LMU GUT Course

Lecture XVIII

15/1/2021

UNIFICATION

SU(5) + generic feature

"running" $(\mu \leq E_1)$

$$\frac{1}{\alpha(E_2)} = \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln E_2/E_1$$

$$\Rightarrow \quad b = \frac{11}{3} T_{GB} - \frac{2}{3} T_F - \frac{1}{3} T_S \text{ (complex)}$$

(real $\rightarrow$ 1/2)

$$T\delta_{ab} = \mathrm{Tr}\, T_a T_b$$

$$\left.\begin{aligned} T(\mathrm{Fund}) &= 1/2 \\ T(\mathrm{Adj}) &= N \end{aligned}\right\} \quad SU(N)$$

$$\boxed{b_3 = \frac{33}{3} - \frac{4}{3} n_g} - \cancel{0} \; (>0)$$

$$b_2 = \frac{22}{3} - \frac{4}{3} n_g - \frac{1}{6} n_H$$

$$b_1 = 0 - \frac{4}{3} n_g - \frac{1}{10} n_H$$

$\boxed{SM}$

<u>SU(5)</u> : (x, y) gauge bosons

24_H : $\Sigma_3, \Sigma_8 \; (Y=0)$

5_H : $T \, (3_c, 1_w, Y = -2/3)$

- $M_U = M_{GUT} \gtrsim M_X \, (= M_Y)$

(i) <u>desert</u> $\quad m_3 = m_8 = m_T = M_X$

only SM : $M_W \le E \le M_X$

$$\left.\begin{aligned}
\frac{1}{\alpha_U} &= \frac{1}{\alpha_3(M_W)} + \frac{b_3}{2\pi}\ln\frac{M_X}{M_W} \\
\frac{1}{\alpha_U} &= \frac{1}{\alpha_2(M_W)} + \frac{b_2}{2\pi}\ln\frac{M_X}{M_W} \\
\frac{1}{\alpha_U} &= \frac{1}{\alpha_1(M_W)} + \frac{b_1}{2\pi}\ln\frac{M_X}{M_W}
\end{aligned}\right\}\ \underline{\underline{\text{UNIFY}}}$$

$$\alpha_U = \alpha_3(M_X) = \alpha_2(M_X) = \alpha_1(M_X)$$

$$\left(\frac{1}{\alpha_1} - \frac{1}{\alpha_2}\right)\Bigg|_{M_W} = \frac{b_2 - b_1}{2\pi}\ln\frac{M_X^{(12)}}{M_W} \qquad (**)$$

$$\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_3}\right)\Bigg|_{M_W} = \frac{b_3 - b_2}{2\pi}\ln\frac{M_X^{(23)}}{M_W} \qquad (*)$$

$$\alpha_i \equiv \alpha_i(M_W)$$

$$(*) \quad \ln \frac{\mu_X}{\mu_W} = 2\pi \left(\frac{1}{\alpha_2} - \frac{1}{\alpha_3} \right) \frac{1}{b_3 - b_2}$$

$$\alpha_3(\mu_W) \simeq 1/10 \qquad \alpha_2(\mu_W) \simeq 1/30$$

small

$$\alpha_{em}(\mu_W) \simeq 1/130$$

$$g' = \sqrt{\frac{3}{5}}\, g_1 \Rightarrow \alpha_1 = \frac{5}{3}\alpha'$$

$$\sin^2\theta_W = \frac{\alpha_{em}}{\alpha_2} \qquad \cos^2\theta_W = \frac{\alpha_{em}}{\alpha'}$$

$$\alpha_1 = 5/3 \frac{\alpha_{em}}{\cos^2\theta_W} = \#$$

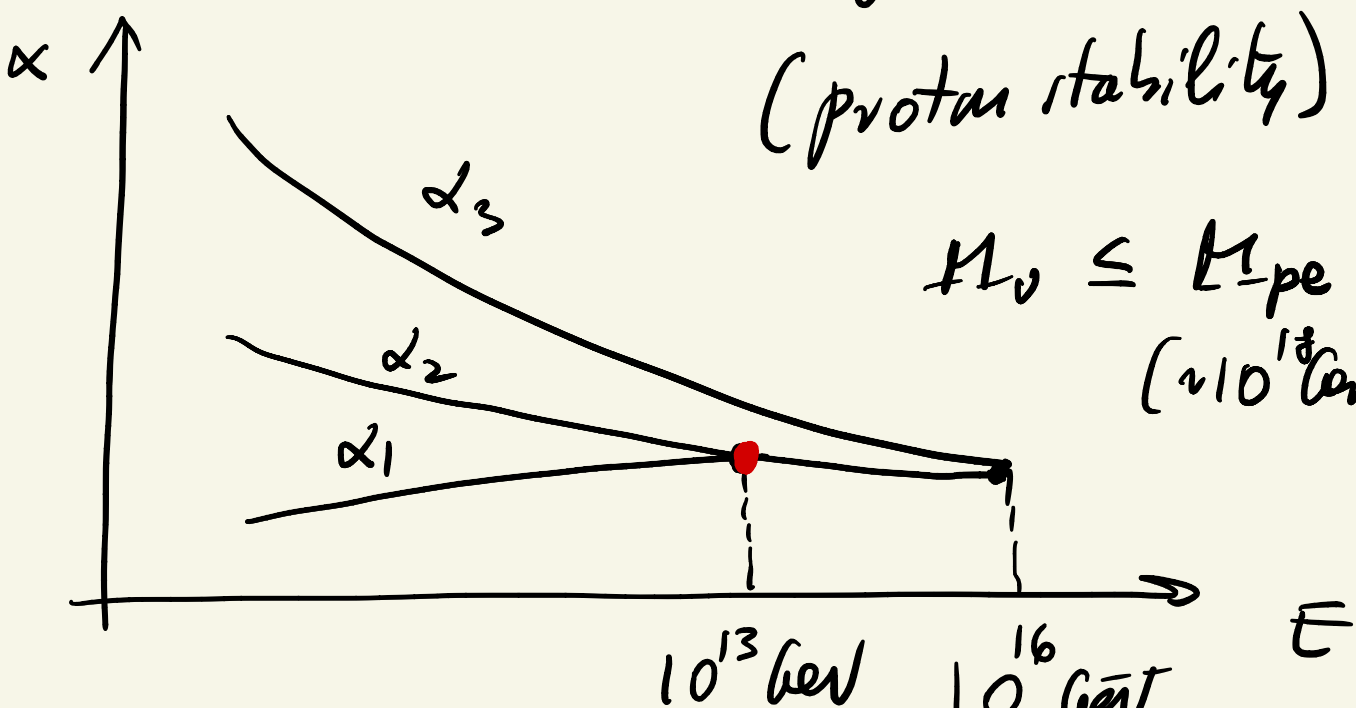
$(*) \longrightarrow$ plug into $(**)$

$$\boxed{\left(\frac{1}{\alpha_1}-\frac{1}{\alpha_2}\right)=\frac{b_2-b_1}{b_3-b_2}\left(\frac{1}{\alpha_2}-\frac{1}{\alpha_3}\right)}$$

"desert"

$M_U > 10^{15}$ GeV
(proton stability)

$M_U \leq M_{pe}$
($\sim 10^{18}$ GeV)



$$\boxed{\text{Desert picture} = \text{wrong}}$$

· lesson 1

SU(5) $\Rightarrow$ $\boxed{\text{slow down } \alpha_2}$

Σ_3 = triplet of $SU(2)$ (1_c)
(3_W)

$$\Rightarrow \boxed{m_3 \ll M_X}$$

ideal : $m_3 \simeq M_W$

$$\boxed{\text{It does not work}}$$

$$\frac{1}{\alpha_2(m_3)} = \frac{1}{\alpha_2(M_W)} + \frac{b_2^{(SM)}}{2\pi}\ln\frac{m_3}{M_W}$$

$$\frac{1}{\alpha_2(M_X)} = \frac{1}{\alpha_2(m_3)} + \frac{\bar{b}_2}{2\pi}\ln\frac{M_X}{m_3}$$

$$\bar{b}_2 = b_2(SM) - \frac{1}{3}\cdot\frac{1}{\cancel{2}}\cdot\cancel{2}$$

" $2/6$

$$b_2 = \frac{22}{3} - 4/3\, y - 1/6$$

$$= 10/3 - 1/6 = 19/6$$

$$\boxed{\text{Explains the failure}}$$

70's — 1991 (LEP) 27km

$e\bar{e}$

$$\sin^2\theta_W^{exp} = 0.2$$

WRONG

slow down α_2 !!!

$$T = \begin{pmatrix} SU(2) \text{ singlet} \\ Y/2 = 1/3 \end{pmatrix}$$

$$\Rightarrow \boxed{m_T \gtrsim M_X}$$

$$\frac{1}{\alpha_3(m_8)} = \frac{1}{\alpha_3(M_W)} + \frac{b_3}{2\pi} \ln \frac{m_8}{M_W}$$

$$\frac{1}{\alpha_3(M_X)} = \frac{1}{\alpha_3(m_8)} + \frac{\bar{b}_3}{2\pi} \ln \frac{M_X}{m_8}$$

$$\bar{b}_3 = b_3 - \underbrace{\frac{1}{3} \cdot \frac{1}{2} \cdot 3}_{1/2}$$

$$b_3 = \frac{33}{3} - \frac{4}{3} n_g = 7$$

$$\boxed{SU(5) \text{ is ruled out}}$$

- NO unification
- $m_\nu = 0$ (later)

24_H = real $\quad (24_H^{+} = 24_H)$

more symmetry (global)

$$24_H \to e^{i\alpha}\, 24_H$$

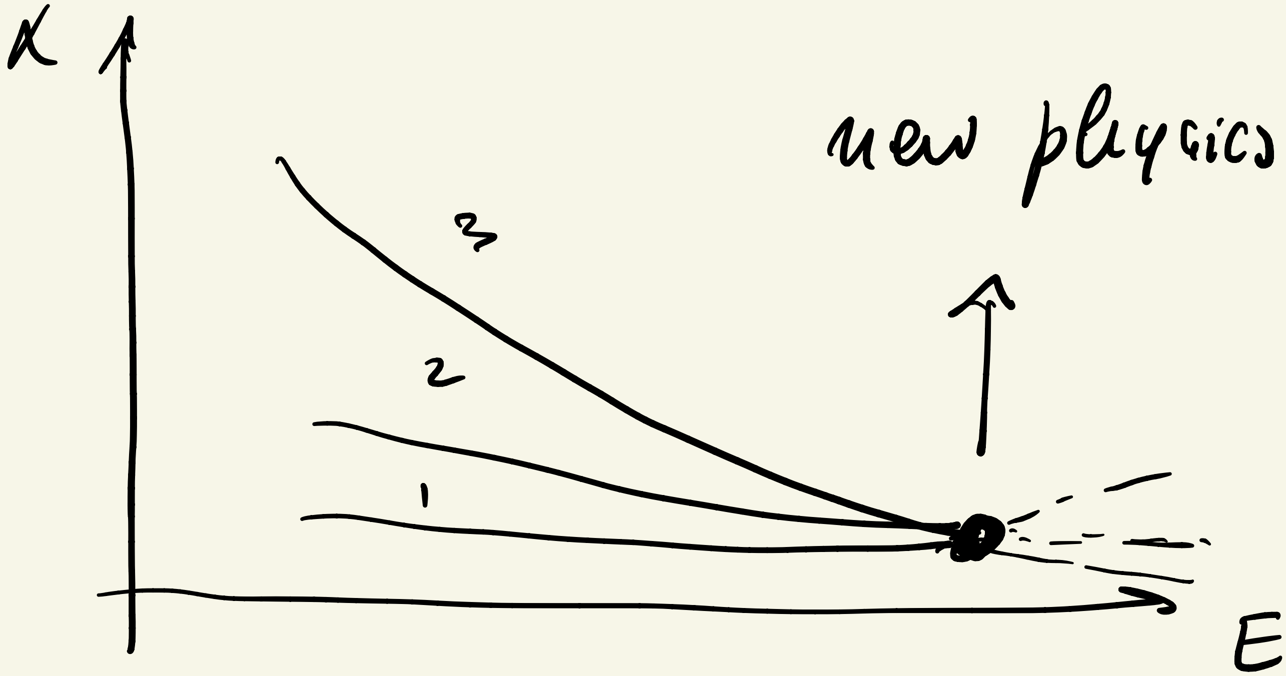
$\Leftrightarrow$ 24_H = complex

$\Leftrightarrow$ 2 such 24_H

$\Rightarrow$ factor of 2!

$\boxed{\text{NO !!}}$

SU(5) ! ruled out in all of parameter space



do couplings stay unified above M_X?

YES : above M_X : all run!

b^{unif} : $(X, Y)^\alpha + (\bar{X}, \bar{Y})^\alpha$

$\Sigma_3, \Sigma_8, \Sigma_X^\alpha, \Sigma_Y^\alpha$ in 24_H

new states

T in 5_H

$\alpha = r, y, b$

$$\underline{b_3^{\text{unit}}} = \underbrace{\underset{\overset{\shortparallel}{\frac{33}{3}}}{b_3} + \frac{11}{3}\cdot\cancel{\frac{1}{2}\cdot 2}\cdot 2}_{\text{gauge bosons}} = \frac{55}{3} \checkmark \qquad T_x = \frac{1}{2}$$

GB: $\frac{11}{3}\cdot N = \frac{11}{3}\cdot 5$ (in $SU(5)$)

$$b_2^{\text{unit}} = \underbrace{\underset{\overset{\shortparallel}{22/3}}{b_2} + \frac{11}{3}\cdot\cancel{\frac{1}{2}}\cdot 3\cdot\cancel{2}}_{\text{gauge bosons}} = \frac{33}{3} = \frac{55}{3} \checkmark$$

<u>scalars</u>

$$b_3^{\text{unit}}(\text{scalar}) = b_3 - \left(\frac{1}{3}\right)\underset{\uparrow\ \Sigma = \text{real}}{\frac{1}{2}}\underbrace{\left[3 + \frac{1}{2}\cdot 2\cdot 2\right]}_{5/2}$$

$$b_2^{\text{mit}}(\text{scalar}) = b_2 - \frac{1}{3}\cdot\frac{1}{5}\underbrace{\left[2 + \frac{1}{2}\cdot 3\cdot 2\right]}_{5/2}$$

add T $\Rightarrow$ same contribution to all couplings (with ϕ)

affects α_3, α_1

$\boxed{\text{HW: Complete}}$

$$b_1 = \ldots \underbrace{4/3\, n_g}_{-2/3\, T_F}$$

F: $5_F + 10_F$ (AS)

$$\boxed{\begin{matrix} T(5) = \frac{1}{2} \\ T(10) = ? \quad T(AS) \end{matrix}}$$

$SU(N)$ $\quad N \times N = S + AS$

$\# \quad \frac{N(N+1)}{2} \qquad \frac{N(N-1)}{2}$

Hint: $\underbrace{SU(2) \text{ and } SU(3)}_{\text{guess } S,\, AS}$

SU(2)

$2 \times 2 = 3 + 1 \Rightarrow S = 3, \ AS = 1$

$2 \times \bar{2} = 3 + 1 \qquad T(1) = 0$

adjoint

$\Downarrow$

$SU(N)$: $T(AS) \propto N-2$

$SU(3)$

$$3 \times \bar{3} = 8 + 1$$

Adjoint $(T = 3)$

$$3 \times 3 = 6 + 3^* \qquad T(3^*) = \frac{1}{2}$$

$$6 = \frac{3 \cdot 4}{2} \qquad 3^* = \frac{3 \cdot 2}{2}$$

$$\Downarrow$$

$$\boxed{T(AS) = \frac{N-2}{2}}$$

Prove!!

$$\boxed{T(S) = \frac{N+2}{2}}$$

$$\boxed{\begin{aligned} T(S) + T(AS) &= \\ &= N \\ &= T(\text{Adjoint}) \end{aligned}}$$

$\Downarrow$ Proof

$$N \times N = S + AS$$

- $T(N \times N) = N\,T(N) + T(N)\,N$

$$= N\tfrac{1}{2} + \tfrac{1}{2}N = N$$

ADJOINT

- $T(N \times \bar{N}) = N\,T(\bar{N}) + T(N)\,(\bar{N} = N)$

$$= N\tfrac{1}{2} + \tfrac{1}{2}N = N$$

$$\boxed{T(\bar{N}) = T(N) = 1/2}$$

$$N \times \bar{N} = \underbrace{\text{Adjoint}}_{N^2-1} + 1$$

$$\boxed{\begin{aligned} T(1) = 0 \Rightarrow T(\text{Adj}) &= \\ = T(N \times \bar{N}) &= N \end{aligned}}$$

Q.E.D.

Check SU(5)

$$5_F + 10_F \quad (\text{gen})$$

$$T: \quad 1/2 + \frac{5-2}{2} = 3/2 = 2$$

$$-2/3\, T_F = -4/3\, n_g$$

SU(5): CONCLUSION

(i) all possible basis

$\Rightarrow$ NO unif.

(ii) above M_X!

$$b_2 = b_3 = b_1 = b_5$$

Low energy supersymmetry

LHC: SM = Higgs, top --

+ exotics = BSM

beyond

+ MSSM

Minimal Supersymmetric Standard Model

$p \Longrightarrow \tilde{p}$ (sparticle)

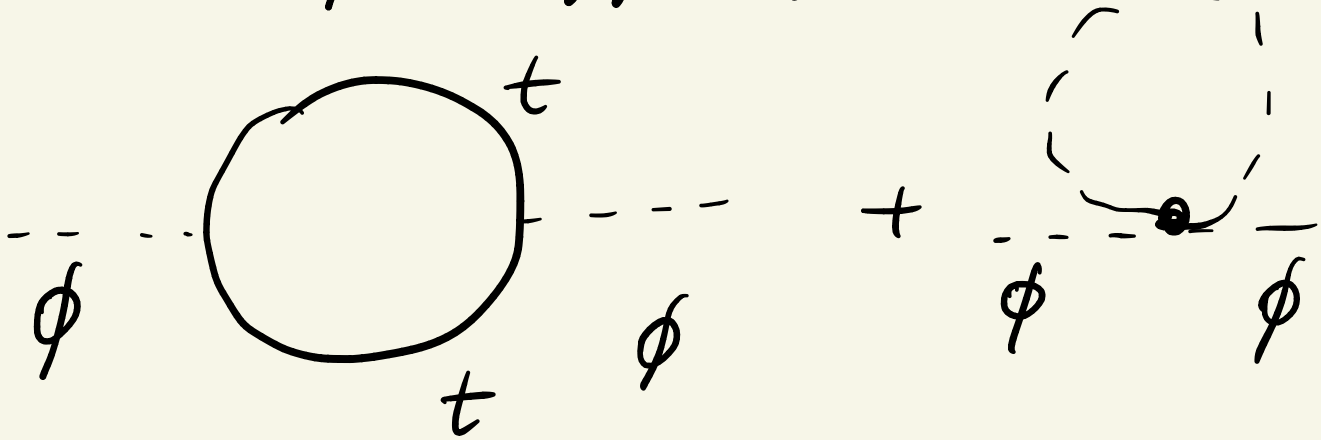
$W \longrightarrow \tilde{W}$ (gaugino)

$f \longrightarrow \tilde{f}$ (sfermion)

$h \longrightarrow \tilde{h}$ (Higgsino)

$$\boxed{\begin{aligned}\Lambda_{SS} &\simeq \mathrm{TeV}\\ m_{\tilde{p}} &\leq \mathrm{TeV}\end{aligned}}$$

keep Higgs light



$$\boxed{m_\phi^{2(1)} = \frac{y_t^2}{16\pi^2}\left(m_t^2 - m_{\tilde{t}}^2 \cancel{+A^2} \cancel{-A^2}\right)}$$

$$\boxed{\text{SS}: \quad \text{couplings} = \text{same}}$$

$\boxed{\text{SS GUT}}$ $\boxed{\text{'70-'80s}}$

$\boxed{\text{"desert"}}$

$$b = \frac{11}{3} T_{GB} - 2/3\, T_F - 1/3\, T_S$$

$$\Downarrow$$

$$b^{SS} = \Big(\underbrace{\frac{11}{3}}_{\text{gauge boson}} - \underbrace{\frac{2}{3}}_{\text{gaugino}}\Big) T_{GB}$$

$$- \Big(\underbrace{\frac{2}{3}}_{\text{fermion}} + \underbrace{\frac{1}{3}}_{\text{sfermion}}\Big) T_F - \Big(\underbrace{\frac{1}{3}}_{\text{Higgs}} + \underbrace{\frac{2}{3}}_{\text{Higgsino}}\Big) T_S$$

quark $\longrightarrow$ squark

$$\begin{pmatrix} u \\ d \end{pmatrix}_L^{\alpha} \longrightarrow \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L^{\alpha}$$ same quantum number

$s = 1/2$ $\qquad$ $s = 0$

$$u_R^{\alpha} \longrightarrow \tilde{u}_R^{\alpha}$$

$$e_R \longrightarrow \tilde{e}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix} \longrightarrow \begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix} \Longleftrightarrow \text{Higgs}$$

lepton doublet $\qquad$ slepton doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\tilde{\Phi} = i\sigma_2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

$$b^{SS} = 3T_{GB} - T_F - T_S$$

$(q, l + \bar{q}, \tilde{l})$ $\quad (h + \tilde{h})$

$\boxed{\text{SS - less AF}}$

$$b_3^{SS} = 3 \cdot 3 - 2n_g - 0 = 3$$

no scalars

$$b_2^{SS} = 3 \cdot 2 \cancel{- 2n_g} - \frac{1}{2} n_H < 0$$

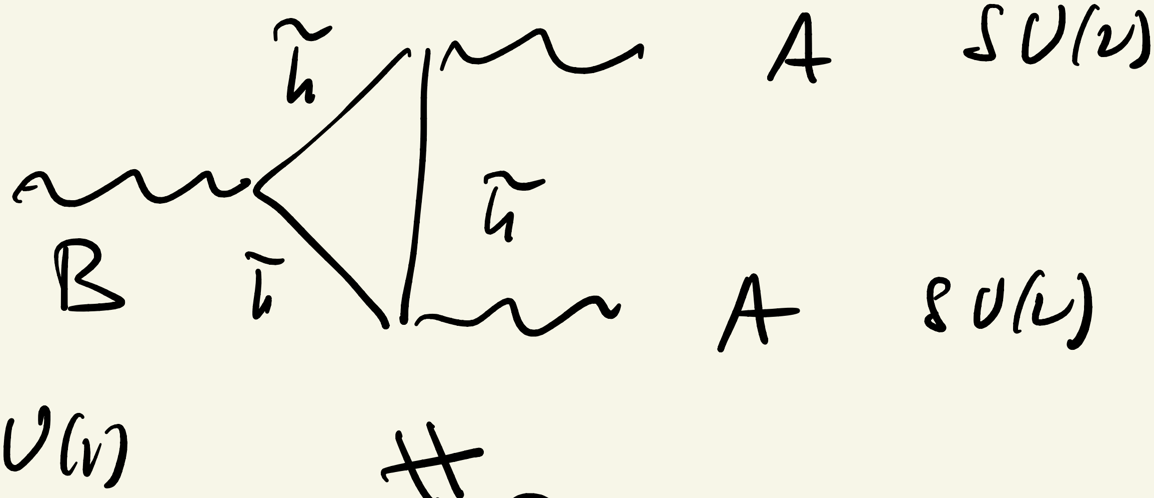
$$b_1^{SS} = -2n_g - \frac{3}{10} n_H$$

side comment: $\boxed{n_H = 2}$

why? ~~Anomalies~~

$$h \to \tilde{h} \quad (Y = +1)$$

↑ anomaly



$$\neq 0$$

$n_H = 2$ $\quad \boxed{\text{anomaly} = 0}$

$$(\sin^2\theta_W)_{exp} = 0.20 \qquad '1981$$

$$\frac{1}{\alpha_1} - \frac{1}{\alpha_2} = \frac{b_2 - b_1}{b_3 - b_2}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_3}\right)$$

$$\sin^2\theta_W = \frac{\alpha_{em}}{\alpha_2} = 1 - \frac{\alpha_{em}}{\alpha_1}$$

$$\boxed{\sin^2\theta_W = f(\alpha_{em}, \alpha_3)}$$

$$\boxed{SS \Longrightarrow \sin^2\theta_W = 0.23}$$

$$\sin^2\theta_W^{exp} = 0.2 + (\rho - 1)$$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

tree

$$\rho = 1 +^{?} \frac{\alpha}{\pi} \frac{m_t^2 - \cancel{m_b^2}}{M_W^2} \nearrow$$

- $m_t \simeq 200\,\mathrm{GeV}$

Marciano, G.S 1981

$\boxed{\text{all knew in 1981! } m_t \leq 20\,\mathrm{GeV}}$

- 1991 LEP $\Rightarrow \sin^2\theta_W = 0.23$

agrees with SS

- 1996 $m_t \simeq 175\,\mathrm{GeV}$

- MSSM
  - Higgs $\Rightarrow -m_\phi^2$
  - hierarchy

• unification

$\Rightarrow$ $\boxed{\text{30 years desperately looking for SS}}$

How to quantify $\Lambda_{SS} = ?$

• $m_\phi^2 = m_0^2 - \frac{y_t^2}{16\pi^2} m_{\tilde{t}}^2$

$\Rightarrow m_{\tilde{t}} \leq \text{TeV}, 2\,\text{TeV}$

$5\,\text{TeV}$??

FT ugly – how ugly?

• unif.

Michael on Monday!

LHC: SS at TeV

$\Lambda_{SS} = 5$ TeV — OK for unification!

100 TeV machine:

$\Lambda_{SS} \simeq 20$-30 TeV

unification!!

desert: $m_3 = m_8 = M_X$