LMU GUT Course

Lecture XVI

8/1/2021

LMU
Winter 2021

SU(5): unification and phenomenology

$$\boxed{\bullet \text{ matter} = \text{fermions} = \\ = \text{quarks} + \text{leptons}}$$

$$5_F = \begin{pmatrix} d \\ \hdashline e^c \\ -\nu^c \end{pmatrix}_R \begin{array}{l} \} \, SU(3) \to \text{gluons} \\ \} \, SU(2) \to W^\pm, Z, \gamma \end{array}$$

$$10_F = \begin{pmatrix} 0 & u^c & & \vdots & u & d \\ & 0 & & \vdots & & \\ & & 0 & \vdots & & \\ \hdashline & & & & 0 & e^c \\ & & & & & 0 \end{pmatrix}_L$$

SU(2) (over the u, d columns), SU(3) (beside the upper rows)

$$\Rightarrow \boxed{Q_\nu = 0, \quad Q_e = 3\, Q_d}$$

$$\boxed{\bullet \;\; \text{gauge bosons} = 24 \qquad 24 = 12 + 12}$$

$\uparrow$ SM (glue, $W^\pm$, Z, γ) $\qquad \uparrow$ new

$$\boxed{\begin{gathered} 12 = 6 + \bar{6} = (x, y)^a + (\bar{x}, \bar{y})^a \\ M_x = M_y + O(M_W) \end{gathered}}$$

$$\mathcal{L}_f = i \bar{5}_F \gamma^\mu D_\mu 5_F + \cdots$$

$$D_\mu = \partial_\mu - i g T_i A_\mu^i$$

$i = 1, 2, \ldots, 24$

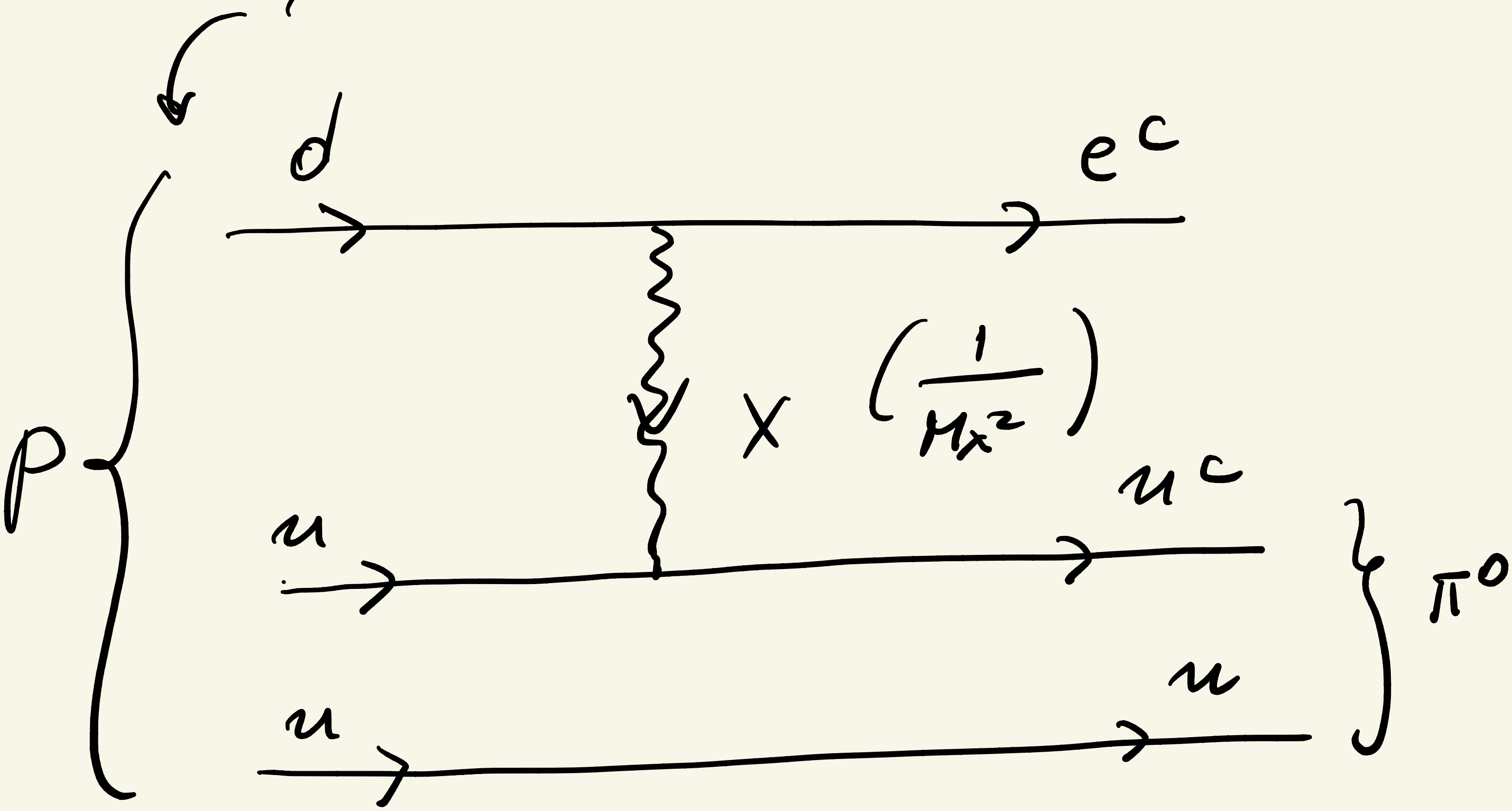
$$10_F = (u^c, u, d, e^c)$$

$$u_L^c \equiv C \bar{u}_R^T$$

$$(-4/3)\, X_\mu \left[\overline{u^c_L}\, \gamma^\mu u_L + \overline{d_L}\, \gamma^\mu e^c_L \right]$$

$$Q: \quad 2/3 \qquad 2/3 \qquad 1/3 \qquad 1$$

$$\overline{X}_\mu \quad \overline{e^c_L}\, \gamma^\mu d_L + \cdots$$



$$\boxed{p \to e^c + \pi^0}$$

$$\tau_p > 10^{34}\,\mathrm{yr} \quad (\mathrm{SK})$$

$\Rightarrow$ X very heavy

$$\Gamma_p \propto g^4 M_X^{-4} m_p^5$$

$$\Gamma_\mu \propto g^4 M_W^{-4} m_\mu^5$$

$$\Rightarrow \quad \Gamma_p / \Gamma_\mu = \left(\frac{m_p}{m_\mu}\right)^5 \left(\frac{M_W}{M_X}\right)^4$$

$$\Rightarrow \quad \tau_p / \tau_\mu = \left(\frac{M_X}{M_W}\right)^4 \left(\frac{m_\mu}{m_p}\right)^4$$

$$= 10^{-5} \left(\frac{M_X}{M_W}\right)^4$$

$$\tau_\mu \simeq 10^{-6} \text{ sec}$$

$$\Rightarrow \quad \tau_p = 10^{-11}\,\text{sec}\left(\frac{M_X}{M_W}\right)^4 \gtrsim 10^{41}\,\text{sec}$$

$$\left(M_X/M_W\right)^4 \gtrsim 10^{52}$$

$$\Rightarrow \quad M_X/M_W \gtrsim 10^{13}$$

$$\boxed{M_X > 10^{15}\,\text{GeV}}$$

$SU(5)$: unified

unification scale M_U

$$M_U \gtrsim M_X > 10^{15}\,\text{GeV}$$

$$10^{15}\,\text{GeV} < M_U \ll M_{pe}$$

M_{pe} = scale when gravity gets strong

$$\alpha(\text{gravity}) \propto G_N E_1 E_2$$

$$\simeq \frac{1}{M_p^2} E_1 E_2$$

$$\boxed{M_p \simeq 10^{18}\,\text{GeV} - 10^{19}\,\text{GeV}}$$

$$\boxed{10^{15}\,\text{GeV} < M_U \ll 10^{18}\,\text{GeV}}$$

- $\alpha_1, \alpha_2, \alpha_3$ — couplings of $U(1)$, $SU(2)$, $SU(3)$
- all 3 must unify at M_U

= miracle

- in a small window:

$$10^{15} - 10^{17}\ \text{GeV}$$

- Symmetry breaking (I)

$SU(5)$ $\boxed{\Sigma \equiv 24_H}$

$\downarrow$ $M_V \simeq M_X$

$SU(3) \times SU(2) \times U(1)$

$$\boxed{\begin{array}{c}\Sigma \to U \Sigma U^{+} \\ \text{Adjoint}\end{array}}$$

$\Sigma^{+} = \Sigma$

$\mathrm{Tr}\,\Sigma = 0$

$$U \equiv e^{i\theta_i T_i} = 1 + i\theta_i T_i + \cdots$$

$$\Sigma \to \Sigma + i[T_i, \Sigma]\,\theta_i$$

$$\Downarrow$$

$$D_\mu \Sigma = \partial_\mu - ig\,[T_i, \Sigma]\,A_\mu^i$$

Reminder: • $\psi \to e^{i\Theta Q}\psi$: $U(1)$

$$D_\mu = \partial_\mu - ig A_\mu Q$$

• $D \to e^{i\Theta_a \sigma_a/2} D$ $\quad SU(2)$

$$\Downarrow$$

$$D_\mu = \partial_\mu - ig\,\frac{\sigma_a}{2} A_\mu^a$$

$$\langle \Sigma \rangle \to U \langle \Sigma \rangle U^+$$

$$= \mathrm{diag}\,(1, 1, 1, -3/2, -3/2)\, v_x$$

$v_x = ?$

$\cdot\ \mathrm{Tr}\, D_\mu \langle \Sigma \rangle D^\mu \langle \Sigma \rangle \Rightarrow$

$$M_X = M_Y = \frac{5}{2} g v_X$$

$$M_{\text{gluon}} = M_W = M_Z = M_\gamma = 0$$

Stage 1 of symm. breaking

(II) $SU(3) \times SU(2) \times U(1)$

$$\Big\downarrow \langle 5_H = \cancel{\Phi} \rangle = M_W$$

$SU(3) \times U(1)_{em}$

$$5_H = \Phi = \begin{pmatrix} T^\alpha \\ \hdashline \phi^+ \\ \phi^0 \end{pmatrix} \begin{matrix} \leftarrow \text{new} \\ \\ \neq \Phi \end{matrix}$$

$$V = V_\Sigma + V_\Phi +$$

$$+ \alpha (\mathrm{Tr}\,\Sigma^2)\, \Phi^+ \Phi + \boxed{\beta\, \Phi^+ \Sigma^2 \Phi}$$

irrelevant crucial

$$\boxed{\Sigma \to \langle \Sigma \rangle}$$

$$\alpha : (\mathrm{Tr}\langle \Sigma \rangle^2)\, 5_H^+ 5_H \longrightarrow \delta m_5^2\, 5_H^+ 5_H$$

(change in V_Φ)

$$V_5 = \mu^2\, 5_H^+ 5_H + \lambda (5_H^+ 5_H)^2$$

$$\mu^2 \to \mu^2 + \alpha \, Tr(\Sigma)^2 = \mu_{new}^2$$

$$\beta: \quad 5_H^+ \langle \Sigma^2 \rangle 5_H$$

$$= (T^+ \; \Phi^+) \, v_x^2 (1,1,1,9/4,9/4) \begin{pmatrix} T \\ \Phi \end{pmatrix}$$

doublet (SM)

$$= \beta \, v_x^2 \left[T^+ T + 9/4 \; \Phi^+ \Phi \right]$$

$$\Downarrow$$

$$\boxed{\begin{aligned} m_T^2 &= m_5^2 + \beta v_x^2 \gtrsim (10^{15}\,\text{GeV})^2 \\ m_\Phi^2 &= m_5^2 + 9/4 \, \beta v_x^2 \simeq (100\,\text{GeV})^2 \end{aligned}}$$

- $T = ?$ who is this?

$\downarrow$ Symmetry arguments

- weak singlet, $SU(3)$ triplet

$$Y = -2/3 \quad \left(Q_T = Y/2 = -1/3 \right)$$

- Lorentz scalar

$$\text{(**)} \quad T^{\alpha} \Big[\overset{\text{? coeff.}}{u_R^{\beta\,T} C\, d_R^{\gamma}\, \epsilon_{\alpha\beta\gamma}} + \overset{\text{coeff.?}}{\underbrace{u_L^{\beta\,T} C\, d_L^{\gamma}\, \epsilon_{\alpha\beta\gamma}}_{\text{singlet?}}} \Big]$$

$Q: -1/3 \qquad +1/3$

$$\boxed{f_1^T\, C\, f_2 = \text{Lorentz singlet}}$$

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \Rightarrow q_L^T C i\sigma_2 q_L \quad \text{is}$$

// SU(2) singlet

$$u_L^T C d_L$$

$\boxed{\text{quark - lepton}}$

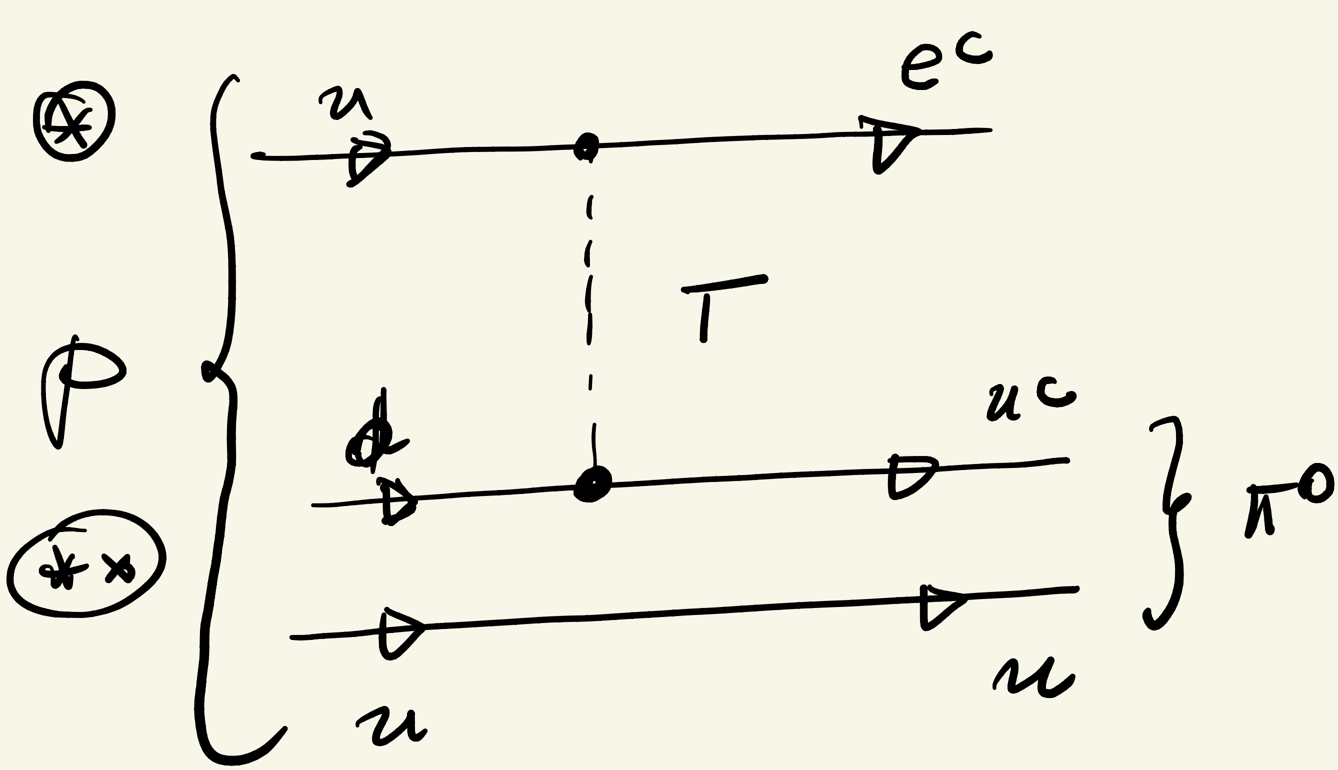
coeff. ?

$$(1/3) \quad T^* \underbrace{u\, e}_{-1/3} = T^*_\alpha \Big[u_R^{\alpha T} C e_R \quad (*)$$

$$\underbrace{+ u_L^{\alpha T} C e_L \pm d_L^{\alpha T} C \nu_L} \Big]$$

$$l_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \Rightarrow q_L^T C i\sigma_2 l_L$$

$$= u_L^T C e_L - d_L^T C \nu_L$$



$$\Rightarrow \quad m_T > 10^{12}\ \text{GeV}$$

$$(\text{unif.}) \Rightarrow \boxed{m_T \gtrsim M_X}$$

· mass terms (T, Φ)

$$\Downarrow$$

$$T: \quad m_5^2 + \beta v_X^2 \simeq (10^{15}\ \text{GeV})^2 \qquad (1)$$

$$\Phi \quad m_5^2 + 9/4\, \beta v_X^2 \simeq 0 \qquad (2)$$

$$m_5^2 \simeq -\frac{9}{4}\beta v_X^2$$

$$\Rightarrow \boxed{m_T^2 = -\frac{5}{4}\beta v_X^2 \quad (\beta < 0)}$$

T is heavy

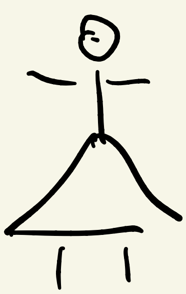
<u>price:</u> fine-tuning (FT)

$$\left.\begin{array}{l} m_{15} \simeq 10^{15}\,\text{GeV} \\ \beta v_X \simeq 10^{15}\,\text{GeV} \end{array}\right\} \boxed{m_5^2 + \frac{9}{4}\beta v_X^2 = 0}$$

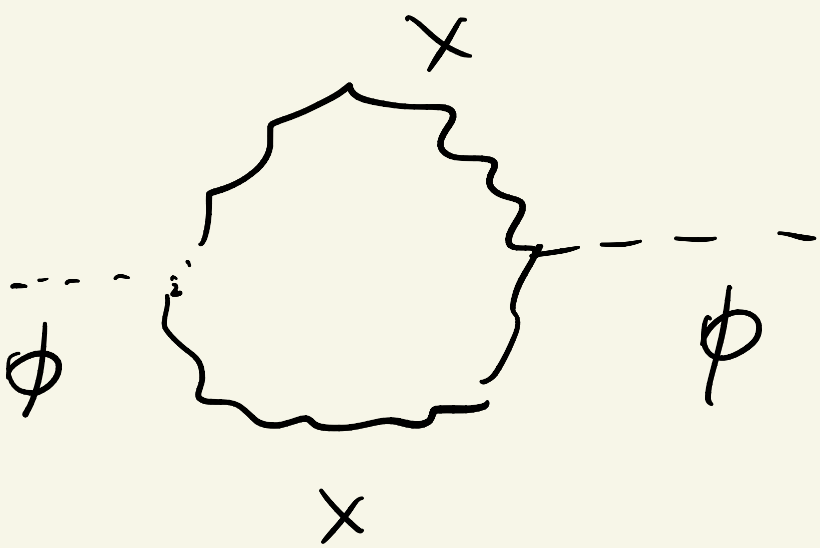
Is this a problem??

Alice: FT is impossible to maintain!

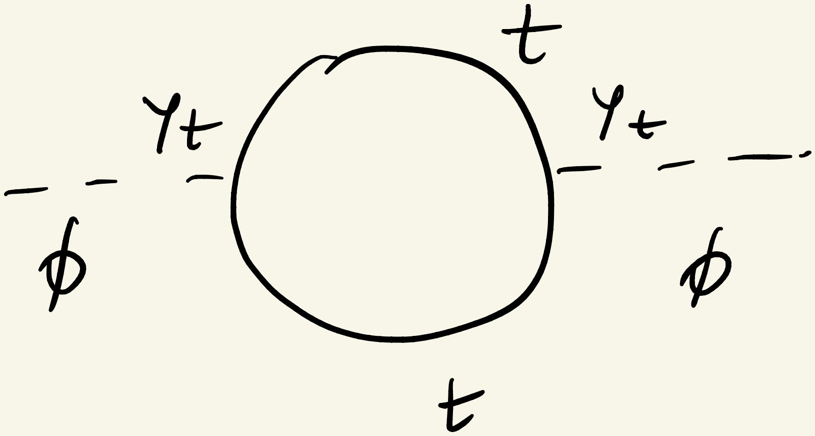
Bob: Just do it!



tree-level ?



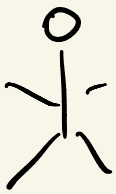
$$m_\phi^2 \propto \frac{\alpha}{\pi} M_X^2$$



(*)

$y_t = 1$

$$\Rightarrow m_\phi^2 \simeq \frac{y_t^2}{16\pi^2}\left(\underbrace{\Lambda^2}_{\gtrsim M_X} + m_t^2\right)$$

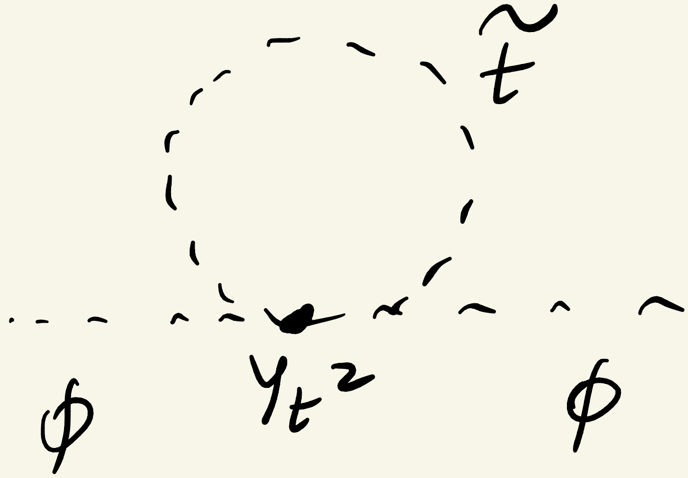


Bob ! $\boxed{\text{do it at 1-loop!}}$

(**) $\tilde{t}$ (scalar) colored triplet

scalar top = stop

$\boxed{\text{same couplings !}}$



$$m_\phi^2 = \frac{y_t^2}{16\pi^2}\left(\cancel{\Lambda^2} + m_t^2 - \cancel{\Lambda^2} + m_{\tilde{t}}^2\right)$$

$m_{\tilde{t}} \leq \text{TeV} \Rightarrow$ Higgs mass protected

$$\boxed{\begin{array}{ccc} p & \longrightarrow & \tilde{p} \\ f & \longrightarrow & b \\ b & \longrightarrow & f \end{array}}$$

1970's

Supersymmetry

$e \longrightarrow \tilde{e}$ (selectron)

$m_{\tilde{e}} \gtrsim 100\,\text{GeV}$

(broken) supersymmetry

Λ_{SS} = scale of SS breaking

= masses of superpartners ($\lesssim$ TeV)

- beautiful protective mechanism, but
- a collection of models

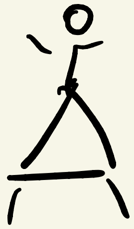
$$\boxed{\begin{gathered} m_{\tilde{t}} > m_{\tilde{e}} < m_{\text{gauge}} \; ?? \\ \sim 100 \text{ new parameters} \end{gathered}}$$

- m_{partners} = the same
  = at same large scale

$$\boxed{\begin{gathered} \text{Supersymmetry} = \\ \text{the same FT} \end{gathered}}$$

but

$$\boxed{\begin{gathered} \text{do it at tree level and you} \\ \text{are free forever} \end{gathered}}$$



Can I find a rationale to Higgs = light?

Idea: Higgs = (pseudo)

= Nambu - Goldstone boson

- Anselm ---
- Berezhiani, Dvali, ---

$\boxed{SU(6)}$

$\Rightarrow$ change physics at the unif. scale (M_U)

$\Rightarrow$ this theory $\Longleftrightarrow$ usual FT

$W \longrightarrow \tilde{W}$ (wino)

$Z \longrightarrow \tilde{Z}$ (zino)

$\gamma \longrightarrow \tilde{\gamma}$ (photino)

$f \longrightarrow \tilde{f}$ (sfermion)

$h \longrightarrow \tilde{h}$ (Higgsino)

(Higgs)

$\boxed{\text{2 Higgs doublets}}$

FT — because

$m_T \gg m_\Phi$

(Tmediates p decay)

$\Rightarrow$ split m_T from m_ϕ

$$\boxed{\text{FT - independent of Super Symmetry}}$$

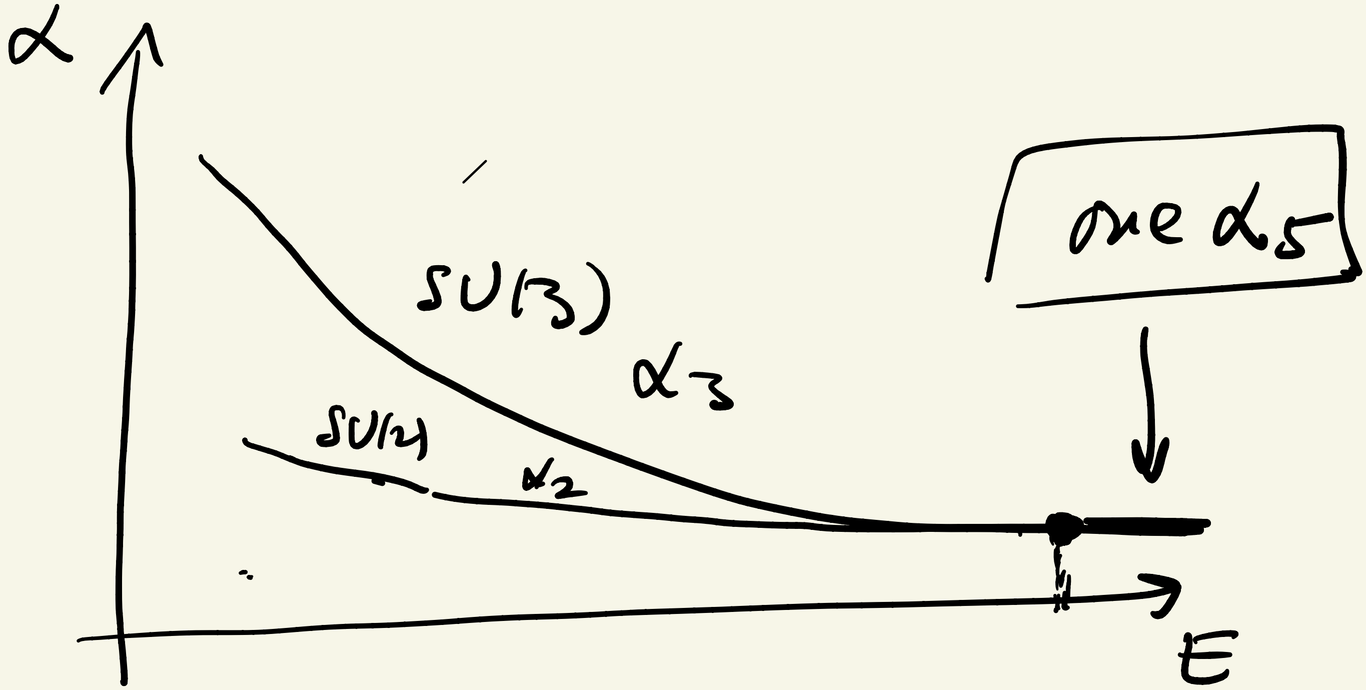
$$\left.\begin{array}{l} M_x > 10^{15}\,\text{GeV} \\ M_Y > 10^{15}\,\text{GeV} \end{array}\right\} \quad \boxed{\begin{array}{c} M_x = M_Y \\ SU(2)\text{ sym.} \end{array}}$$

$$M_x \propto g\, v_x$$

$$M_Y \propto g\, v_x$$

- couplings "run" $\Leftarrow$

$$\alpha = \alpha(E)$$



$M_U \gtrsim M_X$

$$\boxed{M_U \simeq M_X}$$

- $m_f = \underset{\text{tree}}{m_f^0}\left[1 + \frac{\alpha}{\pi}\ln\frac{\Lambda}{m}\right]$
  "small"

$$\boxed{m_f^0 = 0 \Rightarrow \text{chiral sym.} \Rightarrow m_f = 0}$$

$\Downarrow$

Super Symmetry : makes bosons
~ fermions

$\Rightarrow$ Higgs ~ Higgsino

$\Rightarrow$ everybody protected

• <u>GUT</u> $\Rightarrow$ $\boxed{\text{FT}}$

(1) SS $\longrightarrow$ NG picture

large scale $\Lambda \gtrsim M_X$

$$\Rightarrow \quad m_\phi^2 \simeq m_{\phi_0}^2 + \underbrace{\frac{\alpha}{\pi}\Lambda^2}_{\text{large correction}}$$

- $\boxed{\text{no GUT}}$

SUSY ~ they at $\gtrsim M_W$

$$\boxed{\Lambda \simeq 1-10\ \text{TeV}}$$

(ii) unification of couplings

- ordinary $SU(5) \Rightarrow$

  no unif.

- SUSY $SU(5) \Rightarrow$

  unify