LMU GUT Course

Lecture XIV

18/12/2020

LMU
Fall 2020

SU(5): what did we achieve and what is a challenge?

Motivation:

1. Unification
2. miracles:

   a) charge is quantized

   $$q \propto n q_0$$

   b) $q_\nu = 0$ $\qquad \left(q_{em} \equiv Q_{em} \right)$

$$5_F = \begin{pmatrix} d^\alpha \\ \hline e^c \\ -\nu^c \end{pmatrix}_R \Leftrightarrow \begin{pmatrix} d^c \\ \hline \nu \\ e \end{pmatrix}_L = \bar{5}_F$$

α = color = 1, 2, 3

a = weak = 1, 2

$$10_F = \begin{pmatrix} 0 & u^c & u^c & u & d \\ & 0 & u^c & u & d \\ & & 0 & u & d \\ & & & 0 & e^c \\ & & & -e^c & 0 \end{pmatrix}_L$$

(AS)

$$Q\,10 = Q\,5 + Q\,5$$

$$10 = 5 \times 5$$

$$Q_{e^c_L} = Q_u + Q_5 = Q_{e^c_R} + Q_{\nu^c_R}$$

$$\boxed{\text{miracle}} \Rightarrow \boxed{Q_{\nu^c} = 0 = Q_\nu}$$

$$Q_e = 3\,Q_d$$

$$\bar{5}_L : \quad Q_e + \cancel{Q_\nu} + 3\,Q_{d^c} = 0$$

$\boxed{\text{Miracle}}$ $Q_e = -3\,Q_{d^c} = 3\,Q_d$

charge $\equiv$ em charge

$$\boxed{Q^L_{em} = Q^R_{em}}$$

$SU(5) \Rightarrow$ parity violation
(maximal)

$$\bar{5}_F = \begin{pmatrix} d^c \\ \nu \\ e \end{pmatrix}_L \quad \begin{matrix} \rightarrow \text{singlet} = R \\ \leftarrow \boxed{\text{doublet} = L} \end{matrix}$$

$$10_F = \begin{pmatrix} u^c & u\,d \\ & e^c \end{pmatrix}_L \quad \begin{matrix} \rightarrow u_R = (R) \\ \rightarrow \text{doublet} = L \end{matrix}$$

$$\boxed{\begin{aligned}\text{singlets} &= R \\ \text{doublets} &= L\end{aligned}}$$

$L\,(R) = \text{left}\,(\text{right})$

$\boxed{SM}$

$$\begin{pmatrix}u\\d\end{pmatrix}_L \qquad \cancel{\begin{pmatrix}u_R\\d_R\end{pmatrix}} \Leftarrow P$$

$$\begin{pmatrix}\nu\\e\end{pmatrix}_L \qquad \cancel{\begin{pmatrix}\nu\\e\end{pmatrix}_R} \Leftarrow P$$

$$\Downarrow$$

$$\boxed{\exists\, \nu_L \Rightarrow \exists\, \nu_R \Rightarrow m_\nu \neq 0}$$

SM can accomodate both P and $\not{P}$

$$\boxed{SU(5) \Rightarrow \text{maximal } \not{P}}$$

$$n \to \underbrace{p + e} + \bar{\nu}_e$$

$$-Q_p \simeq Q_e \Rightarrow Q_\nu \simeq 0$$

$$(\sim 10^{-20})$$

$$\boxed{\begin{array}{c} \text{SM + anomaly} = 0 \\ \Downarrow \text{ claim} \\ Q_\nu = 0 \end{array}}$$

$\boxed{\text{WRONG}}$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L, \; e_R \; ; \; \begin{pmatrix} u \\ d \end{pmatrix}_L, \; u_R, \; d_R$$

$$\boxed{\text{no } \nu_R \Rightarrow m_\nu = 0}$$

$\boxed{\text{Anomaly}}$

$$\boxed{\begin{array}{l} \bullet \; Q = T_3 + \frac{Y}{2} \; \checkmark \\ \bullet \; Q_L = Q_R \end{array}}$$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad E_R \; ; \quad q = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad U_R, D_R$$

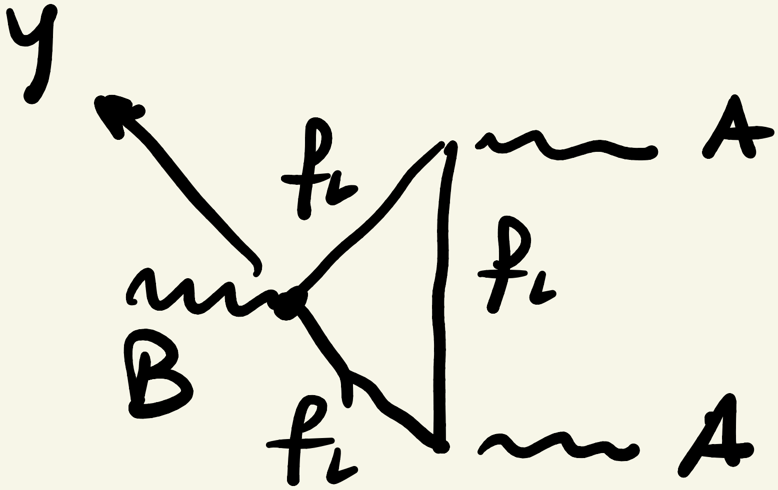
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$Y_l \qquad\qquad Y_E \qquad\qquad Y_q \qquad\qquad Y_U, Y_D$$

$$Q_E = Q_e \Rightarrow \frac{Y_E}{2} = -\frac{1}{2} + \frac{Y_l}{2}$$

$$\Downarrow$$

$$\left.\begin{aligned} Y_E &= -1 + Y_e \\ Y_D &= -1 + Y_Q \\ Y_U &= +1 + Y_Q \end{aligned}\right\} \quad Y_Q,\ Y_e \text{ input}$$



A_a (SU(2))

$\Downarrow$

Anomaly $= 0$

$$\mathcal{A}_{abc} \propto \mathrm{Tr}\,\{T_a, T_b\}\, T_c$$

$$\mathrm{Tr}\, Y_L = 0 \Rightarrow \underset{\text{color}}{3} \cdot Y_Q \cdot \underbrace{2 + Y_e \cdot 2}_{\substack{\text{up + down} \\ \nu + e}} = 0$$

$$\boxed{3 Y_Q + Y_e = 0} \; *$$

$$\boxed{Y_q \leftarrow \text{input}}$$

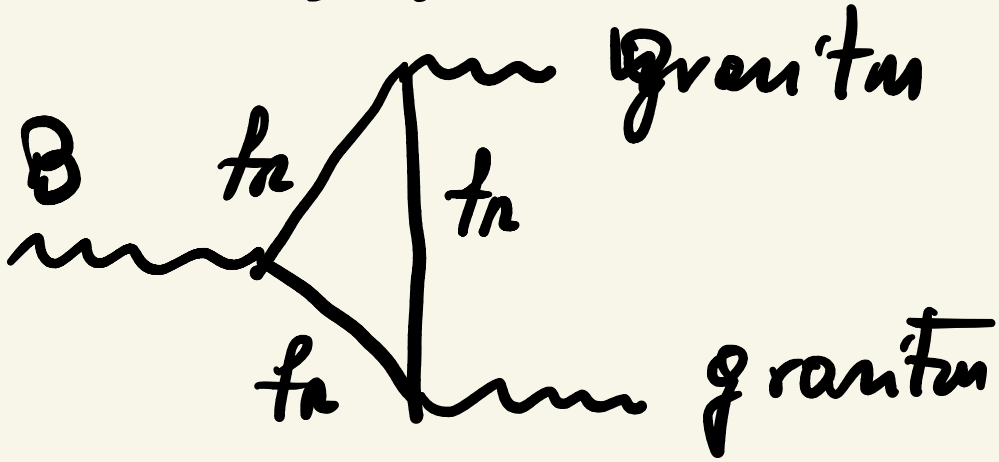
$$Q = T_3 + Y/2 \qquad \mathrm{Tr}\, Y_L = 0$$

$$\Rightarrow \mathrm{Tr}\, Q_L = 0$$

$$Q_L = Q_R \Rightarrow \mathrm{Tr}\, Q_R = 0 \Rightarrow$$

$$\boxed{\mathrm{Tr}\, Y_R = 0}$$

$\Rightarrow$



$$Y_E + (Y_U + Y_D)\,3 = 0$$

$\downarrow$

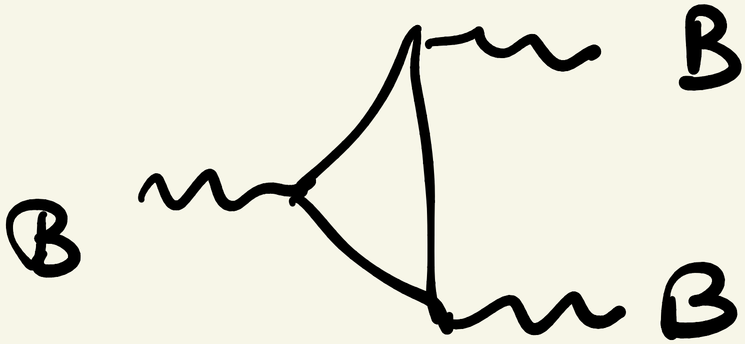
$$-1 + Y_\ell + \left(\cancel{1} + Y_Q + (\cancel{-1} + Y_Q)\right)3 = 0$$

$$-1 + (-3Y_Q) + 6Y_Q = 0$$

$$\boxed{3Y_Q = 1} \Rightarrow \boxed{Y_e = -1}$$

charge quantisation

- cubic anomaly



$$\boxed{\mathrm{Tr}\, Y_L^3 = \mathrm{Tr}\, Y_R^3}$$

SM $\to$ neutrino is massless

$\exists\, \nu_R\ (N)$ — (a)

$\nexists\, \nu_R$ — (b)

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad E_R,\ N_R$$

$$\Rightarrow \boxed{Q_N = Q_\nu}$$

$$Y_N = +1 + Y_l$$

$$Y_E = -1 + Y_l$$

- $\mathrm{Tr}\, Y_L = 0 \Rightarrow \boxed{3Y_q + Y_e = 0}$

- $\mathrm{Tr}\, Y_R = 0 \Rightarrow$

$$\underbrace{Y_E + Y_N}_{\cancel{1} + Y_e - \cancel{1} + Y_e} + \underbrace{(Y_U + Y_D)3}_{1 + Y_q - 1 + Y_q} = 0$$

$$\boxed{2Y_e + 6Y_q = 0}$$

$\Rightarrow Y_q =$ still arbitrary

$$Q_e = -1 + Y_e = -1 - Y_q + Y_e + Y_q$$

$$Q_d = -1 + Y_q$$

$$Q_e = Q_d + Y_q + Y_e$$

$$\Rightarrow \boxed{Q_e = Q_d - 2Y_Q}$$

arbitrary

Y^3 (cubic) anomaly

$$\mathrm{Tr}\, Y_L^3 = \mathrm{Tr}\, Y_R^3 \checkmark$$

$\boxed{\text{charge quantization in SM} \Leftrightarrow \text{neutrino massless}}$

ν massive $\Rightarrow$ its charge is arbitrary in general

$\nu =$ Dirac or Majorana

$m_D \, \bar{\nu}_L \nu_R$ $\qquad$ $m_M \, \nu_L^T C \nu_L$

$\downarrow$ $\qquad$ $\downarrow$

$Q_\nu =$ arbitrary $\qquad$ $Q_\nu = 0$

add N :

$N^T C N$ - gauge + Lorentz invariant

$U(1)_{em}$

$SU(2) \times U(1) \longrightarrow U(1)_{em}$

(b) $\nexists\ \nu_R$, only ν_L

$\Rightarrow$ only $\nu_L^T C \nu_L$

$\Leftrightarrow$ $Q_\nu = 0$

SU(5) · $Q_\nu = 0$

· $m_\nu = 0$ (only ν_L)

if modify SU(5) ∴ $m_\nu \neq 0$

m_D or m_M $\Rightarrow$ $Q_\nu = 0$

in short

$SU(5) \Rightarrow$

(i) $Q_e = 3\,Q_d, \quad Q_\nu = 0$

$Q_u = -2\,Q_d$

(ii) $\not{P}$ maximal

SM

$\not{P}$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R \quad \checkmark$$

SU(5)

$$\bar{5}_F = \begin{pmatrix} d^c \\ \hline \nu \\ e \end{pmatrix}_L \qquad \bar{5}'_F = \begin{pmatrix} d^c \\ \hline \nu \\ e \end{pmatrix}_R$$

$$d^c_R = C \bar{d}_L^T$$

$$10_F = \left(\begin{array}{c|c} u^c & u \; \cancel{d} \, D \\ \hline & E^c \end{array}\right)_L$$

$$10'_F = \left(\begin{array}{c|c} U^c & U \\ \hline & \end{array}\right)_R$$

keep adding new particles

$\boxed{\text{NOT our world}}$

- $m_\nu \neq 0 \Rightarrow \exists\, N = SO(5)$ singlet

in SM: add $N \Rightarrow$ lose charge quantization

in SO(5): $Q_\nu = 0 \Leftarrow Q_N = 0$

$$m_D \bar{\nu} N + N N m_N$$

Q = quantized

$$\text{Cartan} = \{ T_{3c}, T_{8c}, T_{3W} = T_{21}, T_{24} \}$$

$$\Downarrow$$

$$T_{24} = \sqrt{\frac{3}{5}} \left(\begin{array}{ccc|cc} -1/3 & & & & \\ & -1/3 & & & 0 \\ & & -1/3 & & \\ \hline & & & 1/2 & \\ & 0 & & & 1/2 \end{array} \right)$$

$$\mathrm{Tr}\, T_a T_b = \frac{1}{2} \delta_{ab}$$

$$\mathrm{Tr}\, T_{24}^2 = \frac{3}{5} \cdot \left(\frac{1}{9} \cdot 3 + \frac{1}{4} \cdot 2 \right)$$

$$= \frac{3}{5} \left(\frac{1}{3} + \frac{1}{2} \right) = \frac{3}{5} \frac{5}{6} = \frac{1}{2}$$

$$D_\mu = \partial_\mu - i g \, T_i \, A_\mu^i \qquad i = 1, \dots, 24$$

$$= \partial_\mu - \dots - i g \, T_a^W A_\mu^a \qquad a = 1,2,3$$

$$- i g' \frac{Y}{2} B_\mu$$

$$\boxed{\tan\theta_W = g'/g}$$

$$\cancel{SU(2):\ g' = g\ (\theta_W = 45^\circ)}$$

$$5_F = \begin{pmatrix} d^c \\ e^c \\ \nu^c \end{pmatrix}_R \qquad Q = T_3 + \frac{Y}{2}$$

$$\Downarrow$$

$$\frac{Y}{2}(5_F) = \mathrm{diag}\left(-1/3, -1/3, -1/3, 1/2, 1/2\right)$$

$$\Downarrow$$

$$\boxed{T_{24} = \sqrt{\frac{3}{5}}\ \frac{Y}{2}} \leftarrow$$

$$\Downarrow$$

$$\underbrace{g A^\mu_{24} T_{24}}_{SU(5)} = \underbrace{g' B^\mu \frac{Y}{2}}_{SM}$$

$$\boxed{A^\mu_{24} = B^\mu} \quad \boxed{\text{normalised}}$$

$$\mathcal{L}(A,B) = -\frac{1}{4} F_{\mu\nu}{}^a F^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$F_{\mu\nu}{}^a = \partial_\mu A_\nu{}^a - \ldots \qquad B_{\mu\nu} = \partial_\mu B_\nu - \ldots$$

$$\Rightarrow g T_{24} = g' \frac{Y}{2}$$

$$\boxed{g\sqrt{3/5} = g'}$$

$$\Downarrow$$

SU(5) =
Georgi, Glashow

'74

$$\boxed{\tan^2\theta_W = 3/5}$$

$$\Downarrow$$

$$\boxed{\sin^2\theta_W = 3/8}, \quad \cos^2\theta_W = 5/8$$

$$\sin^2\theta_W = 0.2$$

NOT true

$$\sin^2\theta_W^0 = 3/8 \quad \text{true}$$

tree-level $\Longleftrightarrow$ at GUT scale

True at $E \gtrsim 10^{15}$ GeV

coupling "constants"

$\neq$ constants as $f(E)$

$$\sin^2\theta_W(M_{GUT}) = 3/8$$

$$\sin^2\theta_W(M_W) = ?$$

0.23 LEP

Georgi, Quinn, Weinberg '74

$\sin^2\theta_W(M_W) \Leftarrow$ by "running" α's

(renormalization group equation)

$$\boxed{\frac{1}{\alpha(E_2)} - \frac{1}{\alpha(E_1)} = \frac{b}{2\pi} \ln E_2/E_1}$$

$$\frac{1}{\alpha(M_W)} = \frac{1}{\alpha(M_{GUT})} + \frac{b}{2\pi} \ln \frac{M_W}{M_{GUT}}$$

$$M_{GUT} \gtrsim 10^{15}\,\text{GeV}$$

$$\underbrace{\sin^2\theta_W(M_W)}_{\text{exp}} = \sin^2\theta_W(M_{GUT})\left(=3/8\right)$$

$$+ \frac{b(\theta_W)}{2\pi} \ln \frac{M_{GUT}}{M_W}$$

compute

$$\boxed{\begin{aligned} &\text{'}74 \Longrightarrow \sin^2\theta_W(M_W) \overset{SU(5)}{=} 0.2 \\ &\qquad = (\sin^2\theta_W)_{\text{exp}} \end{aligned}}$$

$$\boxed{\text{exp (LEP)} \Longrightarrow \sin^2\theta_W(M_W) = 0.23}$$

- unification of g and g'

$\Rightarrow$ θ_W at M_{GUT}

$\Rightarrow$ run it to M_W

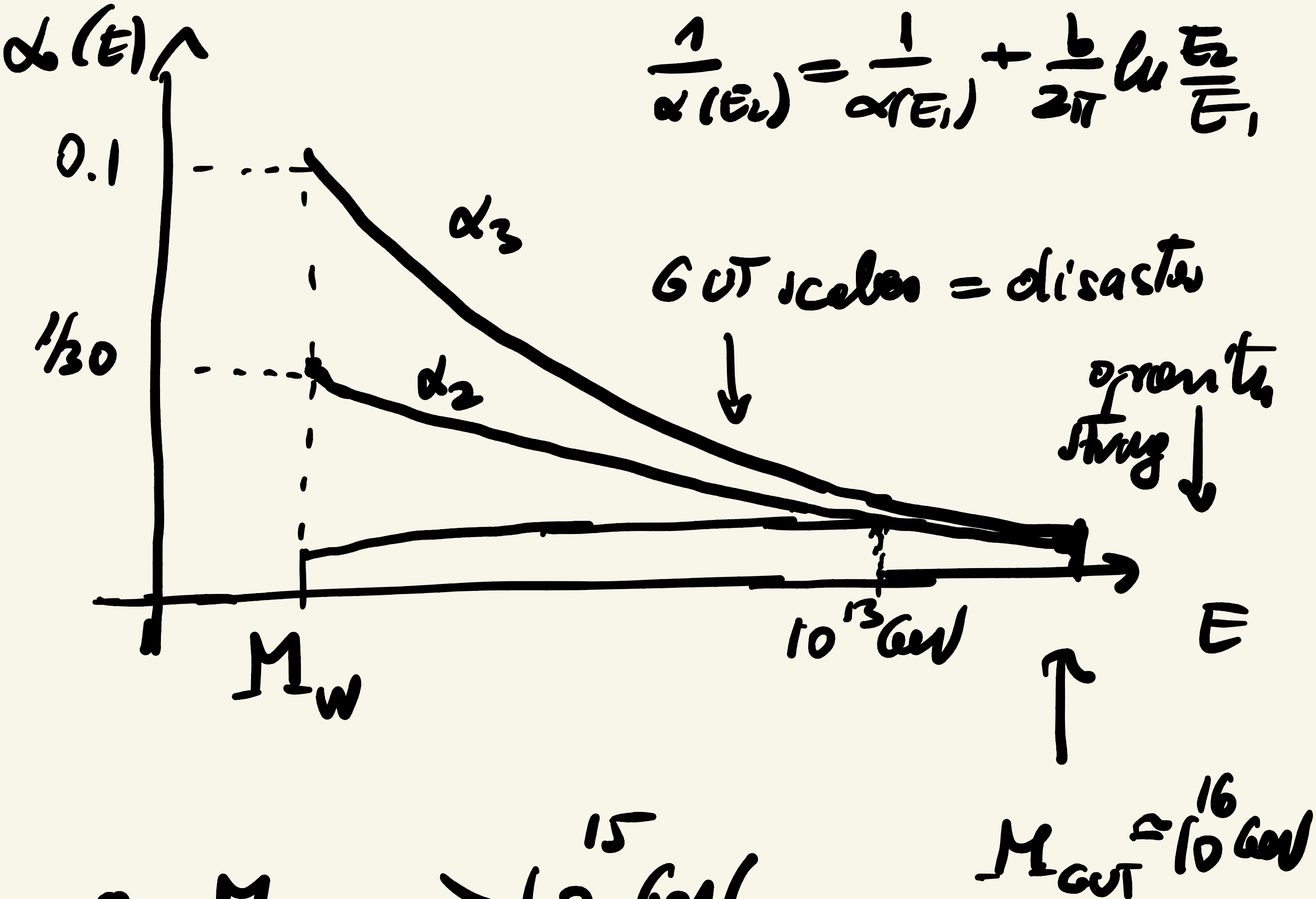
$\Rightarrow$ I fail

$$\boxed{\text{equivalently}}$$

$$\alpha_{em} = \alpha_1, \quad \alpha_2 = \alpha_W, \quad \alpha_3 = \alpha_c$$

em   weak   strong

$$\frac{1}{\alpha(E_2)} = \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln \frac{E_2}{E_1}$$



- $M_{GUT} > 10^{15}$ GeV ($\tau_p > 10^{34}$ yr)

- $M_{GUT} \ll M_{planck}$

$\boxed{\text{in } SU(5) : \text{NO unification}}$

People : $\alpha_1 \leftrightarrow \alpha_2$ meeting M_{GUT}

$\Rightarrow \quad \tau_p \lessapprox 10^{30}\, \mathrm{yr}$

WRONG depiction!

- Natural unification scale M_{GUT}

$= \underline{\text{where } \alpha_2 \leftrightarrow \alpha_3 \text{ meet}}$

$$\Rightarrow \boxed{\exists \text{ new } (X, Y)^{\alpha}} \quad \alpha = 1,2,3$$

weak doublet

$X = X_u$

$Y = X_d$

$$\rightsquigarrow \begin{pmatrix} X_u \\ X_d \end{pmatrix}^{\alpha}$$

di-quarks

lepto-quarks

$\Downarrow$

proton decay

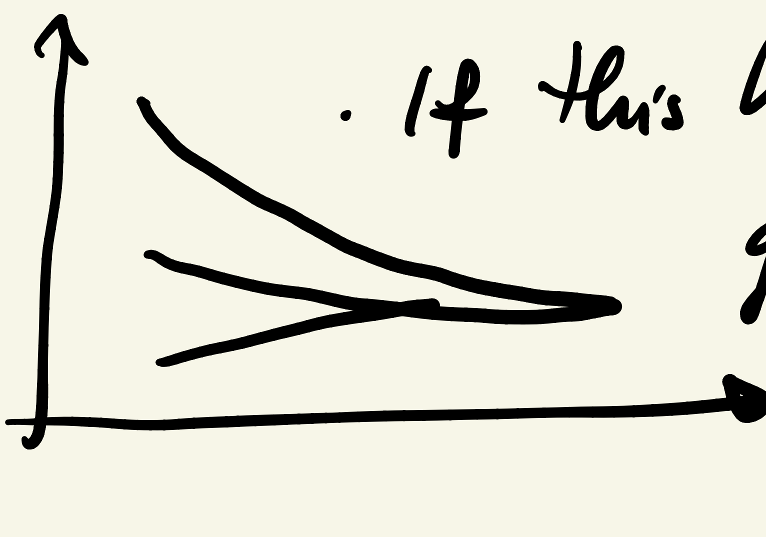
$\alpha_{em} \leftarrow$ runs

$$\alpha_{em}(\text{MeV}) \simeq 1/137$$

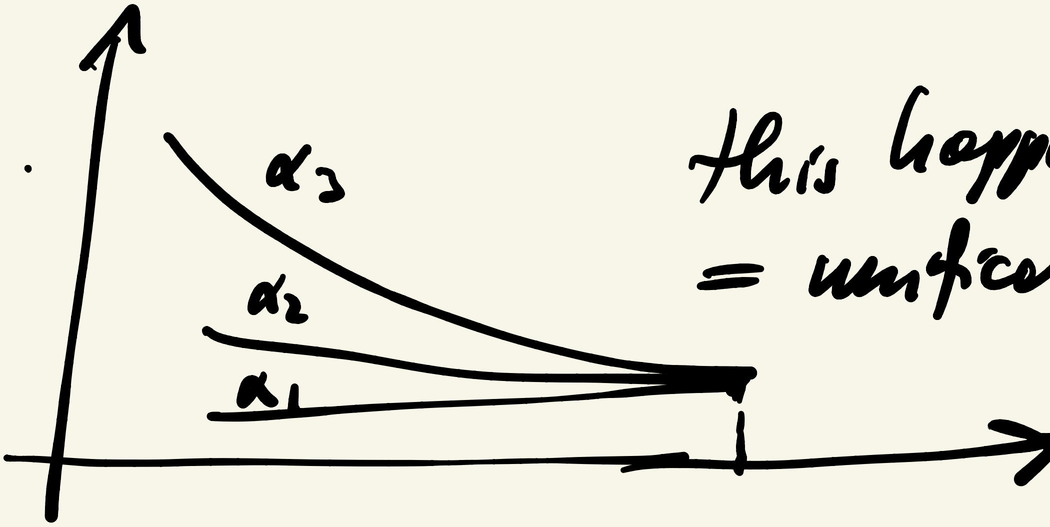
$$\alpha_{em}(M_W) \simeq 1/128$$

$$SU(5) \supseteq SM$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_5 = \alpha_{GUT}$$



· If this happens → go home (shut up)



this happens
= unification

check whether couplings
unify

GQW 74

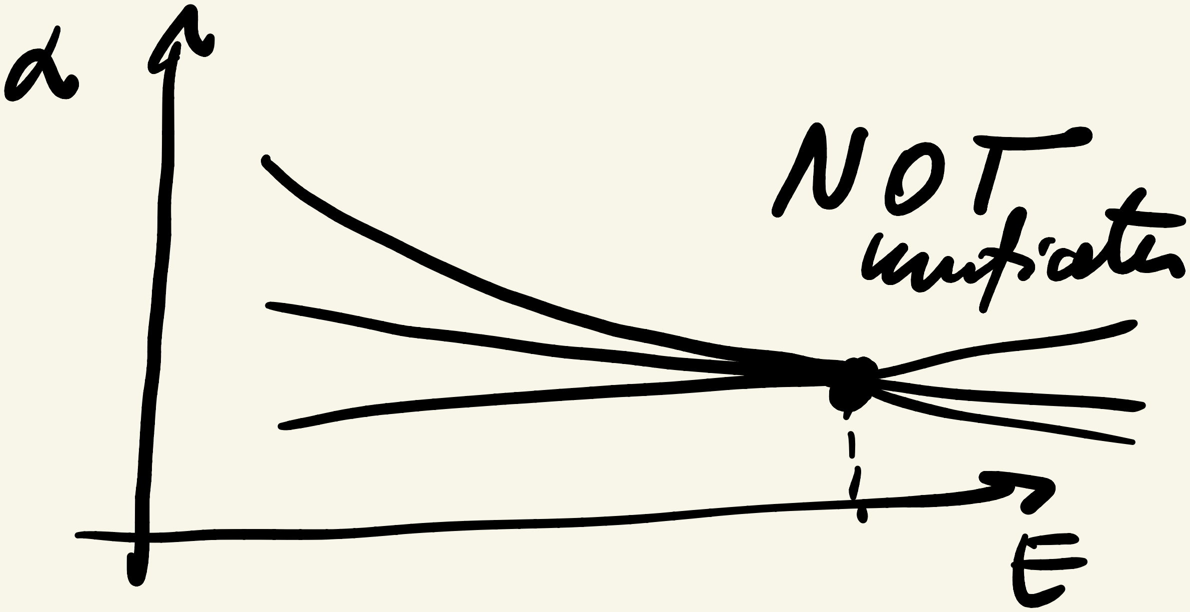
SU(5) $\Leftrightarrow$ unification

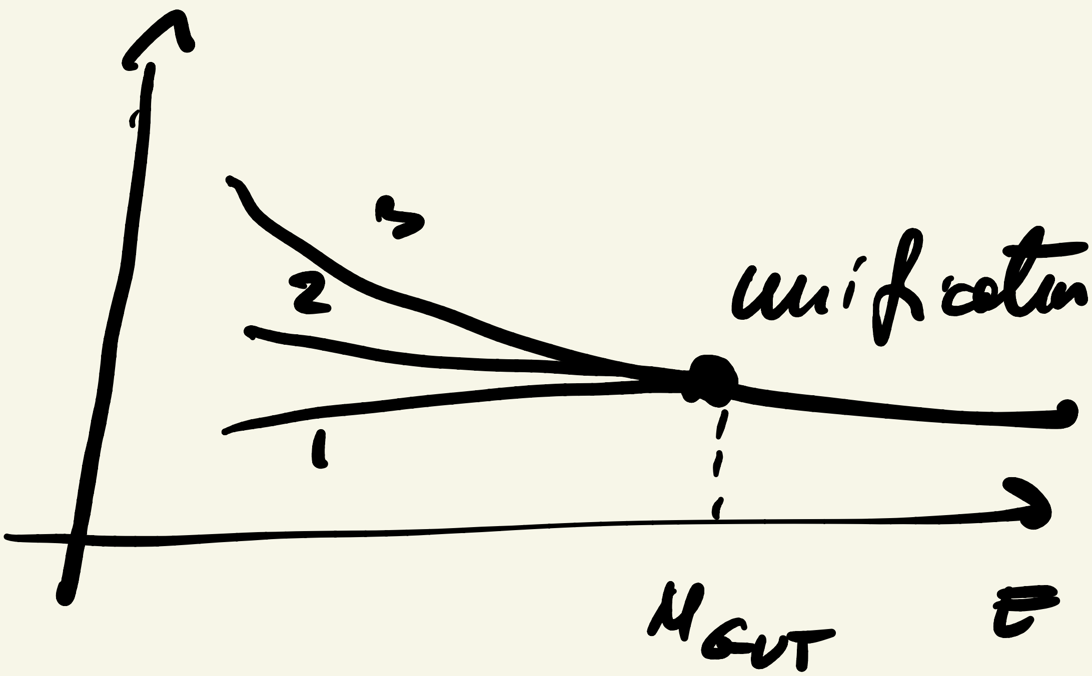
$\Rightarrow \sin^2\theta_W(M_{GUT}) = 3/8$

$\Downarrow$

$\sin^2\theta_W(M_W) = 0.2$

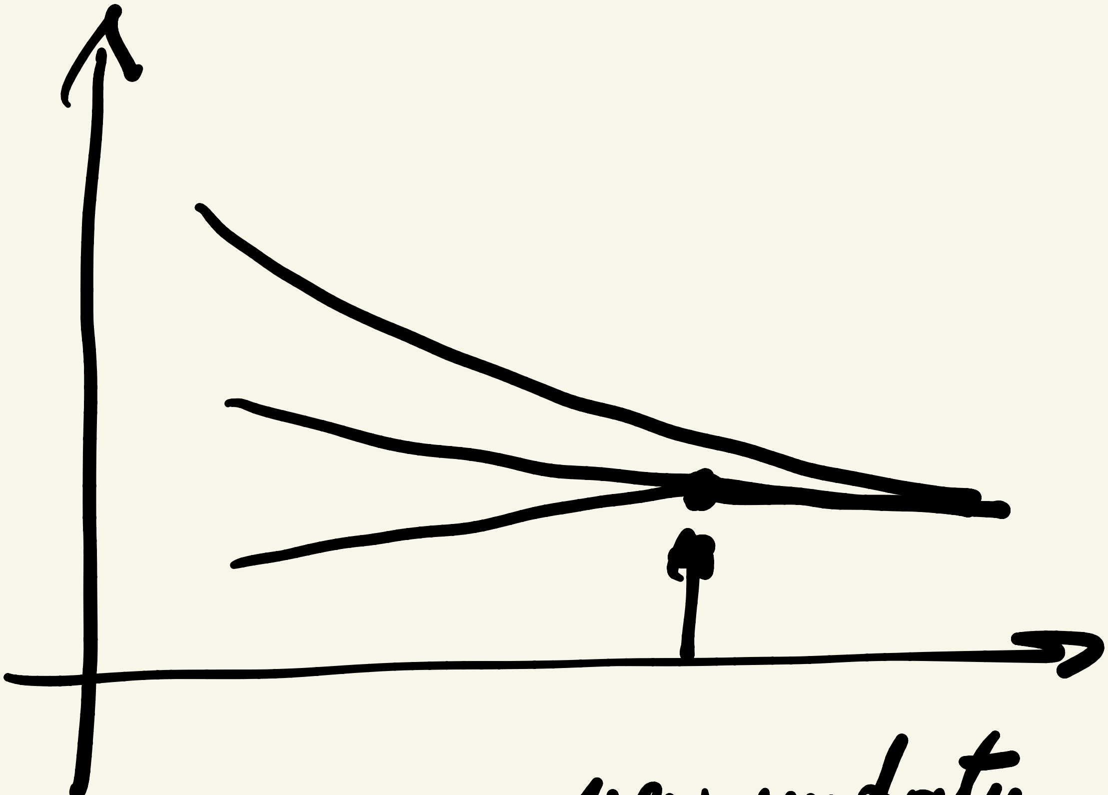
$\downarrow$





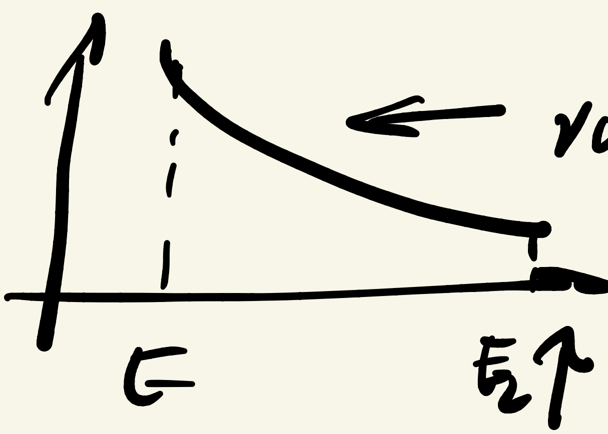
$\Rightarrow$ new physics at M_{GUT}

minimal new physics $= (X, Y)$!!!



near unification

extended theories $\leftrightarrow$ where you have partial unification



particles that contribute

to b = light particles

$(m \leq E,)$

GUT $\Longleftrightarrow$ $G = SU(5)$

light = SM particles

heavy = G new particles

$(m_{new} = M_{GUT})$

$\Downarrow$

do not run

---

running from MeV to M_W

e, A SM

$$m_{new} = M_W$$

$$m_{new} = y\, M_W$$

if I claim in SU(5) :.

$$m_{new} = M_{GUT}$$

$$\Leftrightarrow \quad m_{fermion} = M_W \text{ in SM}$$

$$m_f = y \langle \Phi \rangle = M_W / g$$

$$\boxed{y \ll 1}$$

· it is possible in GUT:

$$m_{new} \ll M_{GUT}$$

People = common ground

$\Longleftrightarrow$ $\boxed{\text{group think}}$

<u>Max</u> : "People"

$\boxed{\text{run} = \text{SM}}$

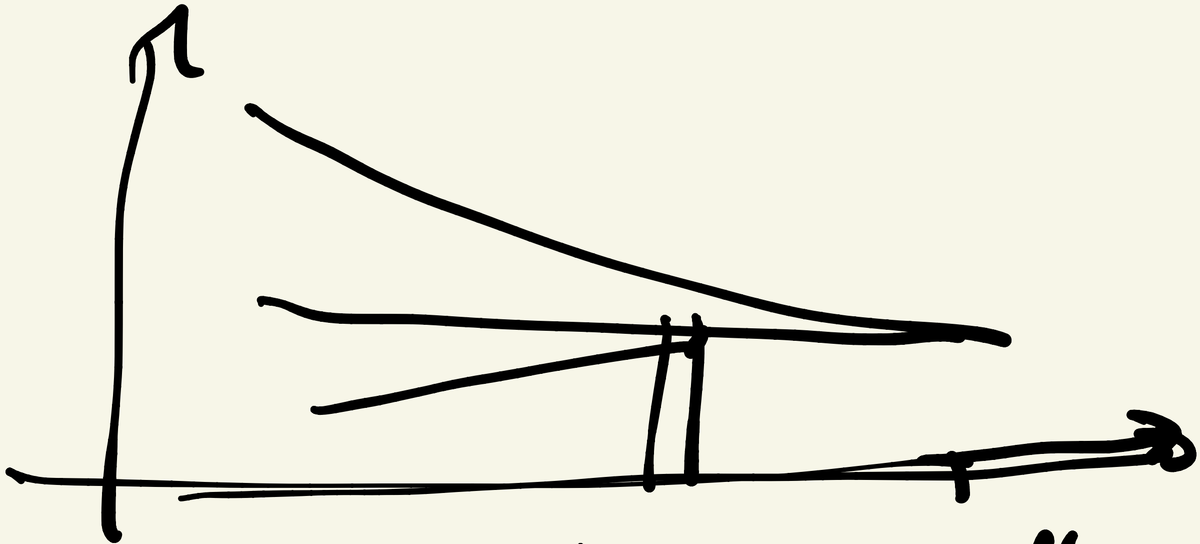
. α_{em} (Mev) $\longrightarrow$ M_W

$$\underbrace{m, \tau, c, b, \ldots}$$

$$m_t = M_W$$

<u>Gon</u> People $\rightarrow$ take into account all the uncertainty

at masses in SU(5)



bad = small

- assume $\alpha_1 = \alpha_2 = \alpha_3 = M_{GUT}$

$\swarrow$ run down

$\alpha_1, \alpha_2, \alpha_3 \, (M_W)$

- $\alpha_1, \alpha_2, \alpha_3$ at M_W

$\longrightarrow M_{GUT}$

bands (<u>not</u> points)

$$\boxed{M_{new} \neq M_{GUT}}$$

$$m_f = g\, M_W$$

$$m_f = m_{top} \simeq M_W$$

$$\text{or } m_e \simeq 10^{-5} M_W$$

$$\boxed{\text{"threshold effects"}}$$

~ 100 new particles

- challenge = check unif. with "threshold effect"