LMU GUT Course

Lecture XV

22/12/2020

Symmetry breaking in SU(5): Higgs and Yukawa sector

$$SM = SU(3) \times \underbrace{SU(2) \times U(1)}$$

$M_W \simeq \langle H \rangle$ Higgs H = $\downarrow$ doublet

$U(1)$

- $SU(5) \Longrightarrow$ (X, Y) gauge bosons

$$m_X = m_Y > 10^{15}\ \text{GeV}$$

$$SU(5) \xrightarrow{\langle \Sigma \rangle} SU(3) \times SU(2) \times U(1)$$

$$M_{GUT} = M_X$$

Who should Σ be?

---

example $\quad$ SM $=$

$$\begin{tikzcd} SU(2)\times U(1) \arrow[d] \\ U(1)_{em} \end{tikzcd}$$

$\ell_L = \begin{pmatrix}\nu \\ e\end{pmatrix}_L \; , \; e_R$

$$\mathcal{L}_y = \underset{\substack{\uparrow \\ \text{doublet}}}{\bar{\ell}_L}\; \underset{\substack{\uparrow \\ \text{doublet}}}{H}\; \underset{\substack{\uparrow \\ \text{singlet}}}{e_R}\; y_e$$

- Σ in $SU(5)$

Σ should <u>not</u> couple to fermions

$f = \bar{5}_L, \ 10_L$ (anti symm)

$$(5_R = \left(\begin{array}{c} d^\alpha \\ \hline e^c \\ \nu^c \end{array} \right) \begin{array}{l} \leftarrow \text{color} \\ \leftarrow \text{weak doublet} \end{array}$$

- $$5_H = \left(\begin{array}{c} T^\alpha \\ \hline H \end{array} \right) \leftarrow \text{weak doublet}$$

$$\langle 5_H \rangle = \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 0 \\ v \end{array} \right) \begin{array}{l} \\ \\ \\ \big\} \text{ weak} \end{array}$$

~~$SU(5) \xrightarrow[\langle 5_H \rangle]{} SU(3) \times SU(2) \times U(1)$~~

NO

$$SU(5) \xrightarrow[\langle 5_H \rangle]{} SU(4)$$

- ~~10_H~~ $\quad \left(10_F = \left(\begin{array}{c:c} u^c & u\,d \\ \hdashline & e^c \end{array}\right)_L\right)$

$\langle 10_H \rangle$ breaks Q_{em}!

no neutral field

- $15_H \leftarrow$ does not work

$$\begin{array}{cccccc} SU(5) & \xrightarrow{\langle\Sigma\rangle} & SU(3) & \times SU(2) & \times U(1) \\ r = 4 & & 2 & 1 & 1 \end{array}$$

$\langle\Sigma\rangle$ does not break rank

Adjoint: $\Sigma \to U \Sigma U^\dagger$

↑ $UU^\dagger = 1$

→ $\det U = 1$

defining 5×5 unitary

$$\boxed{\begin{aligned} \mathrm{Tr}\,\Sigma &= 0 \\ \Sigma^\dagger &= \Sigma \end{aligned}}$$

$\Downarrow$

$\langle \Sigma \rangle \to$ $U \langle \Sigma \rangle U^\dagger =$ diagonal

Hermitian

$\langle \Sigma \rangle =$ diagonal

$\Rightarrow [\Sigma, T_a \in C] = 0$

↑

Cartan

$$\Sigma \to U \Sigma U^{\dagger}$$

$$= \Sigma + i\theta_a [T_a, \Sigma] + \cdots$$

$$\boxed{\langle \Sigma \rangle \text{ preserves rank}}$$

$$\Sigma = T_a \phi_a \qquad a = 1, \dots, 24$$

$$\boxed{\begin{array}{c} \mathrm{Tr}\,\Sigma^2, \\ \mathrm{Tr}\,\Sigma^3, \\ \mathrm{Tr}\,\Sigma^4, \end{array}} \qquad \begin{array}{l} \Sigma^2 \to U \Sigma^2 U^{\dagger} \\ \Sigma^3 \to U \Sigma^3 U^{\dagger} \\ \Sigma^4 \to \cdots \end{array}$$

$m \Sigma^3$ in V

$(m = 0)$

$$V = -\frac{\mu^2}{2} \mathrm{Tr}\,\Sigma^2 \leftarrow \text{mass}$$

$$+ \frac{a}{4}(\mathrm{Tr}\,\Sigma^2)^2 + \frac{b}{2} \mathrm{Tr}\,\Sigma^4$$

$\langle \Sigma \rangle$ = diagonal

$$\langle \Sigma \rangle = v_x \,\mathrm{diag}\left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}\right)$$

↑

preserve $SU(3)_c$, $SU(2)_L$, $U(1)_Y$

let us prove that this is a minimum

Li '1974
Posted

↑
global minimum

<u>SM</u> H = doublet

$$V = -\frac{\mu_H^2}{2} H^+H + \frac{\lambda}{4}(H^+H)^2$$

- $\lambda > 0$ ← boundedness
- $\mu_H^2 > 0$ ← sym. breaking

range of parameters that ensures (proper) sym. breaking

↓

$$\langle \Sigma^2 \rangle = v_x^2 \,\mathrm{diag}\left(1, 1, 1, 9/4, 9/4\right)$$

$$\langle \Sigma^4 \rangle = v_x^4 \,\mathrm{diag}\left(1, 1, 1, 81/16, 81/16\right)$$

$$\mathrm{Tr}\,\Sigma^2 = v_x^2 \, \frac{15}{2}$$

$$\mathrm{Tr}\,\Sigma^4 = v_x^4 \, \frac{105}{8}$$

$$\Downarrow$$

$$V = -\frac{\mu^2}{2} \, \frac{15}{2} v_x^2 + \frac{a}{4} v_x^4 \left(\frac{15}{2}\right)^2 + b/2 \, \frac{105}{8} v_x^4$$

$$\frac{\partial V}{\partial v_x} = 0$$

$$\Downarrow$$

$$\cancel{v_x = 0} \quad \boxed{\text{not physical}}$$

$$\left.\frac{\partial^2 V}{\partial \vartheta_x^2}\right|_0 = -\mu^2 < 0$$ not a minimum

extremum

$$\boxed{\mu^2 = \frac{15a + 7b}{2}\vartheta_x^2}$$

step 1

$(m \ll \mu \; ; \quad m \,\mathrm{Tr}\,\Sigma^3)$

$$\cancel{\mathrm{Tr}\,\Sigma}, \quad \mathrm{Tr}\,\Sigma^2, \quad \mathrm{Tr}\,\Sigma^3, \quad \mathrm{Tr}\,\Sigma^4$$

$$\mathrm{Tr}\,\Sigma^5 \leftarrow \mathrm{Tr}\,\Sigma^2\,\mathrm{Tr}\,\Sigma^3$$

not new

Guth, Weinberg '82

deal with $m \neq 0$

$\Downarrow$ show $\langle \Sigma \rangle$ = local minimum

Study 2nd derivatives matrix

$\Downarrow$

show that the eigenvalues are positive

$$\left.\frac{\partial^2 V}{\partial \Sigma_i \partial \Sigma_j}\right|_{\langle \Sigma \rangle} = \text{matrix of particle masses}$$

(scalar: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2$)

$$m^2 = \left.\frac{\partial^2 V}{\partial \phi^2}\right|_{min}$$

I need to compute the masses in Σ $(\phi_i \equiv \Sigma_i)$

24×24 matrix

$$G \longrightarrow H$$

$SU(5)$ $\qquad\qquad$ $SU(3) \times SU(2) \times U(1)$

use the symmetries of

H

particles connected by symmetry → the same mass

$$\Sigma \to U \Sigma U^{+} \sim 5 \times \bar{5}$$

$$5 \to U\, 5$$

$$\bar{5} \to \bar{5}\, U^{+}$$

$$Q(\Sigma_{ij}) = (Q_i - Q_j)$$

$$Q(10_{ij}) = (Q_i + Q_j)$$

$$\Sigma = \left(\begin{array}{c|c} \Sigma_8 & \overset{\downarrow \quad \downarrow}{\bar{\Sigma}_x \; \bar{\Sigma}_y} \\ (\sim \text{gluons}) & (\sim X, Y) \\ \hline \Sigma_x & \Sigma_3 \\ \Sigma_y & (\sim \vec{W}) \end{array}\right) \begin{array}{l} \left.\vphantom{\begin{array}{c}a\\a\end{array}}\right\} SU(3) \\ \left.\vphantom{\begin{array}{c}a\\a\end{array}}\right\} SU(2) \end{array}$$

$$+\ \Sigma_0 \ (\text{singlet})$$

$$(\sim B \text{ of } U(1)_Y)$$

$$Q(\Sigma_{14}) = Q_1 - Q_4 = -1/3 - 1$$
$$= -4/3$$

$$5_F = \left(\begin{array}{c} d \\ \hdashline e^c \\ \nu^c \end{array}\right)_R$$

(color)

masses! Σ_8 — same mass

Σ_3 — same mass (weak)

Σ_0 — mass

$\underbrace{\Sigma_x^\alpha, \Sigma_y^\alpha}_{\text{weak}}$ — same mass masses: $\underbrace{\Sigma_8, \Sigma_3, \Sigma_0, \Sigma_X}_{\boxed{\text{4 masses}}}$

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$$

$$24 \qquad \underbrace{8 + 3 + 1}_{12}$$

$24 - 12 = 12$ broken generators

Higgs $\Rightarrow$ 12 massive gauge (X^a, Y^a)

$\Rightarrow$ 12 scalars get "eaten"

$\boxed{\text{potential} \Rightarrow \text{global symmetry}}$

- $V(\Sigma) \leftarrow$ masses

12 broken gen. $\Rightarrow$ 12 NG bosons

NG ≡ Nambu - Goldstone

$m = 0$

Guess which fields become NG bosons

$\Downarrow$

$\Sigma_x, \Sigma_y = \text{NG}$

* check

· $m = 0$ (no Σ^3) $\Rightarrow D: \Sigma \to -\Sigma$

$\langle \Sigma \rangle \neq 0 \Rightarrow \cancel{D}$ spont.

$\Rightarrow$ domain walls)

$\Sigma_8, \Sigma_3, \Sigma_0$

Notes:
Beyond

$$m^2_{\Sigma_0} = \frac{15a + 7b}{2} v_x^2$$

$$\mu^2 = \frac{15a + 7b}{2} v_x^2$$

$m^2_{\Sigma_0} = \mu^2$ (Higgs mechanism)

$\Sigma_8 \to$ one states (G)

$\Sigma_3 \to$ -||- (A)

$$\Sigma = \langle \Sigma \rangle + \begin{pmatrix} G/2 & & & & \\ & -G/2 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & +A/2 & \\ & & & & -A/2 \end{pmatrix}$$

$$\Downarrow$$

$$\Sigma = \text{diag}\left(\frac{G}{2} + v_x, -G/2 + v_x, v_x, A/2 - 3/2\, v_x, -A/2 - 3/2\, v_x\right)$$

$$\Sigma^2 = \mathrm{diag}\Big(\sigma^2/4 + \vartheta_x^2 + \sigma\vartheta_x ,\ \sigma^2/4 + \vartheta_x^2 - \sigma\vartheta_x ,\ \vartheta_x^2 ,\ A^2/4 + 9/4\,\vartheta_x^2 - 3/2\,\vartheta_x A ,\ A^2/4 + 9/4\,\vartheta_x^2 + \text{''} \Big)$$

$$\boxed{\Sigma^4 = \text{keep } \sigma^2, A^2 \text{ terms}}$$

$$\Downarrow$$

$$-\frac{\mu^2}{2}\,\mathrm{Tr}\,\Sigma^2 \longrightarrow -\frac{\mu^2}{2}\left(\frac{\sigma^2}{2} + \frac{A^2}{2}\right)$$

$$\frac{a}{4}\left(\mathrm{Tr}\,\Sigma^2\right)^2 \longrightarrow \frac{15}{2}\,a/4\,(\sigma^2 + A^2)\,\vartheta_x^2$$

$$\frac{b}{2}\,\mathrm{Tr}\,\Sigma^4 \longrightarrow \frac{b}{4}\Big[\sigma^2\, 6\,\vartheta_x^2\ (?)$$

$$+ A^2\,\frac{27}{?}\,\vartheta_x^2\ (!)\Big]$$

$$\mu^2 = \frac{15a+7b}{2} v_x^2$$ ①

$$\boxed{15a+7b>0}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} m_G^2 &= \frac{5}{4} b\, v_x^2 \\ m_A^2 &= 5 b\, v_x^2 \end{aligned}}$$

$$\boxed{b>0}$$ ②

ensure that $\langle \Sigma \rangle$ = local minimum

- $b=0 \Longrightarrow m_G = m_A = 0$

$$\Rightarrow \quad m_{\Sigma_8} = m_{\Sigma_2} = 0$$

why massless in $L=0$ limit?

Hint: NG mechanism

$$m_A = 2\, m_G$$

$$(m_3 \equiv m_{\Sigma_3}\,, \quad m_8 \equiv m_{\Sigma_8})$$

$$m_3 = 2\, m_8$$

$$L_i \; : \; \langle \Sigma \rangle_{SM} = v_x \left(1, 1, 1, -3/2, -3/2\right)$$

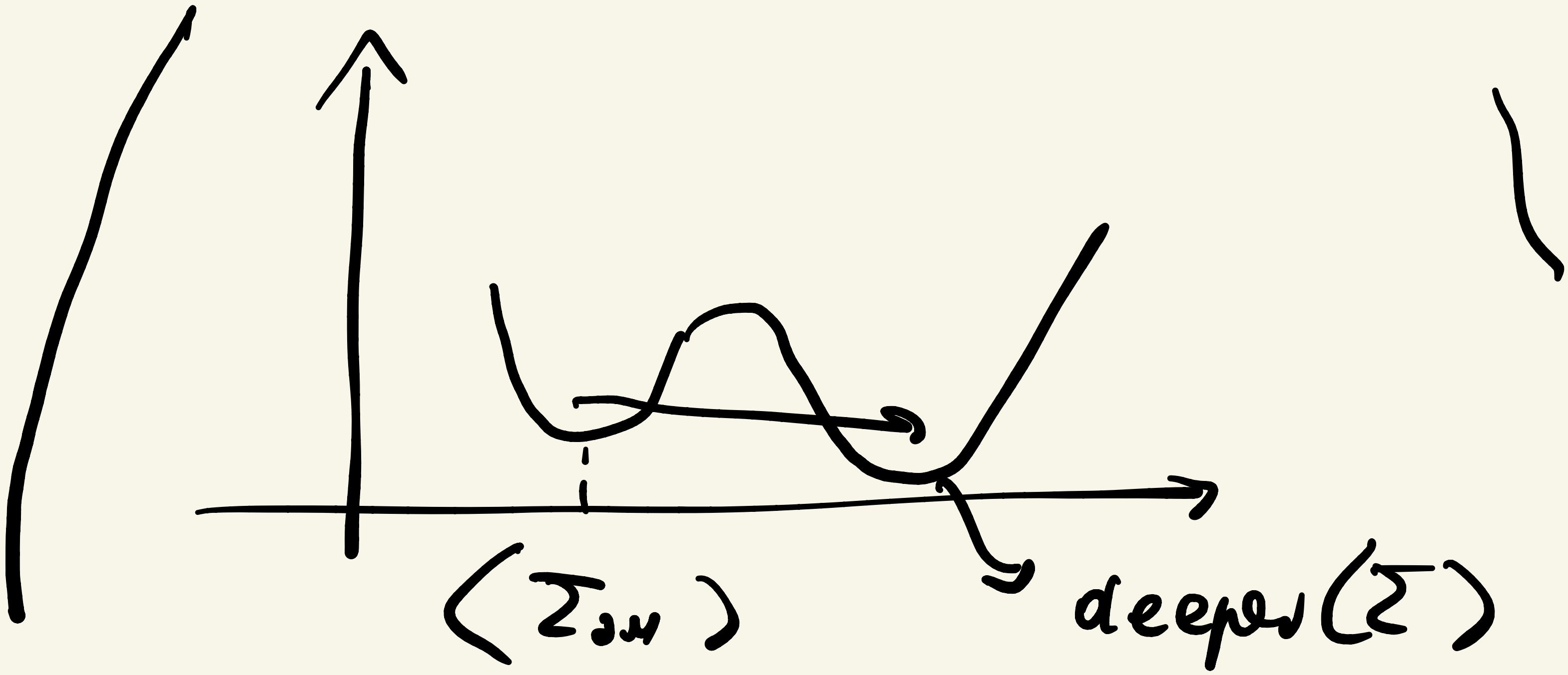
= local minimum ⇒

global minimum

$$\bullet\ \langle \Sigma' \rangle = \vartheta_x' \underbrace{(1, 1, 1, 1, -4)}_{\text{maximum}}$$

$SU(5)$

$\rightarrow SU(4) \times U(1)$



as long as $\tau_{\text{tunneling}} > \tau_U$

$\uparrow$

side comment

age of universe

- $SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$

$$\langle \Sigma \rangle = v_x \,\mathrm{diag}\left(1, 1, 1, -\tfrac{3}{2}, -\tfrac{3}{2}\right)$$

$b > 0, \quad 15a + 7b > 0$

$\mu^2 > 0$

- $SU(2) \times U(1) \xrightarrow[\langle \Phi \rangle]{} U(1)$

$\Phi = ?$

$$\Phi \supseteq H \quad (\text{must!})$$

SM doublet

$$\Downarrow$$

$$\boxed{\Phi = 5_H}$$

$$\Phi = \begin{pmatrix} \left.\begin{matrix} T^1 \\ T^2 \\ T^3 \end{matrix}\right\} \text{color} \\ \left.\begin{matrix} \phi^+ \\ \phi^0 \end{matrix}\right\} \text{weak} \end{pmatrix}$$

$$\Downarrow$$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix} \quad \boxed{\text{must}}$$

$$\boxed{\text{How to achieve this?}}$$

$$\Sigma = \Sigma_8 + \overset{\text{weak}\,\textcircled{3}}{\Sigma_3} + \underbrace{(\Sigma_x , \Sigma_y)}_{\text{eaten}}$$

Color octet

$$+ \Sigma_0$$

singlet

$$\boxed{\text{NO DOUBLET}}$$

$$V = V_\Sigma + V_\Phi + V_{\Sigma\Phi}$$

(above)

$$V_\Phi = -\mu_\phi^2 \Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2$$

$$\boxed{V_{\Sigma\phi} = \alpha\, \Phi^\dagger\Phi \, \mathrm{Tr}\,\Sigma^2 + \beta\, \Phi^\dagger\Sigma^2\Phi}$$

$$\Phi \to U\Phi$$

$$\Downarrow$$

$$\Sigma \to U\Sigma U^{\dagger}$$

$$\underline{\langle \Phi \rangle = ?}$$

$$\boxed{\Sigma \text{ fields} - \text{heavy decouple}}$$

$$\Rightarrow \Sigma \to \langle \Sigma \rangle$$

$$\Downarrow$$

$$V_{\Phi}(\text{physical}) = V_{\Phi} +$$

$$\alpha\, \Phi^{\dagger}\Phi \langle \mathrm{Tr}\, \Sigma^{2} \rangle +$$

$$+ \beta\, \Phi^{\dagger} \langle \Sigma^{2} \rangle \Phi$$

$$\Downarrow$$

$$\cancel{(\Phi^\dagger \Sigma \Phi)}$$

α term: $-\mu_\Phi^2 + \alpha\, \mathrm{Tr}\langle \Sigma^2 \rangle$

$$= -\bar{\mu}_\Phi^2$$

→ only changes the mass term

$$\beta\left[\underline{T^\dagger T}\;\vartheta_x^2 + \frac{9}{4}\,\underline{H^\dagger H}\;\vartheta_x^2\right]$$

$$\vartheta_x = \mathrm{diag}\left(1, 1, 1, -3/2, -3/2\right)$$

$T^\dagger T = T_\alpha^* T_\alpha \quad \alpha = 1, 2, 3$

$H^\dagger H$ = usual SM invariant

$* \; T^{\alpha} \longleftarrow d$

(−1/3 charge)

$$\left(\begin{array}{l} X \xleftarrow{\alpha} 4/3 \\ Y \end{array} \right) \longleftarrow 1/3 \; *$$

$$\Rightarrow \boxed{\text{T mediates proton decay}}$$

$\boxed{\text{Yukawa sector}}$

$\bar{5}_F{}^{(L)} (5_F \, (R))$, $\quad 10_F \, (L)$

$$5_H = \bar{\Phi}$$

$$\Downarrow$$

$$\mathcal{L}_y = \bar{5}_F \, 10_F \, 5_H^* Y_1$$

$$10_F \to U \, 10_F \, U^T$$

$$5_F \to U \, 5_F$$

$$5_H \to U \, 5_H$$

$$\bar{5}_F \underbrace{U^\dagger U}_{1} \, 10_F \underbrace{U^T U^*}_{1} \, 5_H^*$$

= Invariant

$$\underbrace{T_1 \text{ in } 5_H} \qquad T_1^* \, \bar{5}_F^{2} \, 10_F^{21}$$

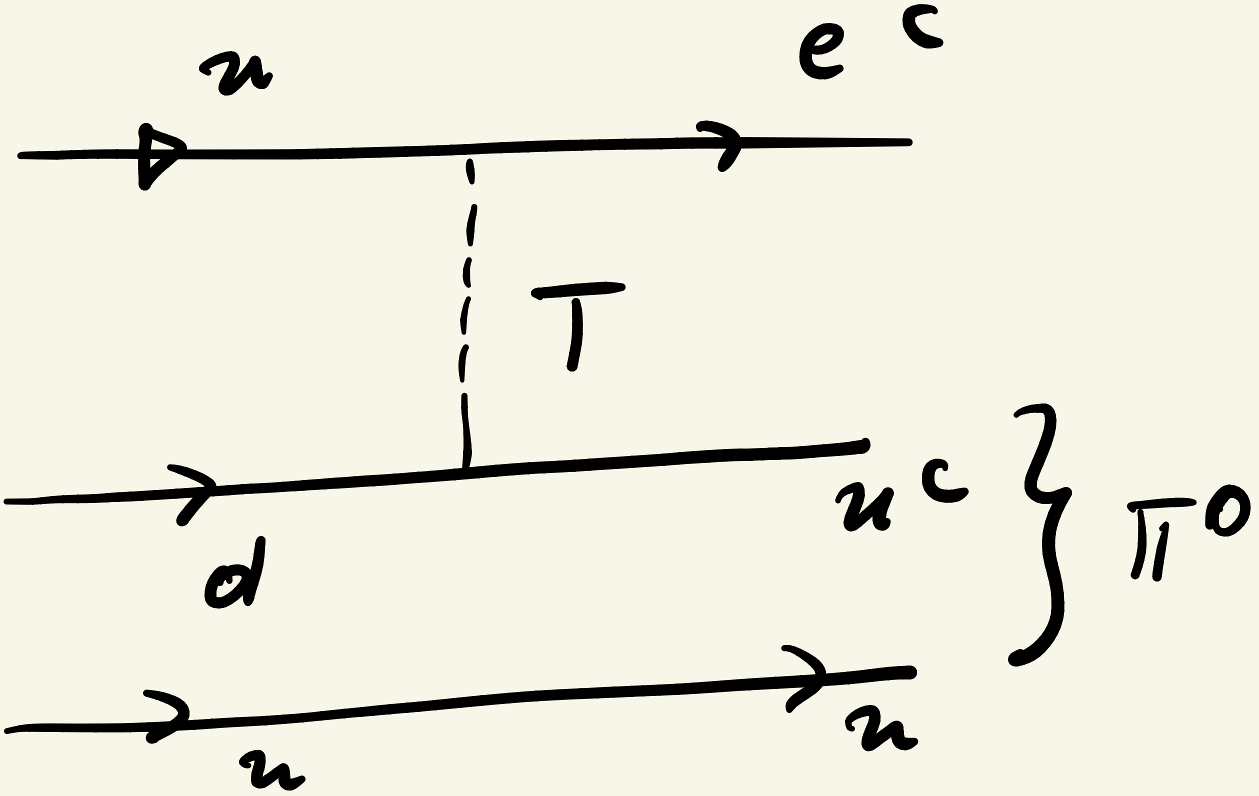
$$\left(= T_1^* \, \bar{d}_{R2} \, u_2^{c\,3}\right)$$

$$+ T_1^* \, \bar{5}_F^4 \, 10_{41} =$$

$$\left(= T_1^* \, \overline{e_R^c} \, u_{L1} \right)$$

$$\Rightarrow \boxed{T \left(\overline{u_L^c} \, d_R + \overline{u_L} \, e_R^c \right) y_1}$$

Q: $-1/3$ $\quad 2/3 \;\; -1/3 \quad -2/3 \;\; +1$



$$\boxed{T \to p \text{ decay}}$$

$$\boxed{m_T > 10^{10}\,\text{GeV}}$$

$$\beta \left[T^+T + \frac{9}{4} H^+H \right] v_x^2$$

$$-\mu^2 (H^+H + T^+T) \quad +$$

$$= \left(-\mu^2 + \beta \, 9/4 \, v_x^2\right)(H^+H) +$$

$$\underbrace{\boxed{\left(-\mu^2 + v_x^2 \beta\right) T^+T}}_{\text{positive and large}}$$

$$\boxed{-\mu^2 + 9/4\,\beta\, v_X^2 \simeq M_W^2 \simeq 0}$$

$$\Big\Uparrow \quad (M_W \ll M_X \sim v_X)$$

Fine Tuning

$$\Big(\underbrace{-\mu^2 + \mathrlap{\overbrace{}^{\beta v_X^2}}9/4\,\beta v_X^2}_{0} - 5/4\,\beta v_X^2\Big) T^\dagger T$$

$$\Downarrow$$

$$\boxed{\beta < 0}$$

T gets a large $(>)$ mass term

$$\Downarrow$$

$$\boxed{\langle T \rangle = 0}$$

$$\Rightarrow \langle h \rangle \neq 0$$

$$-\mu^2 + g/4\, \rho \vartheta_x^2 < 0$$

complete the symmetry breaking

$$m_T \simeq M_{GUT}$$

$$\Rightarrow \rho \vartheta_x \sim M_{GUT}$$

$$\underbrace{\Big(\underset{10^{30}\,\text{GeV}^2}{\underset{\downarrow}{-\mu^2}} + \underset{10^{30}\,\text{GeV}^2}{\underset{\downarrow}{\sim M_{GUT}^2}}}_{\boxed{FT}} \simeq 0\Big)$$

$$= -M_W^2$$

$$(100\,\text{GeV})^2$$

Summary

$$\Sigma \Rightarrow \Sigma_8, \Sigma_3, \boxed{\Sigma_0\ (m_0)}$$

$$\boxed{m_3 = 2\,m_8 = ?}$$

$$m_0 = ?$$

Φ : T : $m_T \simeq M_{GUT}$

H : light

↓

complete the Yukawa story

and study fermion masses

$$\boxed{\bar{5}_F^{\,i}\, 10_{F\,ij}\, 5_H^{*\,j} = \text{inv.}}$$

$$\boxed{\epsilon_{ijklm}\, 10_F^{ij}\, 10_F^{kl}\, 5_H^{m}}$$

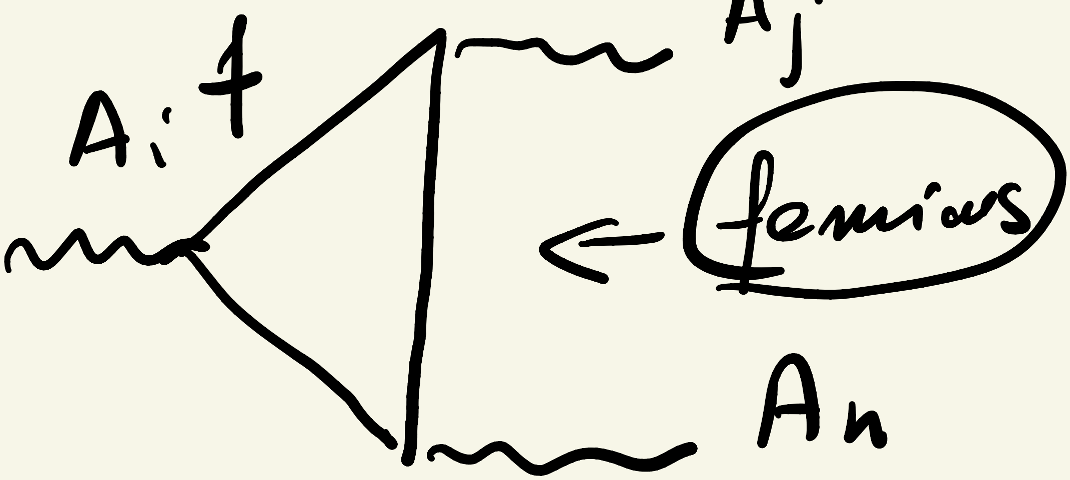
Proton decay

$$\underbrace{(X, Y)}_{\text{color}}{}^{\alpha}_{1/3}$$

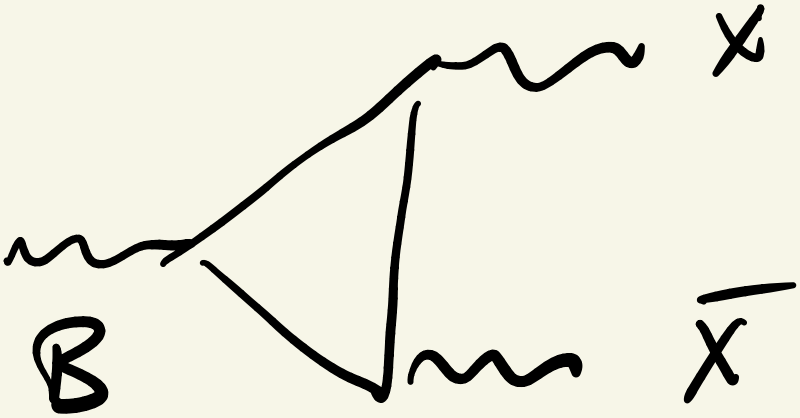
$$\underbrace{T}_{\text{color}}{}^{\alpha} \quad (1/3)$$

fractional charge

$$\boxed{\underset{-1/3}{T_\alpha}\;\underset{-2/3}{\bar{u}^\alpha}\;\underset{1}{e^c} + T^*_\alpha\;\underset{1/3}{\bar{d}_\beta}\;\underset{-2/3}{u^c_\gamma}\;\epsilon_{\alpha\beta\gamma}}$$



scalars = irrelevant



anomaly = 0 (?)

- $m_h = \sqrt{\lambda}/g \; M_W$

  $\lambda^2 \sim g \Rightarrow m_h \sim M_W$

- $M_T^2 = \rho \, v_X^2 \not\Rightarrow \rho \to 1$

  (p decay)

$$m_3 = 2m_8 \quad \alpha\sqrt{6}\, v_x$$

v_0 arbitrary

a little "oasis"

octet of color ; triplet of vector