LMU GUT Course

Lecture XIII

15/12/2020

LMU

Fall 2020

SU(5) GUT

$$\begin{pmatrix} u \\ d \end{pmatrix}_L^{\alpha} \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad W$$

$$u_R^{\alpha},\ d_R^{\alpha},\ e_R^{\alpha} \qquad \alpha = r, y, b$$

$$5_F = \begin{pmatrix} d^r \\ d^y \\ d^b \\ e^c \\ -\nu^c \end{pmatrix}_R \quad \begin{matrix} \left.\vphantom{\begin{matrix} a \\ a \\ a \end{matrix}}\right\} SU(3)_c = QCD \\ \left.\vphantom{\begin{matrix} a \\ a \end{matrix}}\right\} SU(2)_L = \text{weak} \end{matrix}$$

$$\boxed{\psi_R^c = C\,\bar{\psi}_L^T}$$

$$15_F = 5_F + 10_F$$

$$10_F = \underbrace{(5 \times 5)_F}_{\text{Anti-symmetric}}$$

all "charges" = sum of "charges" of 5

5_F: $\quad \mathrm{Tr}\, Q_{em} = 0$ $\qquad$ reminder

$$\Downarrow$$

$$\left[\, 3 Q_d + \underbrace{\overset{1}{Q_{e^c}} + \overset{0}{Q_{\nu^c}}} = 0 \,\right]$$

$$Q_{e^c} = Q_{\nu^c} + 1$$

$$10_F = \begin{pmatrix} 0 & u^c & u^c & u & d \\ & 0 & u^c & u & d \\ & & 0 & u & d \\ & & & 0 & e^c \\ & & & -e^c & 0 \end{pmatrix}_L \begin{array}{l} \to (15) \\ \downarrow \text{color} \\ \\ \to (45) \\ \\ \end{array}$$

$$Q_{u^c} = 2\,Q_d \quad (= Q_d + Q_d)$$

$$Q_u = Q_d + Q_{e^c}$$

$$Q_d = Q_d + Q_{\nu^c}$$

$$\Rightarrow \boxed{Q_{\nu^c} = 0 \Rightarrow Q_\nu = 0}$$

$$Q_{e^c} = Q_{e^c} \cancel{+ Q_{\nu^c}} \quad (= 0)$$

$$Q_{\nu^c} = 0 \Rightarrow Q_{e^c} = 1$$

$$(Q_e = -1)$$

$$\boxed{Q_d = -1/3} \qquad \boxed{Q_{u^c} = -2/3}$$

$$\boxed{\Rightarrow Q_u = 2/3}$$

SU(5) $\Rightarrow$ quantization of charge

<u>SM</u>

$$Q_{em} = T_3 + \frac{Y}{2}$$

arbitrary

Anomaly $= 0 \Rightarrow$

add $\underbrace{F_L, \; F_R}_{\text{same "charges"}}$

$\Rightarrow$ Anomaly $= 0$

SU(5) : all charges are quantized

- $5_L, \ 5_R$ ⟸ same charge quantization
- $10_L, \ 10_R$
- $15_L, \ 15_R$

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digression:

$$SU(3) \times SU(2) \times U(1)$$

→ $\begin{pmatrix} u \\ d \end{pmatrix}_L^\alpha$ $\qquad$ $u_R^\alpha, \ d_R^\alpha$

→ $\begin{pmatrix} \nu \\ e \end{pmatrix}_L^\alpha$ $\qquad$ e_R

singlet under

$\cancel{SU(3)}$, $SU(2)$

real world :

$\exists$ conserved charge Q_{em}

$\longleftrightarrow$ photon

$\longleftrightarrow$ vector-like current

$$\Rightarrow \boxed{Q_L = Q_R}$$

$\Rightarrow$ Anomaly $= 0$

implies SM solution

• however, if you do <u>not</u> input $Q_L = Q_R$

$$\Downarrow$$

$$\boxed{Y_{u_R} + Y_{d_R} = 0}$$

$$Y\begin{pmatrix} u \\ d \end{pmatrix} = Y\begin{pmatrix} \nu \\ e \end{pmatrix} = 0$$

not physical

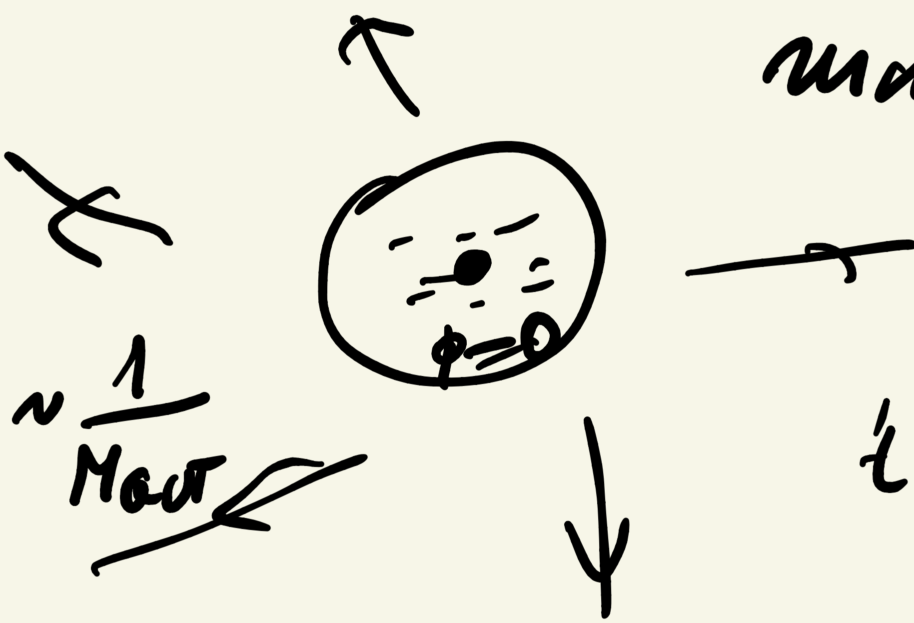
<u>real world</u> : Q_{em}

$$\boxed{Q_L = Q_R}$$

$$\left.\begin{array}{l} \cdot\ Q_e = 3 Q_d \\ \cdot\ Q_\nu = 0 \end{array}\right\} \text{mistery}$$

- $SU(5) \Longrightarrow$ charge quantization of q, l
- SM + anomaly $= 0$ $\Nearrow$

SU(5): difference $\Longrightarrow$ $\exists$ magnetic monopoles



$R \sim \frac{1}{M_{GUT}}$

't Hooft–Polyakov

monopole $\not\Rightarrow$ SU(5) monopole

$$\boxed{\text{there could be point-like Dirac monopole}}$$

$$M_{GUT} \simeq 10^{16}\,\text{GeV}$$

- $$R_{GUT} \simeq 1/M_{GUT} \simeq 10^{-30}\,\text{cm}$$

$$R_e \leq \left(\frac{1}{\text{TeV}} = 10^{-3}\,\text{GeV}^{-1} \simeq 10^{-17}\,\text{cm}\right)$$

(i) Unify weak + strong

$\Downarrow$

charge quantization

(iii) charge quantization $\Rightarrow$

non-Abelian $\Rightarrow$

unification

$\Downarrow$

$$\boxed{\text{new interactions} = \text{new gauge bosons}}$$

$$\begin{array}{c} X \\ Y \end{array} \begin{pmatrix} d \\ e^c \\ -\nu^c \end{pmatrix}_R \Rightarrow \boxed{\begin{array}{l} \bar{d}_R^{\alpha} \gamma^{\mu} e_R^c X_{\mu}^{\alpha} \\ - \bar{d}_R \gamma^{\mu} \nu_R^c Y_{\mu} \end{array}}$$

X^α = 3 states

Y^α = —"—

$(\bar{X}, \bar{Y}) \Rightarrow 6+6=12$ states

$$\underset{\substack{\text{gauge}\\ SU(5)\ \text{bosons}}}{24} = \underset{\text{SM}}{12} + \underset{(X,Y)}{12}$$

$X(-4/3)$

$Y(-1/3)$ $\Rightarrow$ $\underbrace{\begin{pmatrix} Y \\ X \end{pmatrix}^\alpha}_{\substack{SU(2)_L\ \text{doublet}\\ \text{color triplet}}}$ α

$X, Y =$ lepto-quarks

$$\underbrace{\overline{d_R}\,\gamma^\mu\,l^c_R}_{} \qquad l^c = \begin{pmatrix} e^c \\ -\nu^c \end{pmatrix}_R$$

$$B = -\frac{1}{3}, \quad L = -1$$

$$\boxed{\begin{aligned} B(X,Y) &= +1/3, \\ L(X,Y) &= +1 \end{aligned}}$$

$\boxed{B, L \text{ are violated?}}$

NO

$$\boxed{\text{lepto-quarks do } \underline{\text{not}} \text{ mediate } B, L \text{ violation}}$$

$$\boxed{\cdot\ 10_F}$$

$$\begin{array}{cc} & \text{color} \qquad \text{weak} \\ \text{color} & \left(\begin{array}{c|cc} u^c & u & d \\ \hline & & e^c \end{array}\right)_L \begin{array}{l} \}\ \text{color} \\ \}\ \text{weak} \end{array} \end{array}$$

$$\epsilon_{\alpha\beta\gamma}\, \overline{u^c_L}_{\alpha}\, \gamma^\mu\, u_{L\,\beta}\, X_{\mu\,\gamma}(-4/3) \quad \leftarrow$$

$$\overline{u^c_L}\,\gamma^\mu d_L \quad Y\left(-1/3\right) \leftarrow$$

gauge theories

$$\mathcal{L} = i\,\underbrace{\bar{\Psi}\gamma^\mu D_\mu \Psi}_{}$$

$$D_\mu = \partial_\mu - i g T_a A^a_\mu$$

$$\cancel{\bar{\Psi}_1 \gamma^\mu \partial_\mu \Psi_2} \qquad \boxed{\text{by rotation}}$$

SM: $\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad (u^c)_L,\ (d^c)_L$

$$\bar{5}_F = \begin{pmatrix} d^c \\ \hline \begin{pmatrix} \nu \\ e \end{pmatrix} \end{pmatrix}_L$$

L↔R

$$\Leftrightarrow \; 5_F = \begin{pmatrix} d \\ \hline \begin{pmatrix} e^c \\ -\nu^c \end{pmatrix} \end{pmatrix}_R$$

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di-quark

$$\mathcal{L}_{(X,Y)} = \Big(\overline{u^c_L}\, \gamma^\mu u_L \; +$$

$$+ \; \bar{d}_R\, \gamma^\mu e^c_R \Big) X_\mu$$

lepto-quark

$$+ \Big(\overline{u^c_L}\, \gamma^\mu d_L \; +$$

$$+ \; \bar{d}_R\, \gamma^\mu \nu^c_R \Big) Y_\mu$$

$$\bar{d}_R \; e^c_R \quad : \quad B = -\tfrac{1}{3}, \; L = -1$$

$$\bar{u}^c \; u \quad : \quad B = \tfrac{2}{3}, \; L = 0$$

$$\underbrace{B - L = 2/3}_{\text{conserved}}$$

$$\Downarrow$$

$$B, \; L = \text{broken}$$

$$W^+_\mu \underbrace{\left[\bar{u} \gamma^\mu d + \bar{\nu} \gamma^\mu e \right]}_{B=0, \; L=0}$$

$\Rightarrow \quad B(w) = L(w) = 0$

$\Rightarrow \quad B, L =$ conserved in SM

Digression!

① diagonalize $\mathcal{L}_{kin}$

0- step

② Find $\underline{M}$ = mass matrix

③ Diagonize M:

$$V_L \, \underline{M} \, V_R^{+} = \underbrace{m}_{\text{diagonal}}$$

$$m = \mathrm{diag}(m_1, m_2, \ldots)$$

down: $1 = d, \; 2 = s, \; 3 = b$

leptons: $1 = e, \; 2 = \mu, \; 3 = \tau$

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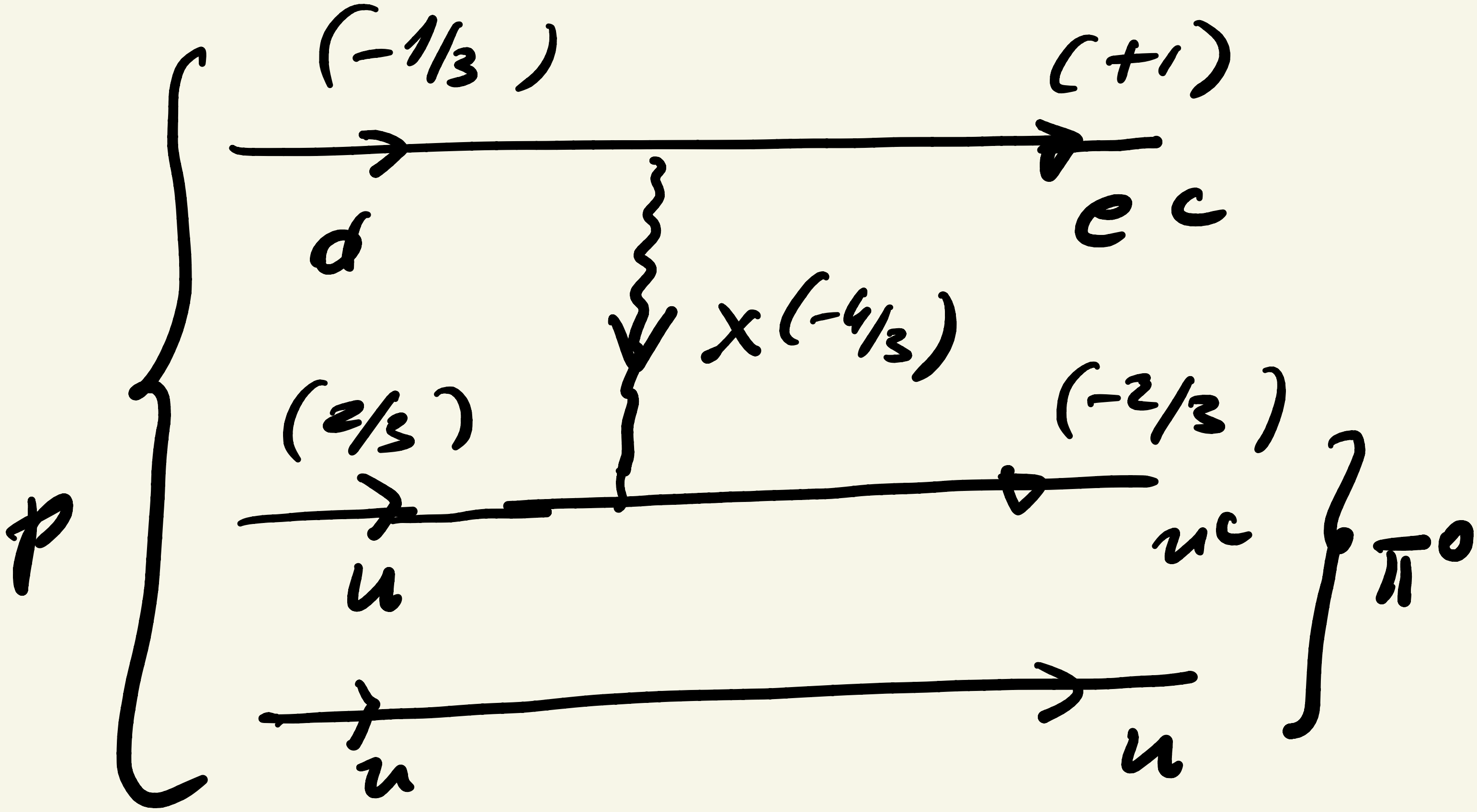
enter u, exit u^c

$$X \left(\overline{u^c}\, u + \bar{d}\, e^c \right) +$$

$$+ \bar{X} \left(\bar{u}\, u^c + \underbrace{\overline{e^c}\, d}_{\substack{\text{enter } d \\ \text{exit } e^c}} \right)$$

$$\Downarrow$$



4 - fermion effective

"super" weak int

$\Longleftrightarrow$ Fermi weak int.

Fermi: $d \to u + e + \bar{\nu}$

SU(5): $d \to u^c + u^c + e^c$

$\Delta B \neq 0 \neq \Delta L$

$\Downarrow$

$$\Delta(B-L) = 0$$

$\frac{1}{3}$ on left $= -\frac{2}{3}+1 = 1/3$ on right

$$\Downarrow$$

$$\not\Downarrow \quad \boxed{p \longrightarrow \pi^0 + e^+}$$

$B-L$: $\quad 1 = -(-1) = +1$ ✓

<u>Ferm.</u> : $\quad G_F \propto \frac{1}{M_W^2}$

(W = messenger)

$$\Downarrow$$

$$\mathcal{H}_{\substack{eff \\ (\Delta B=0)}}^{F} \propto \frac{1}{M_W^2} (\bar{u} d)(\bar{e} \nu)$$

$$\mathcal{H}_{eff}^{\Delta B \neq 0} \propto \frac{1}{M_X^2} (\bar{u}^c u)(\bar{e}^c d)$$

$\mu \to e + \nu_\mu + \bar{\nu}_e$

$\tau_\mu \simeq 10^{-6}$ sec

· proton decay $\neq$ seen

$\tau_p \gtrsim 10^{34}$ yr

$$\tau_p / \tau_\mu \gtrsim \frac{10^{34} \cdot 10^{7}}{10^{-6}} \simeq 10^{47}$$

$$\tau_p = \Gamma_p^{-1} \;; \quad \Gamma_p \propto \frac{1}{M_X^4} m_p^5$$

$$\tau_\mu = \Gamma_\mu^{-1} \;; \quad \Gamma_\mu \propto \frac{1}{M_W^4} m_\mu^5$$

$$\Downarrow$$

$$\frac{\tau_p}{\tau_\mu} = \left(\frac{M_X}{M_W}\right)^4 \left(\frac{m_\mu}{m_p}\right)^5$$

$$= 10^{-5} \left(\frac{M_X}{M_W}\right)^4$$

$$\gtrsim 10^{47} \quad (\text{expeiment})$$

$$\Downarrow$$

$$\left(\frac{M_X}{M_W}\right)^4 \gtrsim 10^{52}$$

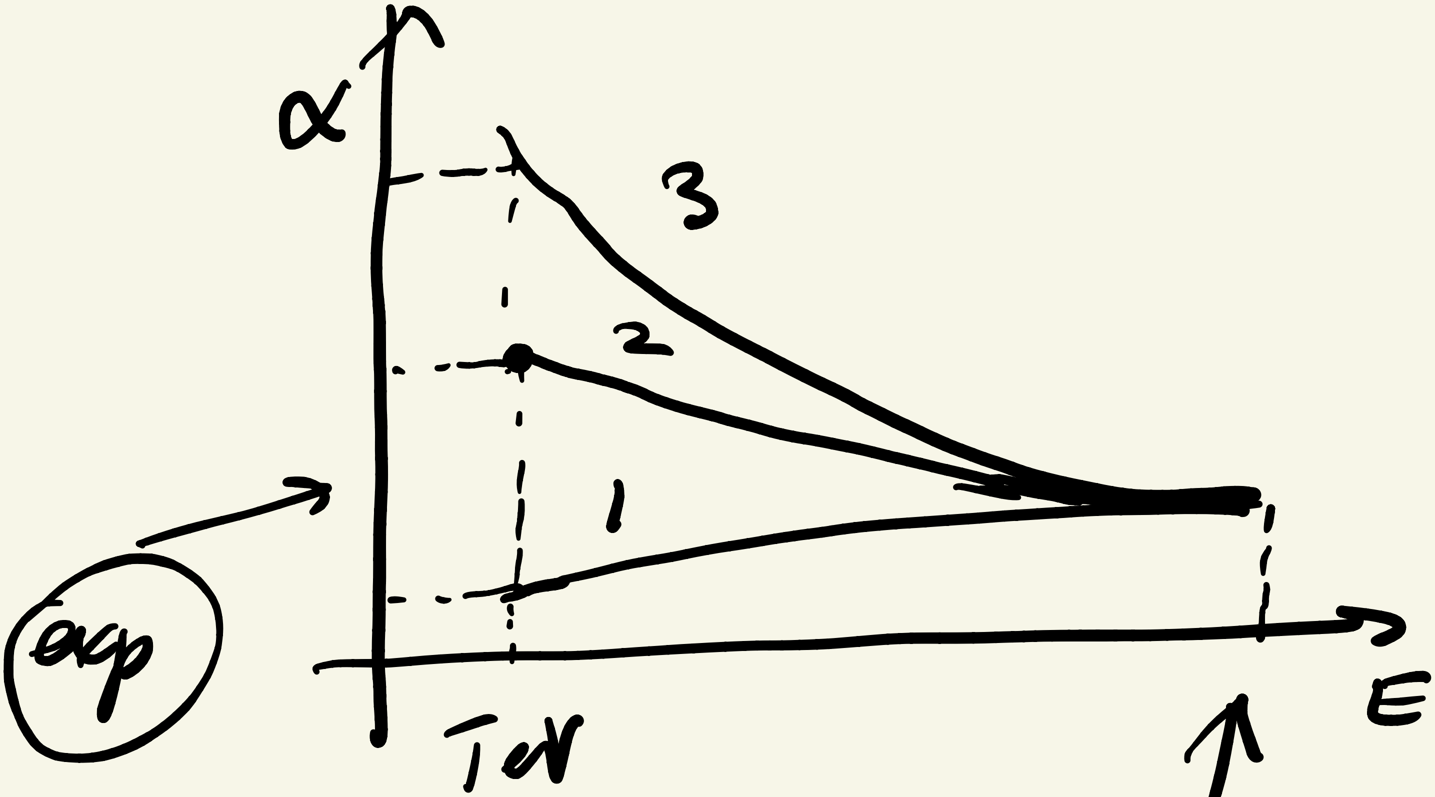
$$\Rightarrow \frac{M_X}{M_W} \gtrsim 10^{13}$$

$$\Rightarrow \boxed{M_X \gtrsim 10^{15}\,\text{GeV}}$$

. if there is unification

$$\Leftrightarrow \text{ if } SM \subseteq SU(5)$$

$$\Rightarrow g_1 = g_2 = g_3 = g_5$$



$$\boxed{M_{GUT} \simeq M_X}$$

we can compute M_X

$$\boxed{M_X \simeq 10^{16}\,\text{GeV}}$$

GUT : tasks

(i) couplings unify

(ii) fermion masses must agree with exp.

(iii) correlate $l \leftrightarrow q$

$\Rightarrow$ compute M_X !

$$\boxed{\text{compute } p \text{ decay branching ratios !}}$$

$\tau_p \propto M_X^4$

small mistake in M_X (~2 factor) $\Rightarrow$

~16 in $\underline{\underline{\tau_p}}$!

$p \to \pi^0 + e^+$, $\pi^+ + \nu$

$\pi^0 + \mu^+$, - - -

$\left(\propto \frac{1}{M_X^4}\right)$ $K^0 + e^+$ - - -

$\boxed{\text{in branching ratios } M_X \text{ uncertainty cancels !}}$

$M_X \simeq 10^{15}$ GeV $= +\infty$

for all, except

p decay

since $\tau_p \gtrsim 10^{34}$ yr!

$$\begin{array}{ccc} G & \xrightarrow{M_{GUT},\ \Sigma\ \text{(new Higgs)}} & H = SM \\ \| & & \downarrow{\scriptstyle \Phi} \\ SU(5) & & U(1)_{em} \times SU(3) \end{array}$$

Φ: SM Higgs

$$\Rightarrow \left.\begin{aligned} M_x^2 &= g^2\left[\langle\Sigma\rangle^2 + ?\langle\Phi\rangle^2\right] \\ M_y^2 &= g^2\left[\langle\Sigma\rangle^2 + ??\langle\Phi\rangle^2\right] \end{aligned}\right.$$

doublet of SU(2)

$\Uparrow$

$$SU(2)_L \Rightarrow M_x = M_y$$

$\uparrow$

broken by $\langle\Phi\rangle = \dfrac{M_W}{g}$

$$\langle\Sigma\rangle = M_{GUT} \simeq 10^{16}\,\text{GeV}$$

$$\langle\Phi\rangle = M_W \simeq 100\,\text{GeV}$$

$$\boxed{M_W^2 \simeq 10^{-28} M_X^2}$$

- why did people (Nobel 1979) believe in SM?

W was discovered in 1983

SM (Glashow) $\Rightarrow$ "branching ratio"

$$\Downarrow$$

$$\frac{g}{\cos\theta_W} Z^\mu \bar{f} \left[T_3 - Q \sin^2\theta_W \right] \gamma_\mu f$$

$$\Downarrow$$

all neutral processes

$$\boxed{\text{"V − A was the key"}}$$

Weinberg '09

Fermi theory

Fermi at GUT effective
(x, y) Theory

$SU(3) \times \underline{SU(2)} \times U(1)$

$\subseteq \underline{SU(4)} \times U(1)$

$$\left(\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \hline \times \\ \times \end{array}\right) \begin{array}{l} \left.\vphantom{\begin{array}{c}\bullet\\\bullet\\\bullet\end{array}}\right\} SU(3) \\ \left.\vphantom{\begin{array}{c}\times\\\times\end{array}}\right\} SU(2) \end{array}$$

$$\Rightarrow \boxed{\begin{array}{c} SU(5) = \text{minimum} \\ \text{GUT} \end{array}}$$

$$3+2+\dots \geq 5$$