LMU GUT Course

Lecture XIX

19/1/2021

Winter 2021

SU(5): fermion mass relations and proton decay

branching ratios

$p \to \pi^0 + e^+ \quad K^0 + e^+$

$\pi^0 + \mu^+ \quad K^0 + \mu^+$

$\cancel{\pi^0 + \tau^+}$

$\Leftrightarrow$ fermion masses and mixings

$X [\bar{u}^c u + \bar{d} e^c]$

mixings?

Standard Model

$m_f \neq 0$ due to $SU(2) \times U(1)$

$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$

$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ Higgs mechanism

$$\mathcal{L}_Y = \bar{q}_L^0 Y_d \langle \Phi \rangle d_R^0 + \bar{q}_L^0 Y_u i\sigma_2 \langle \Phi \rangle^* u_R^0$$

$$+ \bar{l}_L^0 Y_e \langle \Phi \rangle e_R^0 + \text{h.c.}$$

$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\Downarrow$$

$$M_f = Y_f \, v \qquad f = u, d, e$$

$$\overline{f_L^0} M_f f_R^0 \longrightarrow \overline{f_L} U_{Lf}^\dagger M_f U_{Rf} f_R$$

$$f_{L,R}^0 \longrightarrow U_{L,R} f_{L,R} \quad \therefore$$

$$\longrightarrow \boxed{U_{Lf}^{+} M_f \, V_{Rf} = D_f \equiv \tilde{m}_f}$$

$$D_f = \text{diagonal } (= m_f)$$

- $M_f^{+} = M_f$

$$\Longrightarrow \quad U_{Lf} = V_{Rf}$$

- $M_f^{T} = M_f \Rightarrow U_{Lf} = V_{Rf}^{*}$

$$\begin{pmatrix} V_u = U \\ V_d = D \\ V_e = E \end{pmatrix}_{L,R}$$

$$\left.\begin{array}{l} \longrightarrow \; U_L^{+} M_u V_R = \tilde{m}_u \\ \longrightarrow \; D_L^{+} M_d D_R = \tilde{m}_d \\ \longrightarrow \; E_L^{+} M_e E_R = \tilde{m}_e \end{array}\right\} \text{ diagonal matrices}$$

$$\tilde{m}_d = \mathrm{diag}\,(m_d, m_s, m_b)$$

- <u>no</u> relations between m_f
- <u>no</u> relations between mixings
- <u>no</u> information on RH mixing

$$\mathcal{L}_W = \bar{u}_L \gamma^\mu V_q\, d_L\, W_\mu^+ \frac{g}{\sqrt{2}}$$

$$+ \bar{\nu}_L \gamma^\mu V_\ell\, e_L\, W_\mu^+\, g/\sqrt{2}$$

$$V_q \equiv V_{KM} \; (V_{CKM})$$

$$V_\ell \equiv V_{PMNS}$$

$$d_L = \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \quad \cdots$$

$$\boxed{\begin{aligned} V_q &= U_L^\dagger D_L \\ V_e &= N_L^\dagger E_L \end{aligned}}$$

$$\left(N_L^\dagger M_\nu N_R = \tilde{m}_\nu \right)$$

neutrino mixing

- $D_L = 1 \quad \left(U_{dL} \equiv D_L \right)$

$$\Rightarrow \quad V_q = U_L^\dagger$$

- $U_L = 1 \quad \Rightarrow \quad V_q = D_L$

$$\Downarrow$$

$$\boxed{\text{In SM it is impossible to probe } \mathcal{M}_f}$$

Fermion mass "problem":

$$\mathcal{M}_f = ??$$

- $m_f = y_f v$

$$\Rightarrow \Gamma(h \to f\bar{f}) \propto y_f^2$$

$$\propto m_f^2$$

we can still test the origin (Higgs) of mass

• $t, b, \tau, W, Z \Leftarrow$ mass from the Higgs

$$A_\mu \bar{f}^0 \gamma^\mu Q f^0 =$$

$$= A_\mu \bar{f} \underbrace{U_f^\dagger U_f}_{1} \gamma^\mu Q f$$

• A_μ, Z_μ, h interactions are flavor diagonal

$$\Downarrow$$

$$\boxed{\text{GUT comes in to help}}$$

$$5_F = \begin{pmatrix} d \\ \hdashline e^c \\ -\nu^c \end{pmatrix}_R$$

$$10_F = \left(\begin{array}{c:cc} u^c & u & d \\ \hdashline & & e^c \end{array}\right)_L$$

$$\Downarrow$$

$$\mathcal{L}_{(X,Y)} = \left[\bar{d}_R^0 \gamma^\mu e_R^{c0} + \bar{u}_L^{c0} \gamma^\mu u_L^0 + \bar{d}_L^0 \gamma^\mu e_L^{c0} \right] X_\mu$$

$$+ \left[\bar{d}_R^0 \gamma^\mu \nu_R^{c0} + \bar{u}_L^{c0} \gamma^\mu d_L^0 + \bar{u}_L^0 \gamma^\mu e_L^{c0} \right] Y_\mu$$

- $\bar{d}_R^{\,0} \gamma^\mu e_R^{c\,0} \longrightarrow \bar{d}_R\, U_{d_R}^\dagger \gamma^\mu U_{e_R^c}\, e_R^c$

$$f_R^c \equiv C \bar{f}_L^T = C\gamma_0 f_L^*$$

$$\longrightarrow \bar{d}_R \underbrace{D_R^\dagger E_L^*}\, e_R^c$$

relative $d-e$ mixing!

- $\bar{d}_L^{\,0} \gamma^\mu e_L^{c\,0} \longrightarrow \bar{d}_L \underbrace{D_L^\dagger E_R^*}\, e_L^c$

- $\bar{u}_L^{c\,0} \gamma^\mu u_L^0 \longrightarrow \bar{u}_L^c \underbrace{U_R^T U_L} \gamma^\mu u_L(x)$

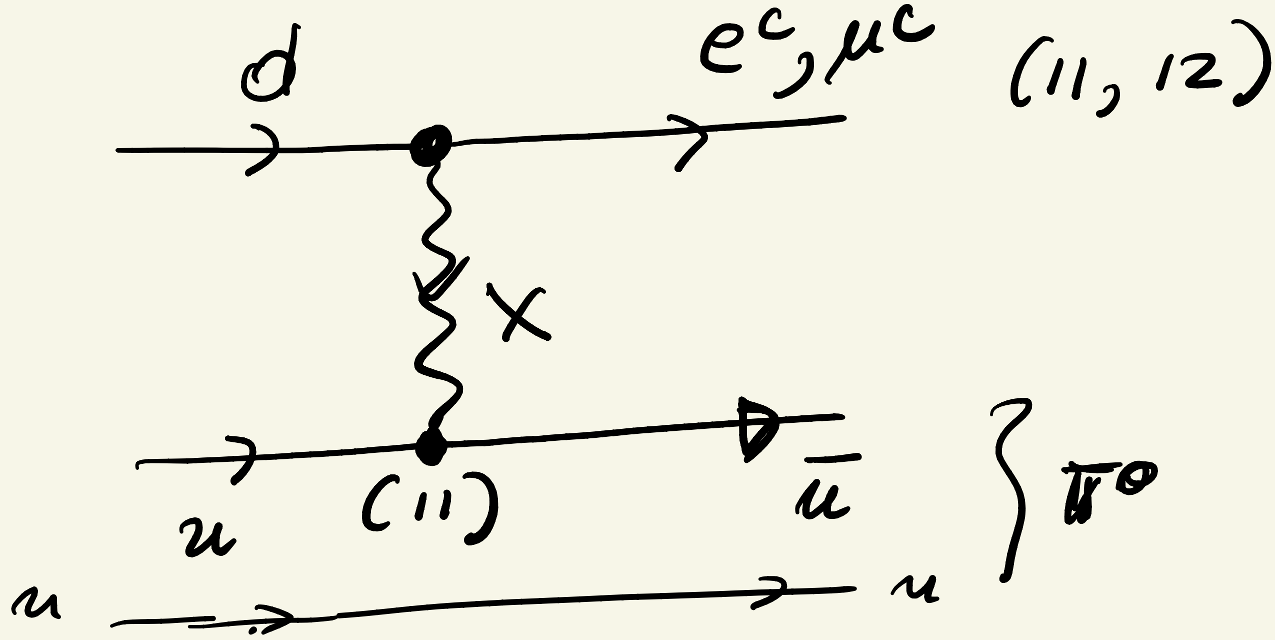
$(f_L^c = C\gamma_0 f_R^*)$ $\qquad u_L = \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}$

$$\bullet\ \overline{u^c_L}^{\,0}\, \gamma^\mu d_L^0 \to \overline{u^c_L}\, \underbrace{U_R^T D_L}\, \gamma^\mu d_L \quad (Y)$$

$$\underbrace{U_R^T U_L}_{(X)}\, \underbrace{U_L^\dagger D_L}_{V_\ell}$$

$$\overline{d_R}\, \gamma^\mu\, \underbrace{D_R^\dagger E_L^*}\, e^c_L\, X_\mu$$

$$+\ \overline{e^c_L}\, \gamma^\mu\, \boxed{E_L^T D_R}\, d_R\, \overline{X}_\mu$$



$$(ij) \longrightarrow (\text{mixing})_{ij}$$

gen gen

Can SU(5) predict the flavor (q, ℓ) mixing?

$$U_{L,R} \; ; \; D_{L,R} \; ; \; E_{L,R}$$

$$\boxed{\begin{aligned} E_L^T D_R &= ? \\ E_R^T D_L &= ? \\ U_R^T U_L &= ? \end{aligned}}$$

$\boxed{\text{YES!}}$

$$\boxed{(5_F)_R \quad (10_F)_L}$$

$$\boxed{\bar{5}_F \quad 10_F \Leftarrow \text{both } L}$$

$\cancel{\bar{5}_F \; 10_F}$ not allowed

$10 = (5 \times 5)_{AS}$

$$\mathcal{L}_Y = \bar{5}^{\,i}_{FR} \, 10_{LF_{ij}} \, Y_d \, 5_H^{*\,j} \, +$$

$i, j = 1, \ldots, 5 \qquad SU(5)$

$$+ \; 10^T_{F,L\,ij} \, C \, Y_u \, 10_{F,L} \, \text{ue} \, 5_{Hm} \, \epsilon_{ijk\,lm}$$

$\left(\bar{f}_R f_L = \text{Dirac inv.} \right)$

$\left(f_L^T C f_L = \text{Majorana inv.} \right)$

analog $SU(2)$: $D_i \, \varepsilon_{ij} \, D_j = \text{inv.}$

$D \equiv$ doublet

$SU(3)$ $T_i T_j T_k \, \epsilon_{ijk} = \text{inv.}$

$T \equiv$ triplet

$SO(N)$: $F_{i_1} F_{i_2} \cdots F_{i_N} \, \varepsilon_{i_1 \cdots i_N}$

$F =$ fundamental $= \text{inv.}$

$(\propto \det U = 1)$

$\boxed{\text{- only } Y_u, Y_d}$

in SM: $\boxed{Y_u, Y_d, Y_e,} (Y_\nu)$

$$F_L^{\dagger} M_f F_R = \widetilde{m}_f$$

$$F_{L,R} \equiv U_{f\,L,R} = \text{unitary}$$

Q. <u>Why unitary?</u>

A. To <u>preserve kinetic energy</u> !

$$\bar{f}_L^{\,0} \gamma^\mu D_\mu f_L^0 + \bar{f}_R^{\,0} \gamma^\mu D_\mu f_R^0$$

$$= \bar{f}_L^{\,0} \gamma^\mu \partial_\mu f_L^0 + (\text{int.})$$

$$+ \bar{f}_R^{\,0} \gamma^\mu \partial_\mu f_R^0 + \text{int.}$$

$$\Rightarrow \bar{f}_L \gamma^\mu \partial_\mu f_L + \bar{f}_R \gamma^\mu \partial_\mu f_R + \text{int.}$$

diagonal, orthogonal

$$\bar{f}_L^{0} \gamma^\mu \partial_\mu f_L^{0} \longrightarrow \bar{f}_L \underbrace{U_{Lf}^{+} U_{Lf}}_{1} \gamma^\mu \partial_\mu f_L$$

$$\boxed{U_{Lf}^{+} U_{Lf} = 1 = U_{Rf}^{+} U_{Rf}}$$

fermion mass matrices can be diagonalised by bi-unitary transformations

$$W^{\pm} = \frac{A_1 \mp i A_2}{\sqrt{2}}$$

$$W_\mu^+ W^{\mu -} = \frac{A_1^\mu A_{1\mu} + A^2_\mu A^{2\mu}}{2}$$

$$M_W^2 W_\mu^+ W^{\mu -} = M_W^2 \frac{A_1^2 + A_2^2}{2}$$

$$\boxed{\text{W and } A_i \text{ are mass eigenstate}}$$

$$\boxed{W^{\pm} = \text{charge eigenstates}}$$

$$\bar{u} \gamma^\mu d \, W_\mu^+$$

generates

$$SU(2) \times U(1) \longrightarrow U_{em}(1)$$

$$\underbrace{A_1,\ A_2,\ A_3}_{SU(2)} \qquad \underbrace{B}_{U(1)}$$

$$Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\bar{Q}\gamma^\mu D_\mu Q \to \bar{u}\gamma^\mu d \, \frac{(A_1 - iA_2)_\mu}{\sqrt{2}} \frac{g}{\sqrt{2}}$$

W^+

giving names = respect
the fundamental symmetries

f = fermions $s = 1/2$

h = scalar $s = 0$

A_μ = gauge bosons $s = 1$

colour $\longrightarrow$ $f = q$ or lepton

em charge $\rightarrow$ $q^i = (u, d)_i$

$\ell^i = (e, \nu)_i$

$\hookrightarrow$ $W^\pm = \dfrac{A_1 \mp i A_2}{\sqrt{2}}$

A_3, B

$\Downarrow$ mass eigenstates

(u) $\tilde{u} = u, c, t$

$\tilde{d} = d, s, b$.

$A_3, B \longrightarrow A, Z$

photon

Z boson

$\boxed{\text{unitary rotations}} =$

= keep kin. energy canonical

$\mathcal{L}_y = \bar{5}_R^{(F)} \, 10_L^{(F)} \, 5_H^{*} \, (Y_d) \leftarrow$

$10_L^{(F)} \, 10_L^{(F)} \, 5_H \quad (Y_u)$

$$\langle 5_H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ v \end{pmatrix} \begin{array}{l} \} \text{ color unbroken} \\ \\ \} \text{ by } SU(2) \end{array}$$

$$\left(\rho\, 5_H^+ \, 24_H^2 \, 5_H \; \therefore \; \rho < 0 \right)$$

$$5_R = \begin{pmatrix} d \\ e^c \\ \nu^c \end{pmatrix}_R$$

$$10 = \begin{pmatrix} u^c & u & d \\ & & e^c \end{pmatrix}_L$$

Y_d: $\langle 5_H \rangle^5 = v$

$$\bar{5}_R^{\,i} \, 10_{i5} \, v$$

$$i = \alpha = 1,2,3 \longrightarrow \bar{d}_R \, d_L \, v$$

$$i = 4 \longrightarrow \overline{e^c_R} \, e^c_L \, v$$

$$\Downarrow$$

$$v \left[\bar{d}_R^{\,0} \, Y_d \, d_L^0 + \overline{e^c_R}^{\,0} \, Y_d \, e_L^{c\,0} \right] + h.c.$$

$$= v \left[\left(\bar{d}_R^{\,0} \, \bar{s}_R^{\,0} \, \bar{b}_R^{\,0} \right) Y_d \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L^0 + \cdots \right] + h.c.$$

$\Rightarrow$ • $M_d = v\, Y_d$ $\boxed{C \equiv i\gamma_2\gamma_0}$

• $\overline{e^c_R}^{\,0} M_d\, e^{c\,0}_L =$

$= \overline{C\, \bar{e}_L^{0T}}\, M_d\, C\, \bar{e}_R^{0T} =$

$= (C\gamma_0 e_L^{0*})^{\dagger}\gamma^0 M_d\, C\, \bar{e}_R^{0T} =$

$= e_L^{0T}\gamma_0 C^{\dagger}\gamma^0 M_d\, C\, \bar{e}_R^{0T} =$

$= e_L^{0T}(-C^{\dagger}C)\, M_d\, \bar{e}_R^{0T} =$

$(-1\ 1$

$= + \ \bar{e}_R^{0} M_d^T e_L^{0}$

$$\boxed{M_e = M_d^T}$$

$\Downarrow$

• $m_e = m_d$???

$\Downarrow$

$m_b = m_\tau$ wrong??

$m_s = m_\mu$

$m_d = m_e$

$$D_L^\dagger M_d D_R = \tilde{m}_d \quad (1)$$

$$\left(E_L^\dagger M_e E_R = \tilde{m}_e \right)^T \quad (2)$$

$$\Rightarrow E_R^T M_e^T E_L^* = \tilde{m}_e$$

$$\Rightarrow \boxed{E_R^T = D_L^\dagger, \quad D_R = E_L^*}$$

$$X_\mu \left[\bar{d}_R^{\,0} \gamma^\mu e_R^{c\,0} + \bar{d}_L^{\,0} \gamma^\mu e_L^{c\,0} \right]$$

$$\Big\downarrow \qquad f_R^c \propto f_L^*, \quad f_L^c \propto f_R^*$$

$$X_\mu \left[\bar{d}_R \, D_R^\dagger E_L^* e_R^c + \bar{d}_L \, D_L^\dagger E_R^* e_L^c \right]$$

$$\begin{array}{cc} \| & \| \\ (D_R^\dagger D_R) & (D_L^\dagger D_L) \\ \| & \| \\ \mathbb{1} & \mathbb{1} \end{array}$$

$$\Downarrow$$

$$\boxed{\underline{\text{no}} \text{ mixing}}$$

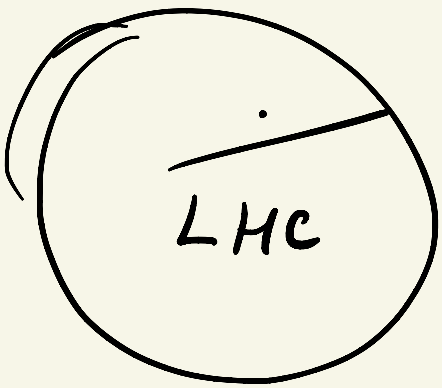
$$X_\mu \left[(\bar{d}_R \; \bar{s}_R \; \bar{b}_R) \gamma^\mu (\mathbb{1}) \begin{pmatrix} e_R^c \\ \mu_R^c \\ \tau_R^c \end{pmatrix} + \cdots \right.$$

$$\underbrace{\bar{d}_R \gamma^\mu e^c_R} + \cancel{\bar{s}_R \gamma^\mu \mu^c_R} + \cdots$$

$\boxed{\text{Only positron}}$

'1950

Fermi



$\boxed{R_{Fermi} = 6000\,\text{km}}$

· Size of new collider to produce X ($M_X \simeq 10^{16}$ GeV)?

$$\left[\overline{u^c_L}\,\gamma^\mu u_L\right] X_\mu$$

$$u^c_L \propto u_R^*$$

$$\overline{u^c_L}\left(U_R{}^T U_L\right) u_L$$

flow

$$10_F\ 10_F\ 5_H$$

$$\varepsilon_{ijklm}\ 10_{F_{ij}}^T\ C\ Y_u\ 10_{F_{kl}}\ 5_{H\,m}$$

$$C^T = -C$$

$$\Rightarrow \boxed{Y_u^T = Y_u}$$

$$\Rightarrow \boxed{M_u = M_u^T}$$

$$U_R^\dagger M_u U_L = \tilde{m}_u$$

$$\parallel$$

$$U_L^T M_u^T U_R^* = \tilde{m}_u$$

↑ diagonal

$$\Longrightarrow \boxed{U_R^\dagger = U_L^T} \Rightarrow U_R^T = U_L^\dagger$$

- $\overline{u^c_L}\, \gamma^\mu\, \underbrace{U_R^\dagger U_L}\, u_L\, X^\mu$

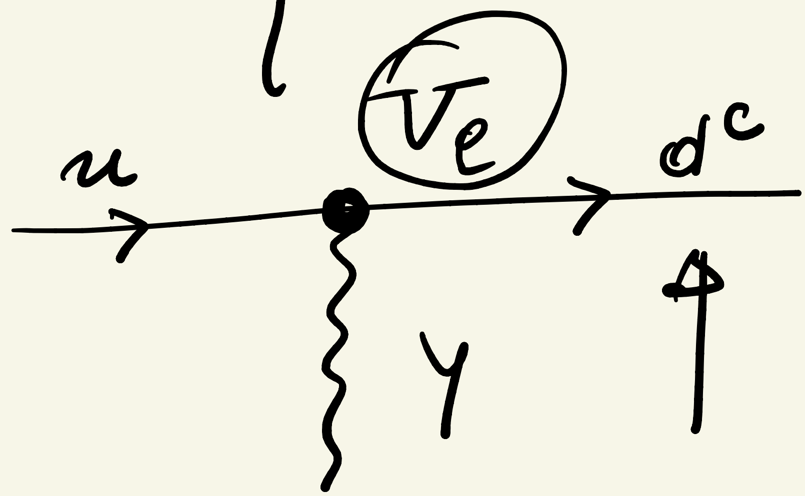
$$\boxed{U_L^\dagger U_L = 1}$$

$$\boxed{\underline{\text{no}}\ \text{mixing}}$$

- $\overline{u^c_L}^{\,0} \gamma^\mu d_L^0 Y_\mu =$

$$= \overline{u^c_L}\; U_R^T \underbrace{(U_L U_L^\dagger)}_{=1} D_L\, d_L\, Y_\mu$$

$= \bar{u}^c_L V_Q d_L \varphi_u$



$d^c \cos\theta_c$

$s^c \sin\theta_c$

d^c: $p \to \pi^0 + e^c$

s^c: $p \to K^0 + e^c$

$\Downarrow$

SU(5) they predicts

all the proton decay
rates

$$\boxed{= f(V_\varepsilon)}$$

<u>SM</u> + decoupling

any new physics $\propto \left(1/M_{new}\right)$

$SU(5)$: $M_{new} = M_X \simeq 10^{16}$ GeV

$\Gamma \propto \left(\frac{1}{M_X^{4}}\right) \Rightarrow$ X decouples

in everything — <u>but</u>

$\boxed{\text{proton decay}}$

$$\boxed{\tau_p \gtrsim 10^{34}\,\text{yr (expl)}}$$

($\tau_p < 10^{-6}$ sec naive!)

<u>rare decays</u>

$$K \to \mu \bar{e} \quad (B \sim 10^{-9})$$

$$\left(K - \bar{K} \text{ mixing} \quad \frac{m_K - m_{\bar{K}}}{m_K + m_{\bar{K}}} \simeq 10^{-15} \right)$$

(X, Y) do not matter

- $m_e = m_d$ — tree level

Q — at which scale ?

A. At $M_{GUT} = M_X$!

$\alpha_2 = \alpha_3 \Leftarrow$ at M_{GUT}

$\Downarrow$

$$\boxed{m_e = m_d \text{ at } M_{GUT}}$$

$\Downarrow$

at M_W: $\boxed{m_d \simeq 3\, m_e \Big|_{M_W}}$

$$\boxed{m_b \simeq 3\, m_\tau \quad \text{good!}}$$

$m_s \simeq 3\, m_\mu$ <u>not good</u>

$m_d \simeq 3\, m_e$ <u>not good</u>

$$\frac{g}{2}\,\frac{m_f}{M_W} \sim y_f \ll 1$$

for 1^{st}, 2^{nd} gen.

SM $\rightarrow$ predictions

SU(5) $\longrightarrow$ more predictions

more symmetry

iff $\underline{\text{minimal } SU(5)}$

$\underline{SO(10)} \supseteq SU(5)$

$10_H = 5_H + \bar{5}_H$

$\Downarrow$

$$\boxed{\begin{aligned} m_e &= m_d \\ m_D^{(\nu)} &= m_u \end{aligned}}$$

$\Rightarrow$ $\boxed{\text{predictions for } p \text{ decay}}$