LMU GUT Course

Lecture XVII

12/1/2021

SU(5): Predictions

1. Unification

2. proton decay —

   branching ratios

3. $q + \ell$ = together $\Rightarrow$

   correlations between m_q, m_ℓ and mixings

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1. Unification:

   scale $M_U \equiv M_{GUT} \equiv M_X$

   $\shortparallel$

$$\Rightarrow g_1 = g_2 = g_3 \equiv g \equiv g_5$$

$$\underset{U(1)}{\uparrow} \qquad \underset{SU(2)}{\uparrow} \qquad \underset{SU(3)}{\uparrow}$$

$$\Downarrow$$

$$D_\mu = \partial_\mu - ig\, T_a A_\mu^a \qquad a = 1, \dots, 24$$

$$= \dots - ig_1 T_{24} A_\mu^{24} \leftarrow$$

$$= \dots - ig' \frac{Y}{2} B_\mu$$

$$Q_{em} \equiv T_3 + \frac{Y}{2} \quad (\text{def})$$

B_μ and A_μ^{24} – normalised canonically

$$\Rightarrow \boxed{A_\mu^{24} = B_\mu}$$

$$T_{24} = \sqrt{\frac{3}{5}} \begin{pmatrix} -1/3 & & & & 0 \\ & -1/3 & & & \\ & & -1/3 & & \\ & & & 1/2 & \\ 0 & & & & 1/2 \end{pmatrix}$$

$$\boxed{\mathrm{Tr}\, T_a T_b = \frac{1}{2}\delta_{ab}}$$

$$\Rightarrow \boxed{T_{24} = \sqrt{\frac{3}{5}}\,(Y/2)}$$

$$5_F = \begin{pmatrix} d \\ - - - \\ e^{c} \\ -\nu^c \end{pmatrix}_R$$

$$g_1 T_{24} = g' \, Y/2 \Rightarrow \boxed{g' = \sqrt{\frac{3}{5}}\, g_1}$$

- $g_1 = g_2$ at M_U

$$\tan^2\theta_W = \frac{g'^2}{g_2^2}$$

$$\Downarrow$$

$$\tan^2\theta_W = \frac{3}{5}\,\frac{g_1^2}{g_2^2}$$

$$\Downarrow$$

$$\boxed{\tan^2\theta_W\,(M_U) = \frac{3}{5}}$$

$$\Downarrow$$

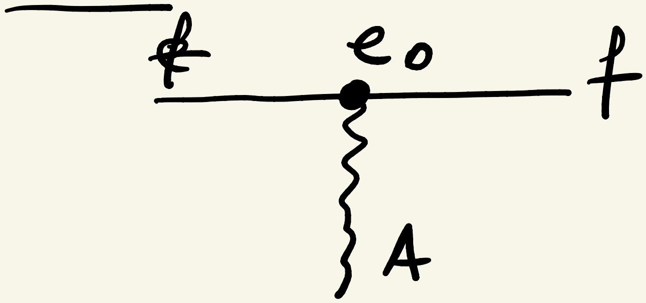
$$\sin^2\theta_W\Big|_{M_U} = 3/8\ ,\quad \cos^2\theta_W\Big|_{M_U} = 5/8$$

- $\sin^2\theta_W\Big|_{M_Z} = 0.23 \quad (\text{exp})$

Q. How to unify?

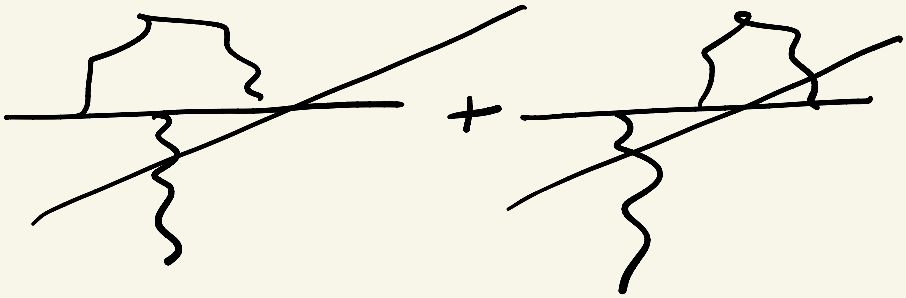
A. Couplings "run" with energy

QED



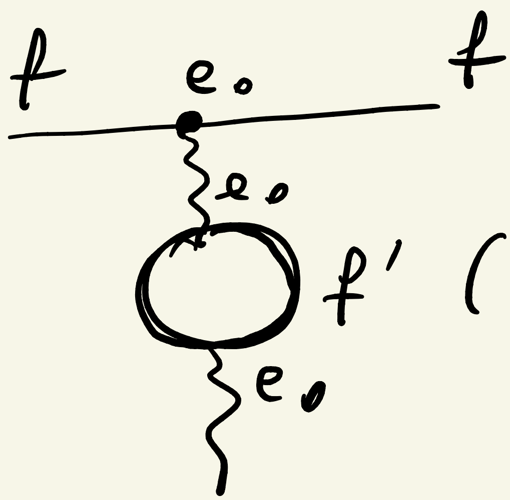
tree - level

$$e = e_0$$

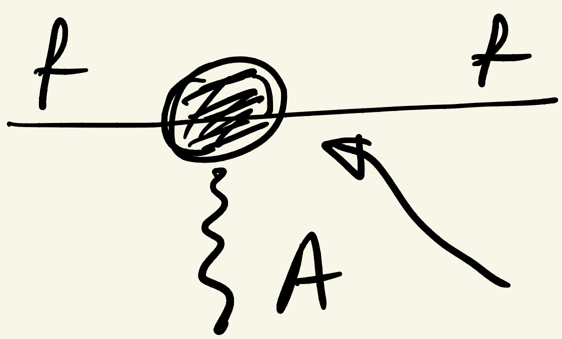


$= 0$

loop



(every charged f)

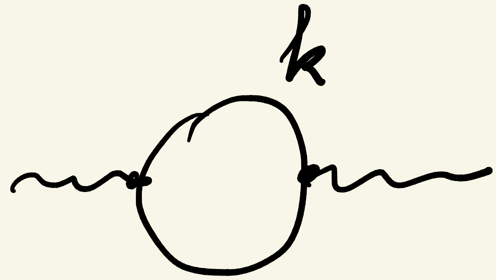


= tree + loop

charge e

$$\underset{\text{tree}}{e_0} \quad \underset{\text{loop}}{e_0^3 a}$$

$$e = e_0 + e_0^3 a$$



$$\int d^4k \, \frac{1}{\not k - m_f} \, \frac{1}{\not k - m_f}$$

$$= \int_0^\Lambda k^2 dk^2 \; \cdots$$

$$\propto \ln \Lambda$$

$$\Rightarrow \quad e = e_0 + e_0^3 \, b \ln \Lambda/?$$

mass scale = energy of the experiment

$$\boxed{e(E) = e_0 + e_0^3 \, b \ln \Lambda/E}$$

- $\Lambda \to \infty$

$$\left.\begin{aligned} e(E_2) &= e_0 + e_0^3\, b \ln \Lambda/E_2 \\ e(E_1) &= e_0 + e_0^3\, b \ln \Lambda/E_1 \end{aligned}\right\} \; -$$

$$\Downarrow$$

$$e(E_2) = e(E_1) + e_0^3\, b \ln E_1/E_2$$

$$\alpha(E_2) = \alpha(E_1) + \alpha_0^2\, c \ln E_1/E_2$$

$$\alpha \equiv e^2/4\pi$$

couplings "run" (crawl) with energy

divide by $\frac{1}{\alpha(E_2)\,\alpha(E_1)}$

$$\Downarrow$$

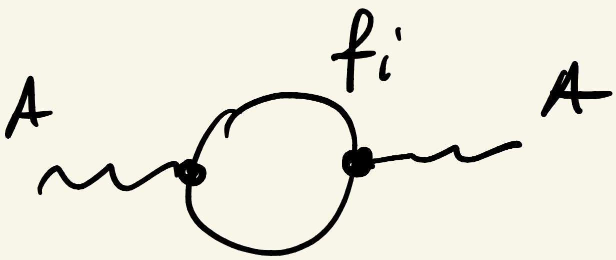
$$\frac{1}{\alpha(E_1)} = \frac{1}{\alpha(E_2)} + \frac{\alpha_0^2}{\alpha(E_2)\,\alpha(E_1)}\, c \ln E_1/E_2$$

$$\Downarrow \qquad\qquad \begin{array}{c} \| \\ 1 + O(\alpha_0^{\cdots}) \end{array}$$

$$\boxed{\frac{1}{\alpha(E_2)} \simeq \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln \frac{E_2}{E_1}} \quad (1)$$

Compute b = Holy Grail

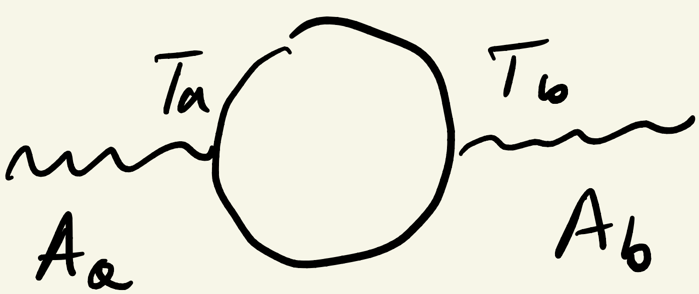
QED



$$\sum q_i^2 = \mathrm{Tr}\, Q_{em}^2$$

$\uparrow$ charges

Yang - Mills



$$\Downarrow$$

$\propto \mathrm{Tr}\, T_a T_b \equiv \frac{1}{2} f_{ab}\, T(R)$

representation

$$\boxed{\cdot \text{ pert. in } \alpha \equiv e^2/4\pi = \text{small}}$$

$\cdot$ From (1) $\Rightarrow$ $\boxed{b > 0 \text{ implies } \alpha \searrow}$

as $E \to \infty \Rightarrow \alpha \to 0$

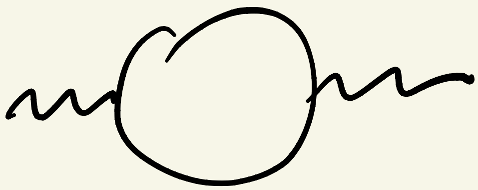
$\boxed{\text{Asymptotic Freedom}}$

$b < 0 \Rightarrow \alpha \nearrow$

$E \to \infty \Rightarrow \alpha \to \infty$

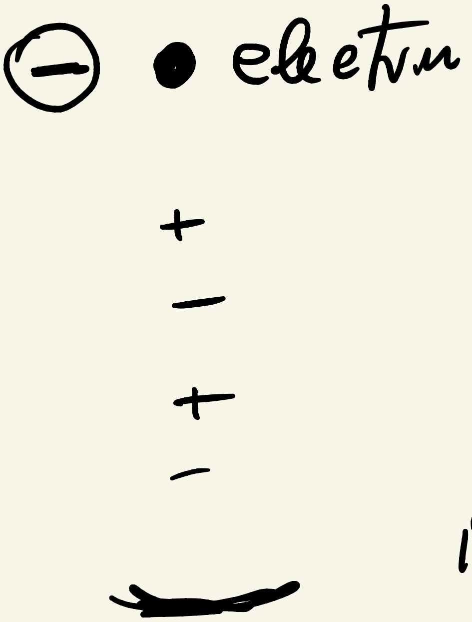
$\cdot$ When do we get AF?

QED

 $\propto \mathrm{Tr}\, Q^2$

$\Rightarrow \; b_{QED} < 0 \Rightarrow \alpha_{em} \uparrow$ with $E \nearrow$

$\alpha_{em} = \infty$ at $E \simeq 10^{120}$ GeV



increase distance =

= less charge

charge screening

large $d \Rightarrow \alpha_{em} \uparrow$
(small E)

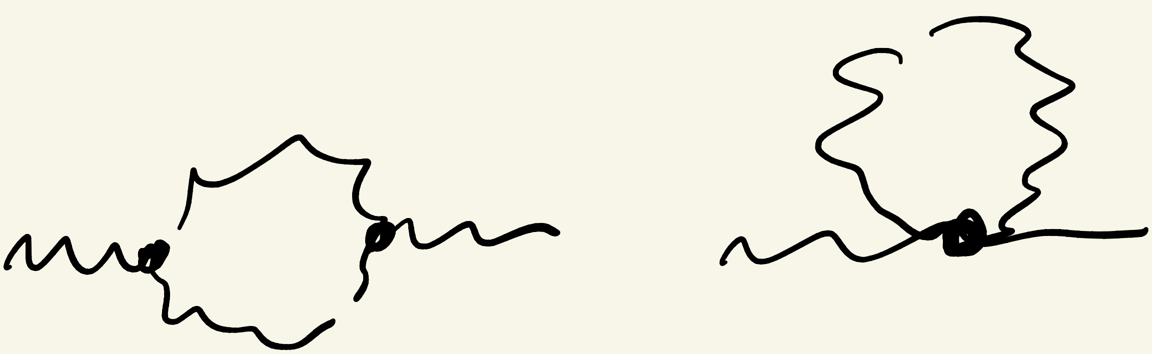
$$\boxed{\text{QED} \neq \text{AF}}$$

- <u>Yukawa</u>

$\bar{\psi}\psi\phi \Rightarrow$ the same

NOT AF

However, in YM (non Abelian)



$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c$$

$$F^a_{\mu\nu} F^{\mu\nu}_a \rightarrow A^3 + A^4$$

$$\Downarrow$$

$$\boxed{b_{YM} > 0}$$

$$\Downarrow$$

$$\boxed{\frac{1}{\alpha(E_2)} = \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln E_2/E_1}$$

complex

$$\boxed{b = \frac{11}{3} T_{GB} - \frac{2}{3} T_F - \frac{1}{3} T_S}$$

$$T\,\delta_{ab} = \mathrm{Tr}\, T_a T_b$$

AF

chiral

exp $\quad E \simeq M_Z$

theory $\quad E \simeq M_{GUT}$

$E = M_Z$: $\quad \alpha_2 \simeq 1/30, \quad \alpha_3 \simeq 1/10$

$\longrightarrow \alpha_{em} \simeq 1/128$

two-loop? $\boxed{\text{Small}}$

$\boxed{SM}$

Fermions

$$\begin{pmatrix} u \\ d \end{pmatrix}_L^{\alpha} \qquad u_R^{\alpha}, \; d_R^{\alpha}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$$

$$\Rightarrow T_F = \frac{1}{2} \; (\underbrace{\text{o}}_{\text{singlet}})$$

Scalar $\Phi \Longrightarrow T_{\bar{\Phi}} = \frac{1}{2}\ (0)$

gauge bosons $=$ $\boxed{\text{Adjoint}}$

$$D_\mu = \partial_\mu - ig\, \underbrace{T_a A_\mu^a}_{\boxed{\text{Adjoint}}}$$

$$\boxed{T_{GB} = ?}$$

$\boxed{\cdot\ \text{induction}}$

$$\boxed{T_{GB} = N \text{ for } SU(N)}$$

Proof:

(i) $N = 2 \Rightarrow$ triplet (vector)

$$T_3 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix} \Rightarrow \mathrm{Tr}\, T_3^2 = 2 \ \checkmark$$

(iii) assume $T_{GB}(N) = N$

<u>Prove:</u> $T_{GB}(N+1) = N+1$

$$A(N) = N \times \bar{N} \quad (A \to UAU^{+})$$

$$\Rightarrow A(N+1) = (N+1) \times (\overline{N+1})$$

$$= \underbrace{N \times \bar{N}}_{A(N)} + \underbrace{N + \bar{N}}_{\text{fundamental}} + 1$$

$$\Rightarrow T(N+1) = T(N) + \frac{1}{2} + \frac{1}{2} + 0$$

$$= N + 1$$

induction

$\boxed{\text{Q.E.D.}}$

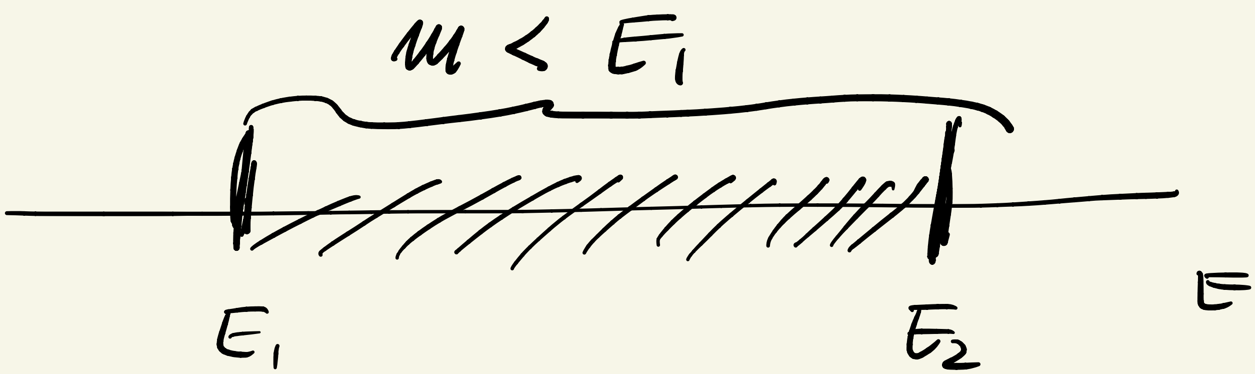
$$A(N+1) = A(N) + F + \bar{F} + 1$$

$$SU(3) \,/\, \boxed{8 = (\vec{\pi}) + K + \bar{K} + \eta}$$

• Compute b?

(i) group theory $\rightarrow$ compute $T(R)$

(ii) who runs in the loops?



decoupling: $M > E_2 \Rightarrow$

then particle with mass M

decouples (<u>does not run</u>)

$$\boxed{\text{only light particles run}}$$

$\Downarrow$

$$\boxed{\text{at least SM particles run}}$$

$E_1 \simeq M_Z$ $\qquad$ $E_2 \simeq M_U$

(who will be running ?)

① SM running

α_3 $\quad b_3 = ?$ $\qquad$ GB (gluons) $= \frac{11}{3} \cdot 3$

$SU(N)$ $\quad b = \frac{11}{3} \cdot N - \frac{2}{3} T_F - \frac{1}{3} T_S$

chiral $\qquad$ complex

$u_L^\alpha, \; d_L^\alpha, \; u_R^\alpha \; d_R^\alpha \rightarrow 4 \cdot 1/2$

$1/2 \quad 1/2 \quad 1/2 \quad 1/2$

u_L^α = triplet of color = F of $SU(3)$

$\Rightarrow \; T_F(QCD) = 2 \left(n_g = \# \text{ of gen.} \right)$

Higgs (Φ) : $\underset{QCD}{T}(\Phi) = 0$

$$\boxed{b_3 = \frac{11}{3} \cdot 3 - \frac{4}{3}\, n_g}$$

$$\boxed{n_g \leq 8 \quad \text{for AF}}$$

α_2 GB: $\frac{11}{3} \cdot (N=2) = \frac{11}{3} \cdot 2$

F: $\begin{pmatrix} u \\ d \end{pmatrix}_L^{\alpha = r, y, b} \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L$

$\quad \frac{1}{2} \qquad\qquad \uparrow \frac{1}{2}$

T_F : $\frac{1}{2} \cdot 4 = 2$

Higgs : $T_2(\Phi) = \frac{1}{2}$

$$\Rightarrow \boxed{b_2 = \frac{11}{3}\cdot 2 - \frac{4}{3} n_f - \frac{1}{6} n_H}$$

(n_H = # of Higgs doublets)

- α_1: $\quad$ ($g_1 = g_2 = g_3 = g_5$ at M_0)

$\alpha_1 \neq \alpha' \;;\; \alpha' = \frac{3}{5}\alpha_1$

$$\boxed{\begin{aligned} g_1 T_{24} &= g' \frac{Y}{2} \\ T_{24} &= \sqrt{3/5}\left(\frac{Y}{2}\right) \end{aligned}}$$

$$\frac{1}{\alpha_1} = \frac{3}{5}\frac{1}{\alpha'}$$

$$\Rightarrow \boxed{b_1 = 3/5\, b'}$$

$$\boxed{b' \leftrightarrow \mathrm{Tr}\left(\frac{Y}{2}\right)^2}$$

α_1: $\underline{\underline{T_{GB} = 0}}$ ($W^\pm$, gluons, Z, A $\leftarrow$ carry $Y = 0$)

$$F: \quad \begin{pmatrix} u \\ d \end{pmatrix}_L^{(1/6)} \quad u_R(2/3),\ d_R(-1/3)$$

$$(-1/2)\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R(-1)$$

$$\frac{Y}{2} = (Q - T_3)$$

$$T_r\left(\frac{Y}{2}\right)^2_F = \underbrace{\frac{1}{36}}_{\begin{pmatrix} u \\ d \end{pmatrix}_L} \cdot \underbrace{2}_{(u,d)} \cdot \underbrace{3}_{\text{color}} + \frac{4}{9} \cdot \underbrace{3}_{\text{color}} + \frac{1}{9} \cdot 3$$

$$+ \underbrace{\frac{1}{4} \cdot 2}_{\begin{pmatrix} \nu \\ e \end{pmatrix}_L} + 1$$

$$= \frac{1}{6} + 4/3 + 1/3 + 1/2 + 1$$

$$= \frac{1 + 8 + 2 + 3 + 6}{6} = \frac{20}{6} = \frac{10}{3}$$

- $b_1(F) = \frac{3}{5} \cdot b'_F = \frac{\cancel{3}}{5} \cdot \frac{10}{\cancel{3}} = 2$

(per gen.)

- Higgs: $\Phi_{,}(Y=1) \Rightarrow b' = \frac{1}{4} \cdot 2 = 1/2$

$$\Rightarrow b_1 = -\frac{4}{3} n_g - \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{5} \cdot \frac{1}{2} n_H$$

$$\boxed{b_1 = -4/3\, n_g - \frac{1}{10} n_H}$$

$$\boxed{\begin{aligned} b_2 &= \frac{22}{3} - 4/3\, n_g - 1/6\, n_H \\ b_3 &= \frac{33}{3} - 4/3\, n_g - 1/10\, n_H \end{aligned}}$$

$$\boxed{\frac{1}{\alpha(E_2)} = \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln E_2/E_1}$$

· new particles:

gauge bosons: (X, Y)

fermions: nothing

scalars: Σ_3 (weak), Σ_8 (strong), T^a near singlet

m_3 $\quad$ 24_H $\quad$ m_8 $\quad$ 5_h

exercise: $m_3 = m_8 = m_T = M_V$
$= M_X = M_Y$

desert picture

compute the 2-3 meeting point

1-2 meeting point

$$\boxed{\alpha_2 = \alpha_1 = \alpha_3 \text{ at } M_U}$$

$$M_W \longrightarrow M_U$$

SM states (desert)