

LMU GUT Course

Lecture XXIII

Minimal P S
Theory

Pati - Salam :

Symmetry breaking

$$\mathfrak{g}_L = \mathfrak{g}_R \equiv \mathfrak{g}$$

- $G_{PS} = SU(2)_L \times SU(2)_R \times SU(4)_C$
($\times LR = P, C$)

- $f_L = (2_L, 1_R, 4_C)$

$$f_R = (1, 2_R, 4_C)$$

$$\textcircled{f_{L,R}} \leftarrow f_{L,R} = \begin{pmatrix} u^a & \nu \\ d^a & e \end{pmatrix}_{L,R} \quad a = u, \gamma, b$$

$$D_\mu = \partial_\mu - ig \cdot (\vec{A}_L \cdot \vec{T}_L + \vec{A}_R \cdot \vec{T}_R)_\mu -$$
$$- ig_c A_c^a T_c^a$$
$$a = 1, \dots, 15$$

$$T_1 T_2^2 = \frac{1}{2} \text{ in fund. repr}$$

(2 for $SU(2)$, 4 for $SU(4)$)

• Higgs sector

$$L_Y = \bar{f}_L Y \Phi \bar{\Phi} f_R + \text{h.c.}$$

$$f_{L,R} \rightarrow U_{L,R} f_{L,R}$$

$$\boxed{\Phi \rightarrow U_L \Phi U_R^\dagger \text{ li-doublet}}$$

$$\Phi: (2_L, 2_R, 1_c)$$

$$\langle \Phi \rangle \neq 0 \Rightarrow M_u, M_d$$

$$\boxed{\langle \Phi \rangle \simeq M_W}$$

$$M_e, M_D$$

(neutrino Dirac)

$$\Delta_L \longleftrightarrow \Delta_R$$

$$0 = \langle \Delta_L \rangle \not\leftrightarrow \boxed{\langle \Delta_R \rangle \neq 0}$$

$$\begin{array}{ccc} \text{Minimality} \Rightarrow & M_{PS} = & M_R \\ & \nearrow & \uparrow \\ & SU(4)_c \text{ breaking} & LR \text{ breaking} \end{array}$$

but not necessary!

$$PS \rightarrow \boxed{SM} \quad (\underline{\text{no}} \nu_R)$$

$$\boxed{\nu_R \text{ very heavy}}$$



$$m_{\nu R} \propto -M_R \quad (M_{ps}?)$$



$$\mathcal{L}_Y^\Delta = f_R^T C i\sigma_2 \Delta_R Y_\Delta f_R + L \leftrightarrow R$$

$$Z_R \quad \times \quad Z_R = \tilde{Z}_R + \cancel{A_R}$$



fixes Δ_R

$$f_R = (1_L, 2_R, 4_C)$$

$$\Delta_R \rightarrow U_R \Delta_R U_R^\dagger \quad SU(2)_R$$

$$\Delta_L \rightarrow U_L \Delta_L U_L^\dagger \quad SU(2)_L$$

$$PS: 4^c = 3_{1/3}^c + 1_{-1}^c$$

$$4^c \times 4^c = (3_{1/3} + 1_{-1}) \times (3_{1/3} + 1_{-1})$$

$$= \underbrace{6_{2/3} + 3_{-2/3} + 1_{-2}}_{10(S)} + \underbrace{3_{-2/3} + 3_{2/3}}_{6(A)}$$

$$\boxed{\frac{Y}{2} = T_{3R} + \frac{B-L}{2}}$$

$$Q_{em} = T_{3L} + \underbrace{T_{3R} + \frac{B-L}{2}}_{\frac{Y}{2}}$$

$$= T_{3L} + \frac{Y}{2}$$

⇓

summary:

b_i
(SM)

(b_2, b_3, b')

↔ $\frac{U(1)}{Y}$

$$4_R \times 4_R = (1_L, 3_R, 10_C) \quad \frac{4.5}{2} = 10$$



$\subseteq SU(2)_R$ triplet

$$B-L = -2$$

$$f_R \Delta_R f_R \Rightarrow$$

$$\Delta_R = (1_L, 3_R, \overline{10}_C)$$

$$\Delta_R = (1_L, 3_R, \overline{10}_C) = \xrightarrow{SU(3)_C}$$

$$= (1_L, 3_R, \overline{6} + \overline{3} + 1)$$

B-L: $-\frac{2}{3}$ $\frac{2}{3}$ $+2$

$$\langle \tilde{\Delta}_R \rangle = (1_L, 3_R, 1_2)$$

"weak"
part

$$G_{PS} = SU(2)_L \times \underbrace{SU(2)_R \times SU(4)}$$

$$\downarrow \langle \Delta_R \rangle$$

$$SU(2)_L \times U(1)_Y \times SU(3)_C = 6SM$$

$$\underline{SU(2)_R} : \tilde{\Delta}_R \rightarrow U \tilde{\Delta}_R U^\dagger$$

$$\hat{T}_R \Delta_R = [T_R, \Delta_R] \quad \curvearrowright$$

$$\hat{T}_R \tilde{\Delta}_R = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{pmatrix} \text{values of} \\ T_{3R} \end{pmatrix}$$

$$(B-L) \Delta_R = 2 \Delta_R$$

$$Q_{em} = T_{3R} + \frac{B-L}{2} \quad (T_{3C} = 0)$$

$$\tilde{\Delta}_R = \begin{pmatrix} \Delta_+ & \Delta^{++} \\ \Delta^0 & -\Delta_+ \end{pmatrix} \rightarrow \boxed{\text{new!}}$$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$



$$M_{WR} \propto v_R, \quad M_{ZR} \propto v_R$$

? ~~$M_{WR} \propto v_R, \quad M_{ZR} \propto v_R$~~

~~$M_{ZL} \propto v_R$~~

$$\langle \Delta_L \rangle = 0$$

$$M_{ps} \propto v_R !$$

$$M_{ps} \propto g_c(g_a) v_R$$

$$M_{WR} \propto g_R v_R$$

$$M_{2R} \propto f(g_R, g_{BL}) v_R$$

$$M_{1R} \propto g_c v_R$$

$$g_{BL} \longleftrightarrow \frac{B-L}{2}$$

$$g' \longleftrightarrow \frac{Y}{2}$$

$$g_c \longleftrightarrow SU(4)_c$$

• $M_R = M_{ps}$ scale: (i) $g_L = g_R$
(L \leftrightarrow R)

(ii) $g_B \propto g_c$

$$T_{15} = N \frac{B-L}{2}$$

$$\boxed{N = ?}$$

NOT a unified theory

$M_R: \underline{g_2, g_4} \Rightarrow$

all is f-on of Θ_w !

$$\tan \Theta_w \equiv g'/g$$

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2}$$

PS: (a) $g \leftrightarrow l$ unif.

(b) origin of P !

charge quantization

LR

$\Downarrow LR$

$$\boxed{\exists v_R}$$

$$\Rightarrow l_R^T C i \sigma_2 \tilde{\Delta}_R \gamma_\Delta l_R \Rightarrow$$

$$\Rightarrow l_R^T C \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}}_{\text{diag}} \gamma_\Delta l_R$$

$$l_R \equiv \begin{pmatrix} v_R \\ e_R \end{pmatrix}$$

$$= v_R^T C \gamma_\Delta v_R v_R$$

$$\Rightarrow \boxed{M_{v_R} = \gamma_\Delta v_R}$$

$$M_{v_R} = f v_R$$

$$\left[\frac{M_{ZR}}{M_{WR}}, \frac{M_{PS}}{M_{WR}} = f(\theta_w) \right]$$

PS: $\alpha_L = \alpha_R$

$$\frac{1}{\alpha_L} = \frac{1}{\alpha_R} + \frac{1}{\alpha_{BL}}$$

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2}$$

$$\alpha_{BL} = ? \alpha_c$$

- $\alpha_L \stackrel{?}{=} \alpha_R$ at M_P

$$SU(2)_L \times SU(2)_R \times SU(4)_c$$

$$P_{\text{odd}} \longleftarrow \downarrow (\sum_i^4) = M_{PS}$$

$$\underline{\underline{SU(3)_c}} \times SU(2)_L \times SU(2)_R \times \underline{\underline{U(1)_{B-L}}} \equiv G_{LR}$$

$$\downarrow \langle D_R \rangle = \delta_{LR}$$

$$\boxed{\Sigma \stackrel{G_{SM}}{\xrightarrow{P}} - \Sigma \quad (L \leftrightarrow R)}$$

- $g_L = g_R$ at M_{ps}

$$\Rightarrow g_L \neq g_R \text{ at } M_R$$

$$\Sigma = ?$$



$$\boxed{\Sigma = (1_L, 1_R, 15_c) \quad \times}$$

$$\dots Tr \Sigma^2, Tr \Sigma^3, Tr \Sigma^4 \dots$$

- LR symmetry breaking

$$\langle \Delta_L \rangle = 0, \quad \langle \Delta_R \rangle \neq 0$$

$$\cancel{\Delta_L}, \cancel{\Delta_L^3}$$

⇓ schematically

$$V = -\frac{\mu^2}{2} (\Delta_L^2 + \Delta_R^2) + \frac{\lambda}{4} (\Delta_L^4 + \Delta_R^4)$$

$$+ \frac{\lambda'}{2} \Delta_L^2 \Delta_R^2$$

$$(\Delta^2 \equiv T_i \Delta^\dagger \Delta)$$

$$= -\frac{\mu^2}{2} (\Delta_L^2 + \Delta_R^2) + \frac{\lambda}{4} (\Delta_L^2 + \Delta_R^2)^2$$

$$+ \frac{\lambda' - \lambda}{2} \Delta_L^2 \Delta_R^2 \leftarrow$$

$$(i) \lambda' - \lambda = 0 \Rightarrow$$

$$\underbrace{\langle \Delta_L^2 \rangle + \langle \Delta_R^2 \rangle}_{\text{who ???}} = \frac{\mu^2}{\lambda}$$

$$(ii) \lambda' - \lambda \neq 0$$

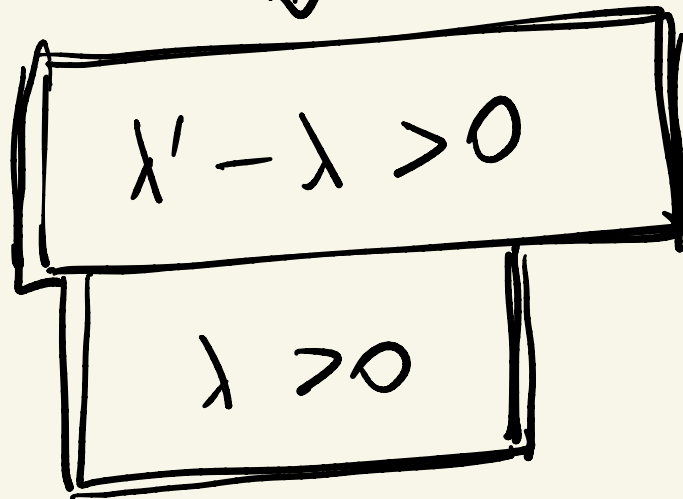
• $\lambda' - \lambda > 0$ • $\lambda' - \lambda < 0$

(a) $\lambda' - \lambda > 0$ $\Rightarrow \langle \Delta_L \rangle = 0$

$\langle \Delta_R \rangle \neq 0$

(b) $\lambda' - \lambda < 0$ $\langle \Delta_L \rangle \neq 0 \neq \langle \Delta_R \rangle$

$\Rightarrow \langle \Delta_L \rangle = \langle \Delta_R \rangle$



$\Rightarrow M_{WR}, M_{ZR} \propto \nu_R$

$M_{VR} \propto \nu_R$



$$\bar{\nu}_R M_D \nu_L \neq 0$$

$$\boxed{m_\nu \neq 0}$$

$$M_{\nu_R} \gg M_D$$

see saw



$$M_\nu = -M_D^T \frac{1}{M_N} M_D$$

$$N_L \equiv C \bar{\nu}_R^T$$

SU(2) $\Delta \rightarrow U \Delta U^\dagger$

$$T_\nu \Delta^3 = ???$$

$$\Delta = T_a \Psi_a \propto \sigma_a \Psi_a$$

$$\text{Tr} \Delta^3 \propto \text{Tr} \sigma_a \sigma_b \sigma_c \Psi_a \Psi_b \Psi_c$$

$$, \propto \text{Tr} \Sigma_{abcd} \sigma_d \sigma_c \Psi_a \Psi_b \Psi_c$$

$$\left(\text{Tr} \Delta^2, \text{Tr} \Delta^4 \propto (\text{Tr} \Delta^2)^2 \right)_{\gamma=1}$$

$$\Downarrow \quad (A1) \quad (S) \\ \propto \Sigma_{abc} \Psi_a \Psi_b \Psi_c = 0$$

$$SU(2)_R * U(1)_{B-L} \Rightarrow \Delta = \text{complex}$$

$$(B-L) \Delta = 2\Delta$$

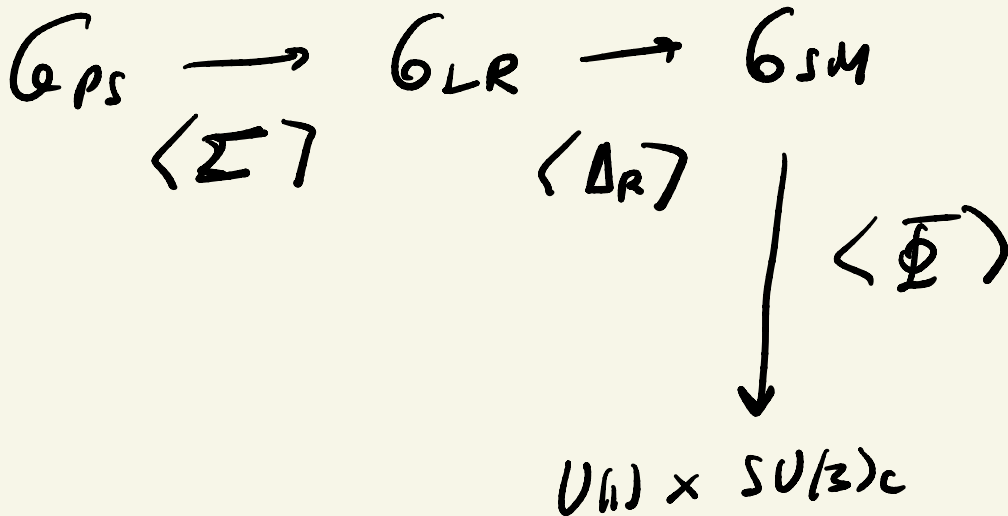
$$\Downarrow$$

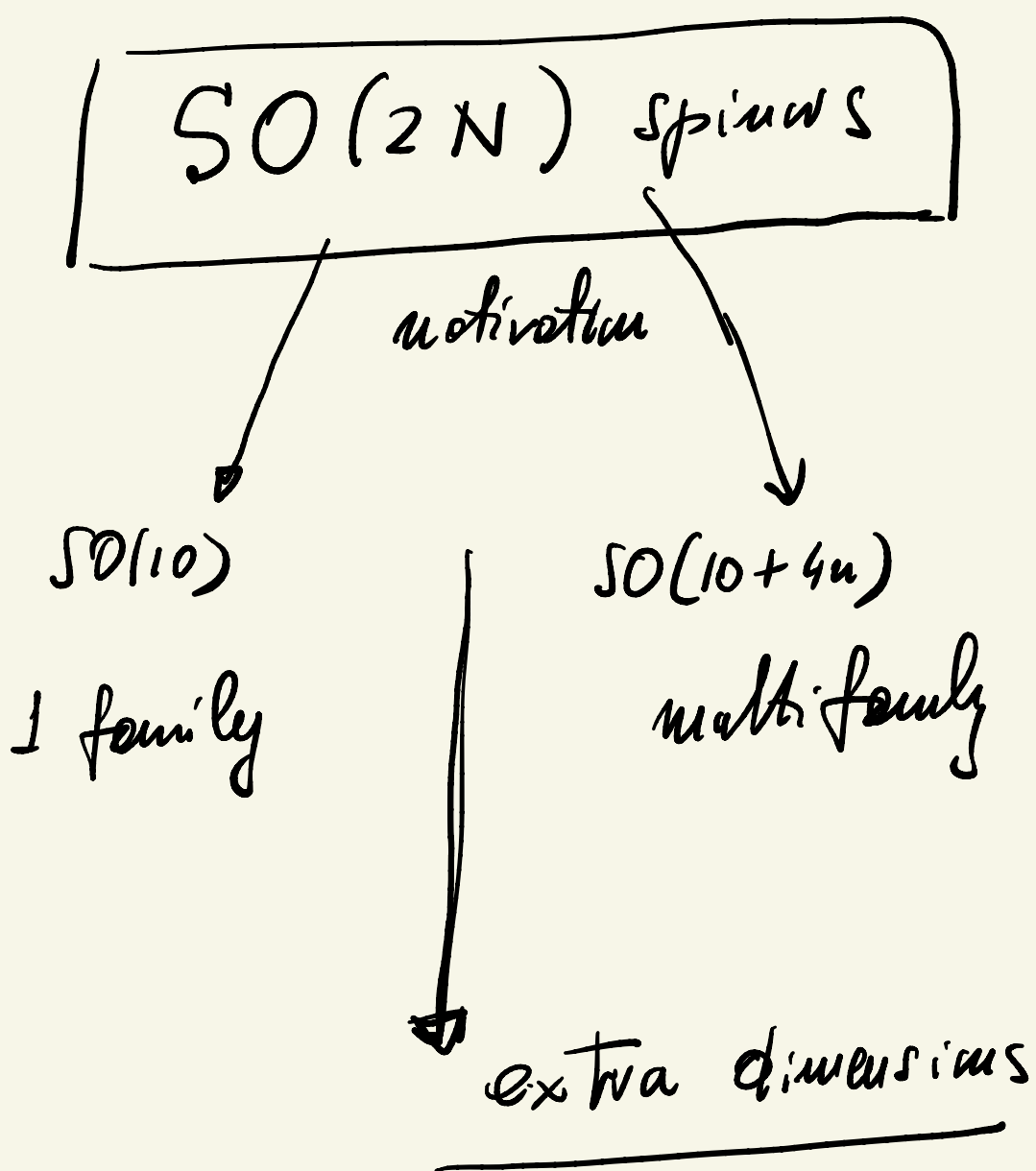
$$\cancel{\text{Tr} \Delta_R^3}, \quad \cancel{\text{Tr} \Delta_R^\dagger \Delta_R \Delta_R}$$

$$\cancel{T, \Delta_L^+ \Delta_R^+ \Delta_R}$$

⇒ change (B-L) forbids odd terms

General breaking





Orthogonal

• $x_1^2 + \dots + x_n^2 = 1 \quad (1Nv)$

$$x_i' = O_{ij} x_j$$

$$x_i' x_i' = O_{ij} x_j O_{in} x_n =$$

$$= O_{ki}^T O_{ij} x_j x_k = x_i x_j$$

$$\Rightarrow O_{ki}^T O_{ij} = \delta_{ja}$$

$$\boxed{O^T O = I = O O^T}$$

$$\det O = 1$$

$$\Rightarrow O = e^{i \theta_{ij}} L_{ij} \quad \boxed{\frac{N(N-1)}{2}}$$

$$\det O = 1 \Rightarrow \text{Tr} L_{ij} = 0$$

$$O^T O = I \Rightarrow L^T + L = 0$$

$$L^* + L = 0$$

$$\Rightarrow (L_{ij})_{ke} = -i (\delta_{ik} \delta_{je} - \delta_{ie} \delta_{jk})$$

$$\theta_{ij} = \frac{N(N-1)}{2} \text{ Euler}$$

SO(3) $L_{12} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$L_{ij} = \epsilon_{ijk} T_k$$

$$\Rightarrow T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

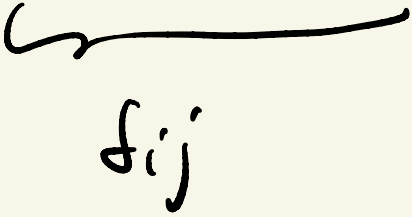
$$\Rightarrow T_3 (\text{diag}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$


$[T_i, T_j] = i \epsilon_{ijk} T_k$

• $[L_{ij}, L_{ke}] = \delta_{ik} L_{je} + \dots$

• $(X_i T_i)^2 = 1$
 \mathbb{R}

$$1 = x_i \Gamma_i \quad x_j \Gamma_j = x_i x_j \frac{1}{2} \{ \Gamma_i, \Gamma_j \}$$





$$\{ \Gamma_i, \Gamma_j \} = 2 f_{ij}$$

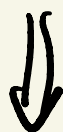
clifford algebra

- $\Gamma_n' = O_{ij} \Gamma_j$

$$\Gamma_n' \Gamma_j' = O_{in} O_{je} \Gamma_n \Gamma_e$$

$$\{ \Gamma_n', \Gamma_j' \}' = O_{in} O_{je} \{ \Gamma_n, \Gamma_e \} =$$

$$= 2 \delta_{ne} O_{in} O_{je} = 2 f_{ij}$$



$$T_i' = S(0) T_i S(0)^{-1}$$

$$S S^{-1} = 1$$

$$\Rightarrow \{T_i', T_j'\} = \{T_i, T_j\}$$

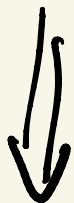
$$\boxed{S(0) T_i S(0)^{-1} = \delta_{ij} T_i} \quad (*)$$

↓ Spinorial analog of 0

$$\psi \rightarrow S(0) \psi \quad \boxed{\psi = \text{Spinor}}$$

$$S(0) = e^{i\theta_{ij} \Sigma_{ij}}$$

$$[\Sigma_{ij}, \Sigma_{ae}] = i(\delta_{ia} \Sigma_{je} + \dots) \quad (**)$$



$$\Sigma_{ij} = \frac{1}{2i} [\Gamma_i, \Gamma_j]$$

$$\bullet \quad \Sigma_{12} = \frac{1}{2i} \Gamma_1 \Gamma_2 \quad \Gamma_1^2 = 1 = \Gamma_2^2$$

$$\Gamma_1 \Gamma_2 = -\Gamma_2 \Gamma_1$$

$$(2\Sigma_{12})^2 = \left(\frac{1}{i}\right)^2 \Gamma_1 \Gamma_2 \Gamma_1 \Gamma_2 = (-1)(-1) = 1$$

$$(2\Sigma_{2n-1, 2n})^2 = 1$$

$$2\Sigma_{2n-1, 2n} : \quad \varepsilon = \pm 1$$

↑
eigenvalues

$$[\Sigma_{12}, \Sigma_{34}] \neq 0$$

$$\psi = |\varepsilon_1, \dots, \varepsilon_N\rangle \quad SO(2N)$$

$$= |\pm 1 \pm 1 \dots\rangle$$

$$\Gamma_{\text{FIVE}} = 2 \Sigma_{12} 2 \Sigma_{34} \dots \Sigma_{2N-1, 2N}$$

$$= \left(\frac{1}{i}\right)^N T_1 \dots T_{2N}$$

$$\bar{\Gamma}_{\text{FIVE}} = (-i)^N T_1 \dots T_{2N}$$

$$\Rightarrow \{ \bar{\Gamma}_{\text{FIVE}}, T_i \} = 0$$

$$[\bar{\Gamma}_{\text{FIVE}}, \Sigma_{ij}] = 0$$

$$\bar{\Gamma}_{\text{FIVE}}^2 = 1$$

$$\bar{\Gamma}_{\pm} = \frac{1 \pm \bar{\Gamma}_{\text{FIVE}}}{2}$$

$$(\bar{\Gamma}_{+})^2 = \bar{\Gamma}_{+}^2$$

$$\bar{\Gamma}_{+} \bar{\Gamma}_{-} = 0$$

projectors

$$\text{(def.) } \psi_+ = \Gamma_+ \psi = \frac{1 + \Gamma_{\text{FIVE}}}{2} \psi$$

$$\Leftrightarrow \Gamma_{\text{FIVE}} \psi_+ = \psi_+$$

$$\text{"} \Gamma_{\text{FIVE}} = 1 \text{"}$$

$$\psi_+ = |\epsilon_1 \dots \epsilon_N\rangle \quad \therefore \quad \boxed{\epsilon_1 \dots \epsilon_N = +1}$$

⇓ examples

SO(2)

" Neutrino mass --- "

G.S. posted

Appendix SO(2N)

$$SO(2) : \{ \Gamma_i \} = \sigma_1, \sigma_2 \quad d=2=2^1$$

$$(gen) \quad \Sigma_{12} = \frac{1}{2i} \Gamma_1 \Gamma_2 = \frac{1}{2} \sigma_3$$

$$(chiral) \quad \Gamma_{FIVE} = \frac{1}{i} \sigma_1 \sigma_2 = \sigma_3$$

$$[\Gamma_{FIVE}, \Sigma_{12}] = 0$$

$$\psi = \begin{pmatrix} u \\ v \end{pmatrix} \Rightarrow \psi_+ = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$\psi_- = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \psi \rightarrow e^{i\theta \Sigma_{12}} \psi = e^{i\theta \sigma_3 / 2} \psi$$

$$\Rightarrow \begin{cases} u \rightarrow e^{i\theta/2} u \\ v \rightarrow e^{-i\theta/2} v \end{cases}$$

$U(1)$

$$SO(2) = U(1)$$

mass terms

$\psi = \text{fermions}$ SM fermions

$$\bullet \quad \underbrace{\psi^T B \psi}_{SO(2)} \Leftrightarrow \underbrace{\psi^T C \psi}_{\text{Lorentz}}$$

$$\psi \rightarrow e^{i\theta \sigma_3/2} \psi$$

$$\Rightarrow B = \sigma_1 \text{ (or } \sigma_2)$$

$$\bullet \quad \psi^T B \psi \rightarrow \psi^T e^{i\theta \sigma_3/2} B e^{i\theta \sigma_3/2} \psi$$

$$= \psi^T B \underbrace{e^{-i\theta \sigma_3/2} e^{i\theta \sigma_3/2}}_1 \psi$$



$$B = \sigma_1 (\sigma_2)$$

(a) $\psi =$ Lorentz scalar

$$\psi^T B \psi \Rightarrow B = \sigma_1 \text{ since}$$

$$(\psi, \psi) = 0$$

(b) $\psi =$ fermion (Lorentz spinor)

$$\Rightarrow \psi_L^T B C \psi_L =$$

$$= -\psi_L^T C^T B^T \psi_L = +\psi_L^T B^T \psi_L$$

$$\Rightarrow \boxed{B^T = B} \Rightarrow \boxed{B = \sigma_1}$$



mass term

$$\psi^T \sigma_1 \psi = (u \nu) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ \nu \end{pmatrix} \\ = 2u\nu = 2\psi_+ \psi_-$$



~~$\psi_+^T \psi_+$~~

$$u \rightarrow e^{i\theta/2} u, \quad \nu \rightarrow e^{-i\theta/2} \nu$$

~~$u u$~~ ~~$\nu \nu$~~ $(u \nu)$

irreducible $\psi = \psi_+ = \begin{pmatrix} u \\ \nu \end{pmatrix}$



no mass term!

↓

$SO(2) =$ chiral theory

good!