LMU GUT Course

Lecture XXII

29/1/2021

Pati – Salam (L R) theory

(quark – lepton) unification

Four colours = r, y, b, v

S M

$(\frac{1}{3})\begin{pmatrix} u \\ d \end{pmatrix}_L$ $\qquad$ $(4/3)\, u_R,\ d_R\ (-2/3)$

(g) SU(2)

$(-1)\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ $\qquad$ $e_R\ (-2)$

$(g')\ U(1)_Y$ $\qquad$ $Q = T_3 + Y/2$

$$\Rightarrow Y = 2\,[Q - T_3]$$

L: $\underbrace{Y_L\,(LH) = \tfrac{1}{3} \text{ for } q\,;\ -1 \text{ for } \ell}$

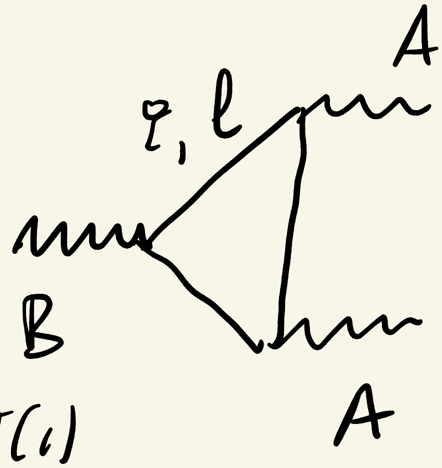
$B \equiv$ Baryon # ; $L \equiv$ Lepton # physics?

$$B_q = \frac{1}{3} ; \quad L_q = 0$$

$$B_e = 0 ; \quad L_e = 1$$

$$\Rightarrow \boxed{Y_L = B - L}$$

anomaly-free



$$\mathcal{A}(Y_L) = 0$$

$$A(B-L) = 0$$

PS's q − l symmetry

$$\underbrace{\begin{pmatrix} u \\ d \end{pmatrix}_L^{\alpha}}_{SU(3)_c} \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \alpha = r, y, b$$

$$M \equiv M_{PQ} = \text{new large scale}$$

$SU(4)_c$ symmetry

$$\begin{pmatrix} u^\alpha & \nu \\ d^\alpha & e \end{pmatrix}_L \longrightarrow r, y, b \,;\, v$$

$$M_{PS} \gg M_W$$

$$\underbrace{SU(4)_c}_{r=3} \xrightarrow[M_{PS}]{} \underbrace{SU(3)}_{r=2}$$

$\underline{SU(3)_c}$:

$$T_3^c = \frac{1}{2}\left(\begin{array}{ccc|c} 1 & & & \\ & -1 & & \\ & & 0 & \\ \hline & & & 0 \end{array}\right)$$

$$\downarrow \qquad T_8^c = \frac{1}{2\sqrt{3}}\left(\begin{array}{ccc|c} 1 & & & \\ & 1 & & \\ & & -2 & \\ \hline & & & 0 \end{array}\right)$$

$$\underbrace{SU(4)_c}_{\begin{array}{c}15 \text{ gen} \\ = 4^2-1\end{array}}$$

$$T_{15}^{c} = \frac{1}{2\sqrt{6}}\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$$

$$= \frac{3}{2\sqrt{6}}\underbrace{\begin{pmatrix} 1/3 & & & \\ & 1/3 & & \\ & & 1/3 & \\ & & & -1 \end{pmatrix}}$$

$$= \frac{3}{\sqrt{6}} \quad \frac{B-L}{2}$$

$$= \sqrt{3/2}\ \boxed{\frac{B-L}{2}}$$

$$\parallel$$

$$\frac{Y_L}{2}$$

$$SU(3)_c \times SU(2)_L \times \underset{Y}{U(1)} \subseteq SU(2)_L \times SU(4)_c$$

$$\times\ ?$$

$$\frac{Y_R}{2} \neq \frac{B-L}{2}$$

• $L \leftrightarrow R$ symmetric

$$\Rightarrow \quad Y_R = Y_L = B-L$$

$$\Downarrow$$

$$SU(2)_L \times U_Y(1) \times SU(3)_c .$$

$$\cap \qquad G_{PS}$$

$$\equiv$$

$$\boxed{SU(2)_L \times SU(2)_R \times SU(4)_c}$$

$$SU(4)_c \xrightarrow[\text{spont.}]{} SU(3)_c$$

$$\not{P} \xrightarrow[\text{spont.}]{} \text{nothing}$$

$$Q_{em} = ?$$

$$Q_{em} = T_{3L} + \frac{Y_L}{2} = T_{3L} + \frac{B-L}{2}$$

$$T_{3L}(f_R) = 0$$

$$T_{3R}(f_L) = 0$$

$$\Downarrow$$

$$\boxed{Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}}$$

$$\left.\begin{array}{l} Q_{em}^{L} = T_{3L} + \frac{Y_L}{2} \\ \quad \| \qquad\quad \| \\ Q_{em}^{R} = T_{3R} + \frac{Y_R}{2} \end{array}\right\} \frac{B-L}{2}$$

$$B-L \subseteq SU(4)_c$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \subseteq G_{PS}$$

(i) $\boxed{\text{gauge group : } G_{PS}}$ ✓

(ii) <u>matter content</u> ✓

$$\begin{pmatrix} u^\alpha & \nu \\ d^\alpha & e \end{pmatrix}_{L,R} \Rightarrow \boxed{\exists\, \nu_R}$$

$$\Rightarrow \boxed{m_\nu \neq 0}$$

(iii) <u>Higgs content</u>

(i) <u>what are new gauge bosons?</u>

SM : 8 gluons, $\underbrace{3\,\vec{A}}_{SU(2)}$, $\underbrace{B}_{U_Y(1)}\Big\} = 12$

PS: $\quad$ 8 gluons, $\underbrace{3\vec{A}_L}_{SU(2)_L}$, $\underbrace{3\vec{A}_R}_{SU(2)_R}$

$$\downarrow$$

$$SU(4)_c : \quad 15 = 8 + 7 = 8 + 6 + 1$$

$$\stackrel{?}{=} 8 + 3 + 3^* + 1$$

$$SU(4)_c : \quad 4 = (3_{1/3} + 1_{-1})$$

$$B-L$$

gauge bosons = adjoint $\subseteq$ $\boxed{4 \times \bar{4}}$

$$(\underset{\bar{4}}{\bar{f}} \gamma^\mu D_\mu \underset{4}{f})$$

$$4 \times \bar{4} = (3_{1/3} + 1_{-1}) \times (3^*_{-1/3} + 1_1)$$

$$= \underbrace{8_0}_{\text{gluons}} + \underset{X_{PS}}{\underset{\uparrow}{3_{4/3}}} + \underset{X^*_{PS}}{\underset{\uparrow}{3^*_{-4/3}}} + \underset{\boxed{B-L}}{\underset{\downarrow}{1_0}} + \ldots$$

$$X_{PS}^{\alpha} = \begin{cases} \text{singlet under } SU(2)_L \times SU(2)_R \\ \text{color triplet} \\ Q_{em} = 2/3 \quad \left(T_{3L} + T_{3R} + \frac{B-L}{2}\right) \end{cases}$$

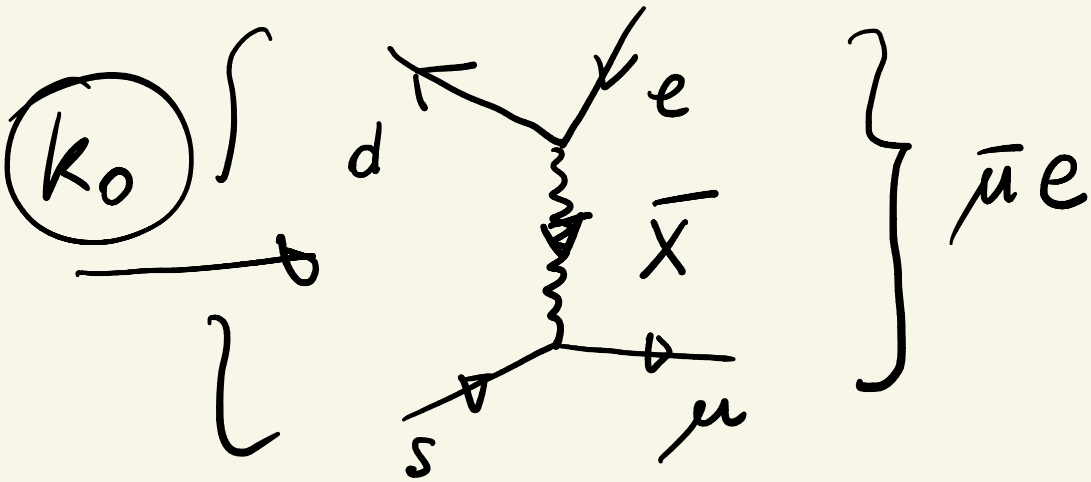
$$\boxed{\begin{aligned} X_{PS\mu} & \begin{cases} [\bar{u}_L \gamma^\mu \nu_L + L \to R] \\ [\underline{\underline{\bar{d}_L \gamma^\mu e_L}} + L \to R] \end{cases} \\ & \| \\ & \text{lepto-quarks} \end{aligned}}$$

- $\cancel{B}, \cancel{L}$?

$$\boxed{\begin{aligned} &\to B(X_{PS}) = \frac{1}{3} \Rightarrow \underline{B} \text{ is conserved} \\ &\to L(X_{PS}) = -1 \Rightarrow L \quad -\|- \end{aligned}}$$

$\underline{\underline{\text{LHC:}}}$ $\boxed{M_X \equiv M_{X_{PS}} \gtrsim \text{TeV}}$

• $X_{PS}\left[\bar{d}e + \bar{s}\mu + \bar{b}\tau\right]$



$\Downarrow$

$$\boxed{\begin{array}{c} K_L \to \bar{\mu} e \\ B(K_L \to \bar{\mu} e) \lesssim 10^{-12} \end{array}}$$

PDG

$$\equiv \frac{\Gamma(K_L \to \bar{\mu} e)}{\Gamma(K_L \to 3\pi)}$$

$\to X_{PS}$

$\to W$

$$A \propto \frac{1}{M_A^2} \to \Gamma \propto 1/M_A^4$$

$$\Rightarrow \mathcal{B}(u_L \to \bar{\mu} e) \simeq \left(\frac{M_W}{M_{X_{PS}}}\right)^4 \leq 10^{-12}$$

$$\Downarrow$$

$$\boxed{M_{X_{PS}} \equiv M_X \gtrsim 10^3 M_W \gtrsim 10^5 \text{GeV}}$$

$\Rightarrow$ X_{PS} can't be directly seen!

$$M_X = ?$$

$u \longrightarrow \nu$ $\quad$ $\bar{u} \longrightarrow \bar{\nu}$

$d \longrightarrow e$ $\quad$ $\bar{d} \longrightarrow \bar{e}$

$$\left.\begin{array}{l} uud \longrightarrow \nu\nu e \\ \bar{u}\bar{u}\bar{d} \longrightarrow \bar{\nu}\bar{\nu}\bar{e} \end{array}\right\} \boxed{p\bar{p} \to e\bar{e}\nu\nu\bar{\nu}\bar{\nu}}$$

limits much worse!

(iii) Higgs sector

Minimal model

$$\underset{(g_L)}{SU(2)_L} \times \underset{(g_R)}{SU(2)_R} \times \underset{(g_c)}{SU(4)_c}, \qquad (g_L) \equiv (g_R)$$

$$\downarrow \; M \equiv M_{PS} \equiv M_R \; (?)$$

$$SU(3)_c \; SU(2)_L \times U(1) \xrightarrow{\; M_W \;} U(1)_{em} \times SU(3)_c$$

must include the SM Higgs doublet

$$l_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$$

$$\text{mass} \longrightarrow y_e \, \bar{l}_L \, \Phi \, e_R \equiv \text{Yukawa}$$

Φ: SU(2) doublet, $Y = +1$

$$f = \begin{pmatrix} u & \nu \\ d & e \end{pmatrix}_{L,R} \equiv f_{L,R}$$

mass : $\overline{f_L}\, f_R \to \overline{\overline{f_L}\; U_R f_R}$

$$SU(2)_L \times \underline{\underline{SU(2)_R}}$$

$$f_L \to U_L f_L \qquad U_L U_L^\dagger = 1$$

$$f_R \to U_R f_R \qquad U_R U_R^\dagger = 1$$

$$f_R \to (f^c)_L \equiv C\, \overline{f_R}^T$$

$$\quad \hookrightarrow U_R^* f_L^c$$

$$\overline{f_L} f_R + \left(\overline{f_R} f_L = f_L^{c\,T} C f_L \right)$$

$$SU(2)_L \times SU(2)_L'$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} d^c \\ u^c \end{pmatrix}_L$$

- $LR \equiv P$: $f_L \to f_R$

Goran

• $LR \equiv C$ ofa $f_L \to (f^c)_L$

$SU(2)_L \times SU(2)_R = SO(4)$

$SU(4)_C = SO(6)$

digression

spinors

$f_L \qquad f_R = $ SM eW $(SU(2))$ singlet

independent

$\mathcal{L}_Y = \bar{f}_L \, \Phi \, f_R = $ inv. (1)

$\Phi \to U_L \, \Phi \, U_R^+$ (bi-doublet)

$$\mathcal{L}_Y \to \bar f_L \overbrace{U_L^{+} U_L}^{1} \Phi \overbrace{U_R^{+} U_R}^{1} f_R = \text{inv.}$$

(1) $\Rightarrow \boxed{(B-L)\,\Phi = 0}$

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2} = T_{3R} = \begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$$

$\boxed{\text{2 SM doublet}}$ $\quad \frac{Y}{2} = \frac{1}{2}; -\frac{1}{2}$

(1) $\Rightarrow$ $SU(4)_c$ property of Φ ?

$$(\bar\nu_L \nu_R) \subseteq \boxed{\bar f_L \,\Phi\, f_R} \qquad \Rightarrow \boxed{\Phi \sim 4 \times \bar 4}$$

$$\bar f_L f_R \sim \underbrace{\bar 4 \times 4}_{SU(4)_c}$$

$$4 = 3 + 1 \Rightarrow 4 \times \bar 4 = 15 + 1$$

Φ : 1_{PS} or 15_{PS}

minimal:

$$\boxed{\begin{array}{c}\Phi\,(2_L,\, 2_R,\, 1_{PS}) \\ SU(2)_L \times SU(2)_R \times SU(4)_{PS}\end{array}}$$

· PS scale : $M \equiv M_{PS} \equiv M_R$

break $SU(4)_{PS}$ ($\to SU(3)_c$)

break P
$SU(2)_L \times SU(2)_R$ ($\to SU(2)_L$)

break: $\boxed{SU(2)_R,\ SU(4)_{PS}}$

$$\boxed{\Delta_R\,(1_L,\ ?_R,\ ?_{PS})}$$

$$SU(2)_L \times SU(2)_R \times SU(4)_c$$

$$r_{PS} = 1 + 1 + 3 = 5$$

$$\Big\downarrow \langle \Delta_R \rangle$$

$$SU(2)_L \times U(1)_Y \times SU(3)_c$$

$$r_{SM} = 1 + 1 + 2 = 4$$

$$\Delta_R \neq \text{adjoint of } SU(4)_{PS}$$

$$\Uparrow$$

$$\langle \Delta_R \rangle : \quad \underbrace{\cancel{T}_{3R} , \; \cancel{T}_{15}}\; (\propto B-L)$$

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2} = \text{unbroken}$$

$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R, d_R \leftarrow$ SM (and LR)

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad e_R, \cancel{\nu_R} \leftarrow$

$\Downarrow$

$\langle \Delta_R \rangle \neq 0 \Rightarrow \begin{cases} 1.\ \text{new gauge bosons} \\ \quad m \propto \langle \Delta_R \rangle \\ 2.\ m_{\nu_R} \propto \langle \Delta_R \rangle \end{cases}$

$\boxed{\begin{array}{c} \text{SM} + \\ m_\nu \neq 0 \end{array}} \Leftarrow$

$\boxed{\text{Mohapatra, G.S.}}$

$\Downarrow$

$\boxed{\Delta_R \text{ couples for } f_R\,!}$

$$f_R = \begin{pmatrix} u & \nu \\ d & e \end{pmatrix}_R$$

$$f_R^T C\, i\sigma_2 \Delta_R f_R$$

(Majorana)
(Lorentz inv.)

$$\hookrightarrow f_R^T U_R^T C\, i\sigma_2\, U_R \Delta_R U_R^\dagger U_R f_R$$

$$= f_R^T C\, i\sigma_2\, U_R^\dagger U_R\, \Delta_R U_R^\dagger U_R f_R$$

$$U_R \equiv e^{i \vec{\theta}_R^{\pi} (\vec{\sigma}/2)}$$

$$= f_R^T C\, i\sigma_2 \Delta_R f_R \checkmark \; (\text{inv.})$$

$$\boxed{\begin{array}{ccc} \Delta_R\,(1_L, & ?_R, & ?_{PS}) \\ \Downarrow & & \updownarrow \\ \Delta_L\,(?_L, & 1_R, & ?_{PS}) \end{array}} \quad *$$

$$Q_{em} = \underbrace{(T_{3L} + T_{3R})}_{\text{quantized}} + \underbrace{\frac{B-L}{2}}_{\text{quantized}}$$

$$\boxed{PS \not\Rightarrow Q_\nu = 0}$$

$\underline{SU(5)}$ • $q_e = 3\, q_d$

• $q_\nu = 0$

SM (minimal) + anomaly $\Big\}$ $\Longrightarrow$ $\boxed{\text{the same}}$

but: $\exists\, \nu_R \not\Rightarrow Q_\nu = 0$

LR: $Q_{em} = (T_{3L} + T_{3R})\, X + \frac{B-L}{2}$

$$\underset{LR}{\uparrow} \qquad \underset{?}{\parallel} \qquad \underset{LR}{\uparrow}$$

$\not\Rightarrow Q_\nu = 0$

$$\boxed{Q_\nu = 0 \iff X = 1}$$

$$\boxed{\begin{aligned} &B-L = LR\text{ "charge"} \\ &(B-L)_\nu \neq 0 \end{aligned}}$$

$Q_\nu = ?$ related to the nature of ν mass

$$\boxed{Q_\nu = 0 \iff \nu = \text{Majorana}}$$

$$Q_\nu \neq 0 \iff \nu = \text{Dirac}$$