# LMU GUT Course

## Lecture XXI

26/1/2021

# Seesaw

- Minimal $SU(5) \Rightarrow m_\nu = 0$
- — " — — " — $\Rightarrow$ NO unif.

$\Downarrow$ Minimal ext

$\boxed{24_F \text{ can cure both}}$

$$\text{SM: } \underbrace{\left[1_F(S_F, N_F) + 3_F(T_F)\right]_L}_{\text{weak}}$$

$\Downarrow$

$$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

$Y = -1$ $\qquad\qquad$ $Y = +1$

$\Downarrow$

$$\mathcal{L}_Y = \underbrace{l_L^T \, i\sigma_2 \, \Phi}_{SU(2)\times U(1) \text{ inv.}} \, (S_F, T_F)$$

LH fermion

- $S_F \equiv N_L$

$$\mathcal{L}_Y = l_L^T \underbrace{i\sigma_2}_{SU(2)} C \, \Phi \, N_L \, Y_D$$

$C^T = -C \Rightarrow$ Lorentz

- $l_L^T i \sigma_2 C \quad \vec{\sigma}_2 \cdot \vec{T}_L \, \Phi \, Y_D$

$$T_F \equiv \vec{\sigma}_2 \cdot \vec{T}_L \quad \text{(Adjoint)}$$

$$T_F \to U T_F U^+$$

$$l^T i\sigma_2 T_F \Phi \to l^T U^T i\sigma_2 U T_F U^+ U \Phi$$

$$= l^T i\sigma_2 \underbrace{U^+ U}_{1} T_F \underbrace{U^+ U}_{1} \Phi = \text{inv.}$$

$N$ = "phantom" particle

$T$ = physical (W, Z boson int.)

can be produced at LHC with $O(1)$ coupling

$$g_W = g_2 = g \simeq 0.6$$

$$\frac{g^2}{4\pi} \equiv \alpha_2 = \alpha_W \simeq 1/30$$

$$\boxed{SU(5) \Rightarrow m_T \simeq M_W}$$

$N$ = generic heavy neutral lepton $\longrightarrow$ see saw

($T^0 \longleftrightarrow N$ in $T$ case)

$$\mathcal{L}_Y = l_L^{\,T}\, i\sigma_2\, C\, Y_D\, \Phi\, N_L \quad + h.c.$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L , \quad \Phi_{un} = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$e \xrightarrow{i\sigma_2} v+h$$

$$\to \nu_L^{\,T}\, i\sigma_2\, C\, Y_D (v+h)\, N_L$$

$$= \nu_L^{\,T}\, i\sigma_2\, M_D^{\,T} \left(1 + \frac{h}{v}\right) N_L$$

$$\boxed{M_D^{\,T} \equiv Y_D v} \qquad \boxed{\text{Higgs int.}}$$

$$\bar{e}e = \bar{e}_R e_L + \bar{e}_L e_R$$

$$\boxed{(e^c)_L \equiv C \bar{e}_R^{\,T}}$$

- $N_L \equiv C \bar{\nu}_R^{\,T}$

RH neutrino

$$\nu_L^T C M_D^T N_L = -N_L^T C^T M_D \nu_L$$

$$= \boxed{N_L^T C M_D \nu_L}$$

$N_L \equiv C \bar{\nu}_R^{\,T}$

def. of Dirac mass matrix

$$\bar{\nu}_R C^T C M_D \nu_L = \boxed{\bar{\nu}_R M_D \nu_L}$$

lepton # $\nu_R$ = lepton # of $\nu_L$

• $N$ = singlet ⇒ mass vector

$$\frac{1}{2} N_L^T C M_N N_L$$

Lorentz inv.

$SU(3)\times SU(2)\times U(1)$ inv.

$$= - N_L^T C^T M_N^T N_L = N_L^T C M_N^T N_L$$

$$\boxed{M_N^T = M_N}$$

Majorana mass : $f_L^T C f_L$

Dirac $\quad \bar{f}_R \cancel{\gamma} f_L \quad (f_R \neq f_L)$

$$\|$$

$$f_L'^T C f_L \qquad \uparrow$$

<u>no</u> connection

$$f_L' \equiv C \bar{f_R}^T$$

$$\Downarrow$$

$$\mathcal{L}_y(SM+N) = N_L^T M_D C \nu_L +$$

$$+ \frac{1}{2} N_L^T C M_N N_L + h.c.$$

$$= \frac{1}{2} N_L^T (M_D + M_D) C \nu_L +$$

$$+ \frac{1}{2} N_L^T C M_N N_L + h.c.$$

$$= \frac{1}{2}\left(N_L^T M_D C \nu_L + \nu_L^T(-C^T) M_D^T N_L\right)$$

$$+ \frac{1}{2} N_L^T C \mathcal{M}_N N_L + h.c.$$

$$= \frac{1}{2}\left[\left(N_L^T C \mathcal{M}_D \nu_L + \nu_L^T C \mathcal{M}_D^T N_L\right) + N_L^T C \mathcal{M}_N N_L\right] + h.c.$$

$$\Downarrow$$

$$\boxed{\begin{array}{cc} \begin{array}{c} \nu \\ N \end{array} & \begin{pmatrix} 0 & \mathcal{M}_D^T \\ \mathcal{M}_D & \mathcal{M}_N \end{pmatrix} \\ & \begin{array}{cc} \nu \qquad & \qquad N \end{array} \end{array}} \equiv \mathcal{M}_{\nu N} \qquad \boxed{\text{mass matrix}}$$

Majorana mass = symmetric

$$\boxed{\mathcal{M}_{\nu N}^T = \mathcal{M}_{\nu N}}$$

$$U^T S \text{ (symm. matrix)} U = D$$
(diagonal)

$$\Downarrow \qquad \boxed{U^\dagger U = 1}$$

$$U^T M_{\nu N} U = D_{\nu N}$$

$\boxed{\text{seesaw}}$ $\boxed{M_N \gg M_D}$

Logic: $M_D = y_D v = y_D \frac{M_W}{g} (2)$

$$M_D \leq M_W$$

$M_N$ = SM singlet

$\rightarrow$ new physics scale

($M_N \gtrsim$ TeV)

- $\Theta_{\nu N}$ = <u>$\nu - N$ mixing</u>

$$\Theta \equiv \Theta_{\nu N} \propto M_D / M_N \ll 1$$

$$U^T M_{\nu N} U = D_{\nu N}$$

$U^\dagger U = 1$ (in order to keep kin. energy)

$$\left.\begin{array}{l} \mathcal{L}_{kin} = i \bar{f} \gamma^\mu \partial_\mu f \\ U^\dagger U = 1 \quad f \to U f \end{array}\right\} \text{inv.}$$

$$U = \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix} \quad (\theta \ll 1)$$

$$\Downarrow$$

$$U^\dagger U = \begin{pmatrix} 1 & -\theta^\dagger \\ \theta & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta^\dagger \\ -\theta & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + \theta^\dagger \theta & \cancel{\theta^\dagger - \theta^\dagger} \\ \cancel{\theta - \theta} & \theta^\dagger \theta + 1 \end{pmatrix}$$

ignore $\theta^2$

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• Show : $\quad \Theta = \frac{1}{M_N} M_D$

$$U^T M_{\nu N} U \simeq \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix} + O(\theta^2)$$

$$\boxed{M_\nu = - M_D^T \frac{1}{M_N} M_D}$$

seesaw

$$
\begin{array}{c|cc}
 & \nu & N \\
\hline
\nu & 0 & M_D^T \\
N & M_D & M_N
\end{array}
\longrightarrow \text{eigenvalues}
$$

$$
\underbrace{M_N, \quad M_\nu = -M_D^T \frac{1}{M_N} M_D}_{\text{lazy person's approach}}
$$

- $M_\nu^T = M_\nu$ (symmetric)
- $M_D \to 0 \Rightarrow M_\nu \to 0$
- $M_N \to \infty \Rightarrow M_\nu \to 0$

$\nu$ couples to $N$ through $M_D^T$

$\rightarrow$ N propagator: $\dfrac{1}{\cancel{\not{p}} - M_N}$

$$x \; \underbrace{M_D^T \frac{1}{M_N} M_D \equiv M_\nu}_{\text{up to a factor } x}$$

$$U^T M_{\nu N} U = D_{\nu N}$$

$$\boxed{\Rightarrow \det M_{\nu N} = \det D_{\nu N} \quad (\det U^T U = 1)}$$

$$\det M_{\nu N} = -\det M_D M_D^T \qquad \det D_{\nu N} = x \det\left(M_D^T \frac{1}{M_N} M_D \, M_N\right)$$

$$\Rightarrow x = -1$$

· electron $\mathcal{L} = i\bar{e}\sigma^\mu \partial_\mu e - m\bar{e}e$

$$\boxed{E^2 = \vec{p}^{\,2} + m^2} \not\!\!\!\!\quad$$

$$\boxed{\begin{array}{lll} \text{Dirac :} & \bar{e}e & (\bar{f}f) \\ \text{Maj} & \nu\nu & (ff) \end{array}}$$

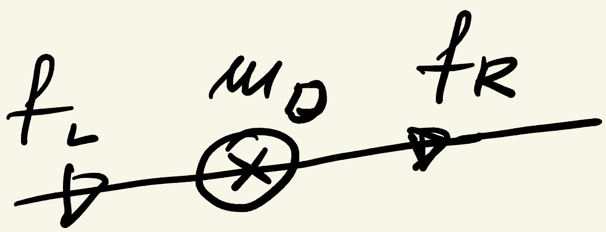
Dirac vs Majorana

| | |
|---|---|
| charges conserved | no charge conserved |
| $\Downarrow$ | $\Downarrow$ |
| lepton # conserved | $\boxed{L \text{ broken}}$ |

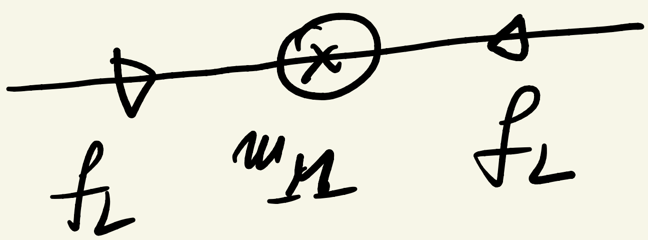
$0\nu 2\beta$

neutrinoless double beta

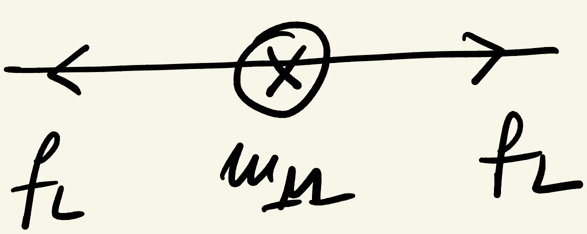
• (D) $\bar{f} f$ :

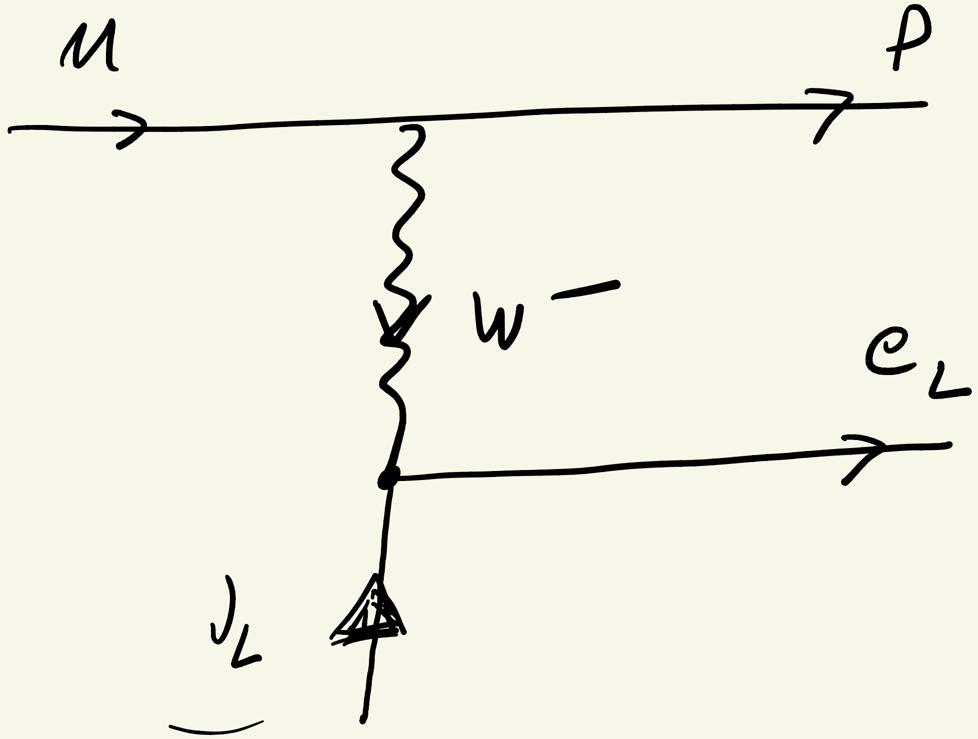



• (M) $f f$




breaks (F, L, ...) by 2 units






$\beta$ decay

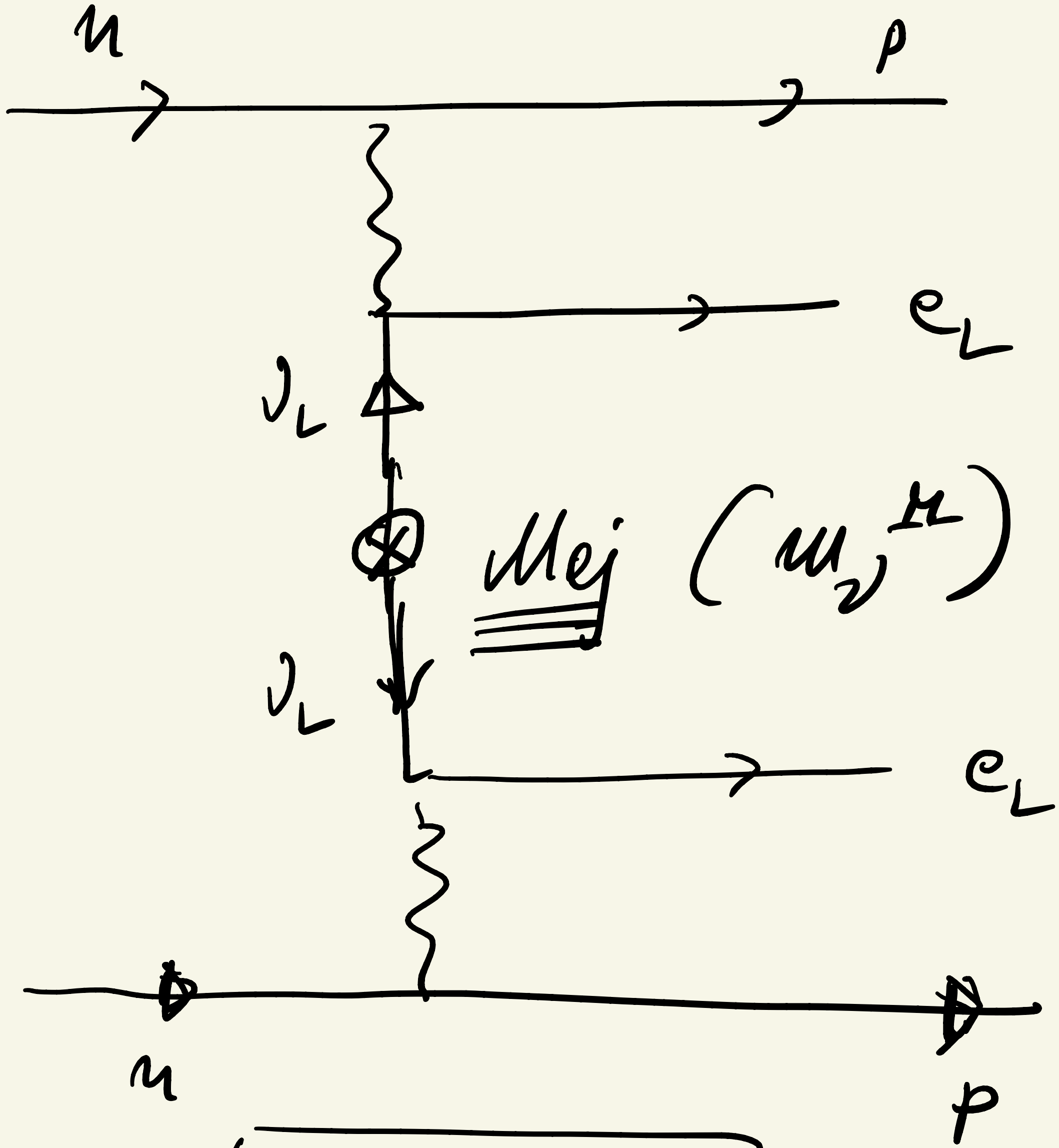
$\beta$ decay: **no** distinction between D & **M**

$$\boxed{m_\nu \leq 10^{-6}\, m_e}$$ puzzle

neutrino is neutral

$m_D \sim m_e$

$$\Rightarrow m_\nu^M \sim m_e^2 / M_N \simeq m_e \left( m_e / M_N \right)$$




$\boxed{\text{double } \beta}$

<u>if $\beta$ is forbidden!</u>

$$\cancel{Ge \to Ar + e + \bar{\nu}_e}$$

$$m_{Ar} > m_{Ge}$$

$$\Downarrow$$

$$Ge \to Se + e + e + \bar{\nu}_e + \bar{\nu}_e$$

$$m_{Se} < m_{Ge} \qquad 2\beta$$

$$\tau_{2\beta} \simeq 10^{21}\ \text{yr}\ !$$

$0\nu 2\beta$ (Majorana)

$$\tau_{0\nu 2\beta} \gtrsim 10^{25}\ \text{yr}$$

$$\Rightarrow m_\nu^{d} \leq 0.1\ \text{eV}$$

## What if we observe $0\nu 2\beta$?

- we observe $0\nu 2\beta$ = correct
- we observe $m_\nu^M$

To discuss

there could be new physics with say $N$ ??

$\Downarrow$

seesaw? How to observe it?

How to observe $N$?

$\Downarrow$

$$\boxed{\text{How to produce } N?}$$

$\Theta_{\nu N}$ = only road to $N$

- all couplings of $N \propto \Theta_{\nu N}$

$$\Theta_{\nu N} \equiv \theta = \frac{1}{M_N} M_D$$

$$M_{\nu N} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

$$\frac{g}{\sqrt{2}} \bar{\nu} e W \Rightarrow \frac{g}{\sqrt{2}} \theta \bar{N} e W$$

$$\Downarrow$$

(i) $M_W > m_N$ : $W \to e + N$

(ii) $m_N > M_W$ : $N \to e + W$

How to produce N ?

(i) produce W → look into decays

(ii) <u>more interesting</u>

$$\sigma(N) \propto \Theta^2$$

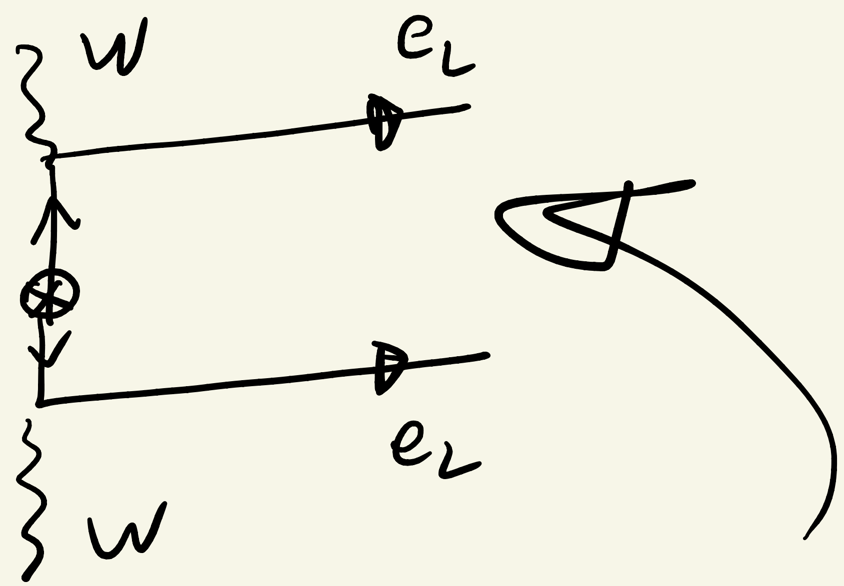
$$|m_\nu| = m_D^2/m_N \qquad \Theta = m_D/m_N$$

$$|m_\nu/m_N| = |m_D^2/m_N^2| = |\Theta|^2$$

$$\boxed{\sigma(N) \propto {m_\nu}/{m_N} \ll 1}$$

in the seesaw

$\underline{0\nu 2\beta}$ in $SM + \nu_M$




- $e = e_R \Rightarrow$ wrong!

new physics a must!!!

• $e = e_L$ ? Dvali, G.S., Maiezza, Tello

ongoing study

effective interactions

fund. (renorm.) interactions

$\boxed{\text{inconclusive!}}$

$\rightarrow$ $\boxed{\text{neutrino Majorana mass}}$