martin

Quigg

"My notes"