LMU GUT Course

Lecture XXIII

$\boxed{\text{Minimal PS Theory}}$

Pati - Salam :

Symmetry breaking

$$g_L = g_R \equiv g$$

- $G_{PS} = SU(2)_L \times SU(2)_R \times SU(4)_C$

  $(\times LR = P, C)$

- $f_L = (2_L, 1_R, 4_C)$

  $f_R = (1, 2_R, 4_C)$

$\exists \nu_R \Leftarrow f_{L,R} = \begin{pmatrix} u^\alpha & \nu \\ d^\alpha & e \end{pmatrix}_{L,R}$ $\alpha = r, y, b$

$$D_\mu = \partial_\mu - ig \cdot (\vec{A}_L \cdot \vec{T}_L + \vec{A}_R \cdot \vec{T}_R)_\mu -$$

$$- i g_C A_C^a T_C^a$$

$a = 1, \ldots, 15$

$\mathrm{Tr}\, T_i^2 = \frac{1}{2}$ in fund. repr

(2 for $SU(2)$, 4 for $SU(4)$)

- <u>Higgs sector</u>

$$\mathcal{L}_Y = \overline{f_L}\, Y_\Phi\, \Phi\, f_R + h.c.$$

$$f_{L,R} \to U_{L,R}\, f_{L,R}$$

$$\boxed{\Phi \to U_L\, \Phi\, U_R^\dagger \quad \text{bi-doublet}}$$

$$\Phi : (2_L, 2_R, 1_c)$$

$$\langle \Phi \rangle \neq 0 \Rightarrow M_u, M_d$$

$$\boxed{\langle \Phi \rangle \simeq M_W} \qquad M_e, M_D$$

(neutrino Dirac)

$$\Delta_L \longleftrightarrow \Delta_R$$

$$0 = \langle \Delta_L \rangle \not\longleftrightarrow \boxed{\langle \Delta_R \rangle \neq 0}$$

Minimality $\Rightarrow$ $M_{PS} = M_R$

$SU(4)_c$ breaking

LR breaking

<u>but</u> <u>not necessary!</u>

PS $\longrightarrow$ SM (<u>no</u> ν_R)

$$\Longleftrightarrow$$

$$\boxed{\nu_R \text{ very heavy}}$$

$$\Downarrow$$

$$m_{\nu_R} \propto M_R \quad (M_{PS}\,?)$$

$$\Downarrow$$

$$\mathcal{L}_y^{\Delta} = f_R^T C\, i\sigma_2\, \Delta_R\, Y_\Delta f_R + L \leftrightarrow R$$

$$2_R \quad \times \quad 2_R = 3_R + \cancel{1_R}$$

$$\Downarrow$$

$$\boxed{\text{fixes } \Delta_R}$$

$$f_R = (1_L, 2_R, 4_C)$$

$$\Delta_R \to U_R\, \Delta_R\, U_R^\dagger \qquad SU(2)_R$$

$$\Delta_L \to U_L\, \Delta_L\, U_L^\dagger \qquad SU(2)_L$$

PS: $4^c = \underset{B-L:}{} 3^c_{1/3} + 1^c_{-1}$

$$4^c \times 4^c = (3_{1/3} + 1_{-1}) \times (3_{1/3} + 1_{-1})$$

$$= \underbrace{6_{2/3} + 3_{-2/3} + 1_{-2}}_{10\,(S)} + \underbrace{3_{-2/3} + 3^*_{2/3}}_{6\,(A)}$$

$$\boxed{\frac{Y}{2} = T_{3R} + \frac{B-L}{2}}$$

$$Q_{em} = T_{3L} + \underbrace{T_{3R} + \frac{B-L}{2}}$$

$$= T_{3L} + \frac{Y}{2}$$

$$\Downarrow$$

summary: $\underset{(SM)}{b_i}$ (b_2, b_3, b') $\longleftrightarrow U(1)_Y$

$$4_R \times 4_R = (1_L, 3_R, 10_c) \qquad \frac{4\cdot 5}{2} = 10$$

$\subseteq SU(2)_R$ triplet

$B-L = -2$

$$f_R \ \Delta_R \ f_R \Rightarrow$$

$$\Delta_R = (1_L, 3_R, \overline{10}_c)$$

$$\boxed{\Delta_R = (1_L, 3_R, \overline{10}_c)} = \underbrace{}_{} \longrightarrow SU(3)_c$$

$$= (1_L, 3_R, \overline{6} + \overline{3} + 1)$$

$B-L$: $-2/3 \quad 2/3 \quad +2$

$$\boxed{\langle \tilde{\Delta}_R \rangle = (1_L, 3_R, 1_2)}$$

"weak" part

$$G_{PS} = SU(2)_L \times \underbrace{SU(2)_R \times SU(4)}$$

$$\downarrow \langle \Delta_R \rangle$$

$$SU(2)_L \times U(1)_Y \times SU(3)_c = G_{SM}$$

<u>$SU(2)_R$</u> : $\tilde{\Delta}_R \to U \tilde{\Delta}_R U^\dagger$

$$\hat{T}_R \Delta_R = [T_R, \Delta_R]$$

$$\hat{T}_{3_R} \tilde{\Delta}_R = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \left(\text{values of } T_{3R} \right)$$

$$(B-L) \Delta_R = 2 \Delta_R$$

$$Q_{em} = T_{3R} + \frac{B-L}{2} \quad (T_{3L} = 0)$$

$$\Downarrow$$

$$\tilde{\Delta}_R = \begin{pmatrix} \Delta_+ & \Delta^{++} \\ \Delta^0 & -\Delta_+ \end{pmatrix} \leftarrow \boxed{\text{new!}}$$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{M_{W_R} \propto v_R, \quad M_{Z_R} \propto v_R}$$

? $\cancel{M_{W_L} \propto v_R}$, $\cancel{M_{Z_L} \propto v_R}$

$$\cancel{m_{\text{gluons}} \propto v_R}$$

$$\boxed{\langle \Delta_L \rangle = 0}$$

$$\boxed{M_{PS} \propto v_R \quad !}$$

$$M_{PS} \propto g_c\,(g_4)\, v_R$$

$$M_{W_R} \propto g_R\, v_R$$

$$M_{Z_R} \propto f(g_R, g_{BL})\, v_R$$

$$M_{PS} \propto g_c\, v_R$$

$$g_{BL} \longleftrightarrow \frac{B-L}{2}$$

$$g' \longleftrightarrow \frac{Y}{2}$$

$$g_c \longleftrightarrow SU(4)_c$$

- $M_R = M_{PS}$ scale: (i) $g_L = g_R$

  $(L \leftrightarrow R)$

  (ii) $g_{BL} \propto g_c$

  $$T_{15} = N\, \frac{B-L}{2} \qquad \boxed{N = ?}$$

NOT a unified theory

M_R: $\underbrace{g_2, g_4}$, θ

all is f-on of θ_W!

$$\tan\theta_W \equiv g'/g$$

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2}$$

PS: (a) $q \leftrightarrow \ell$ unif.

(b) origin of P!

charge quantization

LR

$$\Downarrow LR$$

$$\boxed{\exists\ \nu_R}$$

$$\Rightarrow\ l_R^T C\ i\sigma_2\ \tilde{\Delta}_R\ Y_\Delta\ l_R \Rightarrow$$

$$\Rightarrow\ l_R^T C \overbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}}^{\text{dicey}} Y_\Delta\ l_R$$

$$l_R \equiv \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

$$= \nu_R^T C\ Y_\Delta\ v_R\ \nu_R$$

$$\Rightarrow\ \boxed{M_{\nu_R} = Y_\Delta v_R}$$

$$M_{\nu_R} = f\ v_R$$

$$\boxed{M_{Z_R}/M_{W_R}, \quad \frac{M_{PS}}{M_{W_R}} = f(\theta_W)}$$

PS: $\alpha_L = \alpha_R$

$$\frac{1}{\alpha'} = \frac{1}{\alpha_R} + \frac{1}{\alpha_{BL}}$$

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2}$$

$$\alpha_{BL} \overset{?}{=} \alpha_C$$

- $\alpha_L \overset{?}{=} \alpha_R$ at M_R

$$SU(2)_L \times SU(2)_R \times SU(4)_C$$

P odd $\longleftarrow \downarrow (\Sigma) = M_{PS}$

$$\underline{SU(3)_C} \times SU(2)_L \times SU(2)_R \times \underline{U(1)_{B-L}} \equiv G_{LR}$$

$$\downarrow \langle \Delta_R \rangle = v_R$$

G_{SM}

$$\boxed{\Sigma \xrightarrow{P} -\Sigma \quad (L \leftrightarrow R)}$$

- $g_L = g_R$ at M_{PS}

$\Rightarrow g_L \neq g_R$ at M_R

$\Sigma = ?$

$\Downarrow$

$$\boxed{\Sigma = (1_L, 1_R, 15_c)}$$

$\ldots \; Tr\,\Sigma^2, \; Tr\,\Sigma^3, \; Tr\,\Sigma^4 \; \ldots$

- LR symmetry breaking

$\langle \Delta_L \rangle = 0, \; \langle \Delta_R \rangle \neq 0$

$$\cancel{\Delta_L}\,,\ \cancel{\Delta_L^3}$$

$\Downarrow$ schematically

$$V = -\frac{\mu^2}{2}\left(\Delta_L^2 + \Delta_R^2\right) + \frac{\lambda}{4}\left(\Delta_L^4 + \Delta_R^4\right)$$

$$+ \frac{\lambda'}{2}\Delta_L^2\Delta_R^2$$

$$\left(\Delta^2 \equiv \mathrm{Tr}\,\Delta^\dagger\Delta\right)$$

$$= -\frac{\mu^2}{2}\left(\Delta_L^2 + \Delta_R^2\right) + \frac{\lambda}{4}\left(\Delta_L^2 + \Delta_R^2\right)^2$$

$$+ \frac{\lambda' - \lambda}{2}\Delta_L^2\Delta_R^2 \Leftarrow$$

(i) $\lambda' - \lambda = 0 \Rightarrow$

$$\boxed{\underbrace{\langle\Delta_L\rangle^2 + \langle\Delta_R^2\rangle}_{\text{who ???}} = \mu^2/\lambda}$$

(ii) $\lambda' - \lambda \neq 0$

- $\lambda' - \lambda > 0$ • $\lambda' - \lambda < 0$

(a) $\underline{\lambda' - \lambda > 0} \Rightarrow \langle \Delta_L \rangle = 0$

$\langle \Delta_R \rangle \neq 0$

(b) $\underline{\lambda' - \lambda < 0}$ $\langle \Delta_L \rangle \neq 0 \neq \langle \Delta_R \rangle$

$\Rightarrow \langle \Delta_L \rangle = \langle \Delta_R \rangle$

$$\Downarrow$$

$$\boxed{\lambda' - \lambda > 0}$$

$$\boxed{\lambda > 0}$$

$\Rightarrow \quad M_{W_R},\ M_{Z_R} \propto v_R$

$M_{\nu_R} \propto v_R$

$$\Downarrow$$

$$\bar{\nu}_R \, M_D \, \nu_L \not\Rightarrow$$

$$\boxed{m_\nu \neq 0}$$

$$M_{\nu_R} \gg M_D$$

seesaw

$$\Downarrow$$

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$

$$N_L \equiv C \bar{\nu}_R^T$$

SU(2) $\quad \Delta \to U \Delta U^\dagger$

$$\mathrm{Tr}\, \Delta^3 = ???$$

$$\Delta = T_a \varphi_a \propto \sigma_a \varphi_a$$

$$\mathrm{Tr}\,\Delta^3 \propto \mathrm{Tr}\,\sigma_a \sigma_b \sigma_c \;\; \varphi_a \varphi_b \varphi_c$$

$$\propto \mathrm{Tr}\,\varepsilon_{abd}\,\sigma_d \sigma_c \;\; \varphi_a \varphi_b \varphi_c$$

$$\boxed{\mathrm{Tr}\,\Delta^2 , \quad \mathrm{Tr}\,\Delta^4 \propto (\mathrm{Tr}\,\Delta^2)^2}$$

$$\nu = 1$$

$$\text{(A)} \qquad \text{(S)}$$

$$\Downarrow \propto \;\; \varepsilon_{abc}\, \varphi_a \varphi_b \varphi_c = 0$$

$$SU(2)_R \times U(1)_{B-L} \Longrightarrow \boxed{\Delta = \text{complex}}$$

$$(B-L)\,\Delta = 2\Delta$$

$$\Downarrow$$

$$\cancel{\mathrm{Tr}\,\Delta_R^3} , \quad \cancel{\mathrm{Tr}\,\Delta_R^\dagger \Delta_R \Delta_R}$$

$\cancel{T, \Delta_R^\dagger \Delta_R^\dagger \Delta_R}$

$\Rightarrow$ $\boxed{\text{change } (B-L) \text{ forbids odd terms}}$

General breaking

$$G_{PS} \xrightarrow[\langle \Sigma \rangle]{} G_{LR} \xrightarrow[\langle \Delta_R \rangle]{} G_{SM} \xrightarrow{\langle \Phi \rangle} U(1) \times SU(3)_c$$

$\boxed{SO(2N) \text{ spinors}}$

motivation

SO(10) — 1 family

SO(10+4n) — multi family

extra dimensions

Orthogonal

- $$x_1^2 + \cdots + x_N^2 = 1 \quad (\mathrm{INV})$$

$$x_i' = O_{ij}\, x_j$$

$$x_i'\, x_i' = O_{ij}\, x_j\, O_{ik}\, x_k =$$

$$= O^T_{ki} O_{ij} \, x_j x_k = x_i x_i$$

$$\Rightarrow \quad O^T_{ki} O_{ij} = \delta_{jk}$$

$$\boxed{\begin{aligned} O^T O = 1 = O O^T \\ \det O = 1 \end{aligned}}$$

$$\Rightarrow \quad O = e^{i \theta_{ij} L_{ij}} \qquad \boxed{\frac{N(N-1)}{2}}$$

$$\det O = 1 \Longrightarrow \operatorname{Tr} L_{ij} = 0$$

$$O^T O = 1 \Longrightarrow L^T + L = 0$$

$$L^* + L = 0$$

$$\Rightarrow (L_{ij})_{kl} = -i(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})$$

$$\theta_{ij} = \frac{N(N-1)}{2} \quad \text{Euler}$$

$\underline{SO(3)}$ $\quad L_{12} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$L_{ij} = \varepsilon_{ijk} T_k$$

$$\Rightarrow T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow T_3 \text{ (diag)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{[T_i, T_j] = i \varepsilon_{ijk} T_k}$$

- $[L_{ij}, L_{ke}] = \delta_{ik} L_{je} + \cdots$

- $(x_i T_i)^2 = 1$

$\mathbb{R}$

$$1 = x_i \Gamma_i \; x_j \Gamma_j = x_i x_j \underbrace{\tfrac{1}{2}\{\Gamma_i, \Gamma_j\}}_{\delta_{ij}}$$

$$\Downarrow$$

$$\boxed{\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}}$$

Clifford algebra

- $\Gamma_i' = O_{ij} \Gamma_j$

$$\Gamma_i' \Gamma_j' = O_{ik} O_{jl} \Gamma_k \Gamma_l$$

$$\{\Gamma_i', \Gamma_j'\} = O_{ik} O_{jl} \{\Gamma_k, \Gamma_l\} =$$

$$= 2\delta_{kl} O_{ik} O_{jl} = 2\delta_{ij}$$

$$\Downarrow$$

$$\Gamma_i' = S(0)\,\Gamma_i\,S(0)^{-1}$$

$$SS^{-1} = 1$$

$$\Rightarrow \{\Gamma_i', \Gamma_j'\} = \{\Gamma_i, \Gamma_j\}$$

$$\boxed{S(0)\,\Gamma_i\,S^{-1}(0) = O_{ij}\,\Gamma_j} \quad (*)$$

Spinorial analog of O

$$\psi \to S(0)\,\psi \quad \boxed{\psi = \text{spinor}}$$

$$S(0) = e^{i\theta_{ij}\Sigma_{ij}}$$

$$[\Sigma_{ij}, \Sigma_{ke}] = i(\delta_{ik}\Sigma_{je} + \cdots) \quad (**)$$

$$\Downarrow$$

$$\Sigma_{ij} = \frac{1}{4i}\left[\Gamma_i, \Gamma_j\right]$$

- $$\Sigma_{12} = \frac{1}{2i}\Gamma_1\Gamma_2$$

$$\Gamma_1^2 = 1 = \Gamma_2^2$$

$$\Gamma_1\Gamma_2 = -\Gamma_2\Gamma_1$$

$$(2\Sigma_{12})^2 = \left(\frac{1}{i}\right)^2 \Gamma_1\Gamma_2\Gamma_1\Gamma_2 = (-1)(-1) = 1$$

$$(2\Sigma_{2n-1,2n})^2 = 1$$

$$\boxed{2\Sigma_{2n-1,2n} \ : \ \varepsilon = \pm 1}$$

eigenvalues

$$[\Sigma_{12}, \Sigma_{34}] \neq 0$$

$$\psi = |\varepsilon_1 \cdots \varepsilon_N\rangle \qquad SO(2N)$$

$$= |\pm 1 \ \pm 1 \cdots\rangle$$

$$\boxed{\begin{aligned}\Gamma_{FIVE} &= 2\Sigma_{12}\, 2\Sigma_{34} \cdots \Sigma_{2N-1,2N} \\ &= \left(\frac{1}{i}\right)^N \Gamma_1 \cdots \Gamma_{2N}\end{aligned}}$$

$$\Gamma_{FIVE} = (-i)^N \Gamma_1 \cdots \Gamma_{2N}$$

$$\Rightarrow \{\Gamma_{FIVE}, \Gamma_i\} = 0$$

$$[\Gamma_{FIVE}, \Sigma_{ij}] = 0$$

$$\Gamma_{FIVE}^2 = 1$$

$$\boxed{\Gamma_\pm = \frac{1 \pm \Gamma_{FIVE}}{2}}$$

$$(\Gamma_+)^2 = \Gamma_+^2$$

$$\Gamma_+ \Gamma_- = 0$$

projectors

(def.) $\psi_+ = \Gamma_+ \psi = \frac{1+\Gamma_{FIVE}}{2}\psi$

$\Leftrightarrow \Gamma_{FIVE}\,\psi_+ = \psi_+$

" $\Gamma_{FIVE} = 1$ "

$\psi_+ = |\varepsilon_1 \cdots \varepsilon_N\rangle \;\; \therefore \;\; \boxed{\varepsilon_1 \cdots \varepsilon_N = +1}$

$\Downarrow$ $\boxed{\text{examples}}$

$\boxed{SO(2)}$

" Neutrino mass --- "

G. S. posted

Appendix $SO(2N)$

$SO(2)$: $\{\Gamma_i\} = \sigma_1, \sigma_2$ $d = 2 = 2^1$

(gen) $\Sigma_{12} = \frac{1}{2i} \Gamma_1 \Gamma_2 = \frac{1}{2} \sigma_3$

(chir) $\Gamma_{FIVE} = \frac{1}{i} \sigma_1 \sigma_2 = \sigma_3$

$$[\Gamma_{FIVE}, \Sigma_{12}] = 0$$

$$\psi = \begin{pmatrix} u \\ v \end{pmatrix} \Rightarrow \psi_+ = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$\psi_- = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \psi \to e^{i\theta \Sigma_{12}} \psi = e^{i\theta \sigma_3/2} \psi$$

$$\Rightarrow \boxed{\begin{aligned} u &\to e^{i\theta/2} u \\ v &\to e^{-i\theta/2} v \end{aligned}} \quad U(1)$$

$$\boxed{SO(2) = U(1)}$$

$$\boxed{\text{mass terms}}$$

ψ = ~~fr~~ SM fermions

- $$\underbrace{\psi^T B \psi}_{SO(2)} \Leftrightarrow \underbrace{\psi^T C \psi}_{\text{Lorentz}}.$$

$$\psi \to e^{i\theta\sigma_3/2}\psi$$

$$\Rightarrow B = \sigma_1 \ (\sigma_2)$$

$$\psi^T B \psi \to \psi^T e^{i\theta\sigma_3/2} B e^{i\theta\sigma_3/2}\psi$$

$$= \psi^T B \underbrace{e^{-i\theta\sigma_3/2} e^{i\theta/2\,\sigma_3}}_{1}\psi$$

$$\Downarrow$$

$$B = \sigma_1 \ (\sigma_2)$$

(a) ψ = Lorentz scalar

$\psi^T B \psi \Rightarrow B = \sigma_1$ since

$$[\psi, \psi] = 0$$

(b) ψ = fermion (Lorentz spinor)

$$\Rightarrow \psi_L^T B C \psi_L =$$

$$= -\psi_L^T C^T B^T \psi_L = +\psi_L^T B^T \psi_L$$

$$\Rightarrow \boxed{B^T = B} \Rightarrow \boxed{B = \sigma_1}$$

$$\Downarrow$$

mass term

$$\psi^T \sigma_1 \psi = (u\ v)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

$$= 2uv = 2\psi_+\psi_-$$

$$\Downarrow$$

$$\cancel{\psi_+^T ? \psi_+}$$

$$u \to e^{i\theta/2} u, \quad v \to e^{-i\theta/2} v$$

$\cdot\ \cancel{uu} \qquad \cdot\ \cancel{vv} \qquad uv$

<u>irreducible</u> $\psi = \psi_+ = \begin{pmatrix} u \\ 0 \end{pmatrix}$

$$\Downarrow$$

<u>no mass term !</u>

$$\boxed{SO(2) = \text{chiral theory}}$$

$\boxed{\text{good !}}$