LMU GUT COURSE

Lecture XI

8/12/2020

LMU
Fall 2020

SU(5) GUT

How to build it?

SU(5) for pedestrians

SM in a nut-shell

$$SU(3)_c \times SU(2)_L \times U_Y(1)$$

strong

ew int.

$\boxed{Y}$

$\boxed{T_a^W}$ $a = 1, 2, 3$

T_α^c

$\alpha = 1, \dots, 8$

$$[T_\alpha^c, T_a^w] = [T_\alpha^c, Y]$$

$$= [T_\alpha^w, Y] = 0$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L^{\alpha = r,y,b} \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$(u_R, d_R) \to (u^c, d^c)_L \qquad e_R \to (e^c)_L$$

$$(\psi^c)_L = C \bar{\psi}_R^T$$

$$\alpha = r, y, b$$

<u>SU(2)</u>

$$T_1, T_2, T_3 \to A_1, A_2, A_3$$

$$T_\pm = T_1 \pm i T_2 \qquad \Downarrow$$

$$W^{\pm} = \frac{A_1 \mp i A_2}{\sqrt{2}}$$

$$T_1 A_1 + T_2 A_2 \propto$$

$$\propto \left(T_+ W_+ + T_- W^- \right)$$

flavor = ν, e

u, d

W changes flavor

W = "color blind"

$$T_3 = \frac{\sigma_3}{2} = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

$\Downarrow$ SU(3)

$T_\pm$ $\qquad T^c_a = \frac{\lambda_\alpha}{2} \qquad \alpha = 1, \dots, 8$

$\uparrow$ out of σ_+, σ_-

$$\sigma_\pm \propto \sigma_1 \pm i\sigma_2$$

$$\sigma_\pm \; (\sigma_{1,2}) \longrightarrow \begin{matrix} (12) & (13) \\ (23) & \end{matrix}$$

$$T^c_{\substack{1\\(2)}} = \frac{1}{2}\begin{pmatrix} 0 & 1(-i) & 0 \\ (i)1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^c_{\substack{4\\(5)}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1(-i) \\ 0 & 0 & 0 \\ (i)1 & 0 & 0 \end{pmatrix} \qquad T^c_{\substack{6\\7}} = \dots$$

SU(2) $\qquad [T_a, T_b] = i\,\epsilon_{abc}\,T_c$

$$\boxed{3 \times 2 = 6 \quad (T_\pm)}$$

$$+ \boxed{\text{Cartan}}$$

$$\boxed{\begin{aligned} T_3^c &= \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \\ T_8^c &= \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \end{aligned}}$$

[illegible] "A_L"

$$\longleftarrow \quad \xrightarrow{\text{color}}$$

$$W \curvearrowleft \begin{pmatrix} u \\ d \end{pmatrix}^r_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}^y_2 \qquad \begin{pmatrix} u \\ d \end{pmatrix}^b_L$$

? colored "~~W~~"

? colored "~~W~~"

$$\boxed{T_3^W = \text{unique}} \qquad \begin{pmatrix} u \\ d \end{pmatrix} \sim \begin{pmatrix} \nu \\ e \end{pmatrix}$$

em charge

$$Q_{em} \neq Y$$

$$[Y, T_a^W] \Rightarrow$$

$$Y u_L = Y d_L$$

$$\Rightarrow$$

$$Y \nu_L = Y e_L$$

$$Q_{em} = \cancel{a T_3^c} + \cancel{b T_8^c} + c T_3^W + d Y$$

$$\boxed{\text{photon} = \text{color blind}}$$

- $Q_u - Q_d = +1$

$$Q_u - Q_d = c\left(\tfrac{1}{2}\right) - c\left(-\tfrac{1}{2}\right)$$

$$= c \Rightarrow \boxed{c=1}$$

$$\Downarrow$$

$$\boxed{Q_{ew} = T_3 + \frac{1}{2} Y}$$

$\boxed{\text{arbitrary}}$

$$\boxed{Y = 2 \left[Q_{ew} - T_3 \right]}$$

$$e_R \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_L$$

sym. breaking

(doublet) $\Phi \longrightarrow U_w \Phi$

$$T_a^w \Phi = \frac{\sigma_a}{2} \Phi$$

$$\boxed{Y_\Phi = 1} \qquad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$Q = \left[T_3 = \begin{pmatrix} 1/2 & \\ & -1/2 \end{pmatrix} + \frac{Y}{2} = \begin{pmatrix} 1/2 & \\ & 1/2 \end{pmatrix} \right]$$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \leftarrow$$

$$Q_{em} \langle \Phi \rangle = 0$$

$$T_a \langle \Phi \rangle \neq 0$$

$$\left. \begin{aligned} T_3 \langle \Phi \rangle &\neq 0 \\ Y \langle \Phi \rangle &\neq 0 \end{aligned} \right\}$$

$$T_3 (\Phi) = t_3 \langle \Phi \rangle$$

$$Y \langle \phi \rangle = y \langle \Phi \rangle$$

$$\Downarrow$$

$$\underbrace{(y T_3 - t_3 Y)}_{Q_{em}} \langle \Phi \rangle = 0$$

$$\langle \Phi_c \rangle = \begin{pmatrix} 0 \\ v_c \end{pmatrix}$$

$$\cancel{\langle \Phi_M \rangle = \begin{pmatrix} v_M \\ 0 \end{pmatrix}}$$

$$\cancel{\langle \Phi_0 \rangle \begin{pmatrix} ' \\ v_2' \end{pmatrix}}$$

$$\cancel{Q_{em} \neq f(T_3, Y) = c_1 T_1^W + c_2 T_2^W + c_3 T_3^W + d Y}$$

$$\xcancel{\psi = \begin{pmatrix} a\nu + b e \\ c e + d'\nu \end{pmatrix}}$$

$$\boxed{Y_\Phi = +1}$$

$$\mathcal{L}_Y = \boxed{\overset{+1}{(\bar{\nu}\ \bar{e})_L}\ \overset{+1}{\bar{\Phi}}\ \overset{-2}{\underset{\uparrow}{e_R}}}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv l_L \qquad Y = 2\left[Q - T_3^W\right]$$

$$Y_{e_R} = 2\left[(-1) - 0\right]$$

$$= -2$$

$$\Rightarrow \boxed{Y_{l_L} = -1}$$

$$\Downarrow$$

$$Q_\ell = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$ ecity

$$\Rightarrow \boxed{\ell_\nu = 0}$$ "predicted"

$$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$$

$$\cancel{\bar{e}_L e_R}$$

$$\bar{\ell}_L \bar{\Phi} e_R \Rightarrow$$

anomaly cancellation ⇒
fixes charges

$q_u = 2/3, \quad q_d = -1/3$

$\boxed{q_e = -1} \quad \boxed{q_\nu = 0}$

$$\langle \Phi_G \rangle \leftrightarrow \langle \Phi_M \rangle = \langle \Phi_0 \rangle$$

$$\phi \to U \Phi$$

$$\langle \Phi_0 \rangle \xrightarrow{U} \langle \Phi_G \rangle$$

$$\boxed{U^{+}_{0G} \langle \Phi_0 \rangle = \langle \Phi_G \rangle}$$

$$\langle \Phi_0 \rangle = \underset{\overset{\shortparallel}{?}}{\boxed{U_{0G}}} \underset{\shortparallel}{\langle \Phi_G \rangle}$$

$$\begin{pmatrix}0\\ v\end{pmatrix}$$

$$\Psi_L = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_L \quad \Big\| \quad \psi_R$$

$$\Phi = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$$

$$\langle \Phi_0 \rangle = \begin{pmatrix}v_1\\ v_2\end{pmatrix}$$

- $\bar{\Psi}_L \, \Phi \, \psi_R \longrightarrow$

$$\bar{\Psi}_L \langle \Phi_0 \rangle \psi_R =$$

$$= \underbrace{\left(\bar{\psi}_{1L} \, v_1 + \bar{\psi}_2 \, v_2 \right)}_{\bar{e}_L} \underbrace{\psi_R}_{e_R}$$

$$\psi_L^{G} = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \langle \Phi^{a} \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\langle \Phi^{0} \rangle = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$e_L = (v_1 \psi_{1L} + v_2 \psi_{2L}) \frac{1}{\sqrt{v_1^2 + v_2^2}}$$

$$\nu_L = (v_2 \psi_{1L} - v_1 \psi_{2L}) \frac{1}{\sqrt{}}$$

$$\Downarrow$$

$$\psi_L^{\oplus} = \begin{pmatrix} v_2 \nu + v_1 e \\ \perp \nearrow \end{pmatrix}_L$$

$$\boxed{M_W^2 = \frac{g^2}{4}(v_1^2 + v_2^2)}$$

$SO(3)$

$$\downarrow \quad \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$\vec{\Phi} = (\phi_1, \phi_2, \phi_3)$$

$$V \propto -\Phi^T \Phi + (\Phi^T \Phi)^2$$

$$\Rightarrow \quad \langle \Phi^T \Phi \rangle \neq 0$$

$$\langle \Phi^G \rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} SO(2) \\ \\ \end{matrix}$$

$$\langle \Phi^0 \rangle = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

SU, Φ (doublet)

$$(D_\mu \Phi)^\dagger (D^\mu \Phi)$$

$\langle \Phi \rangle \neq 0$ $\quad$ $V = f(\Phi^\dagger \Phi)$

$$\to (D_\mu \langle \Phi \rangle)^\dagger (D^\mu \langle \Phi \rangle)$$

$$D_\mu = \partial_\mu - i g T_a A_\mu^a$$

$\downarrow$ $T_a = \sigma_a/2$ $\quad$ $T_a = T_a^\dagger$

$$g^2 \langle \Phi^\dagger \rangle T_a A_\mu^a T_b A^\mu_b \langle \Phi \rangle$$

$$= \frac{g^2}{4} \langle \Phi^\dagger \rangle \left(\delta_{ab} + \cancel{i \epsilon_{abc} T_c} \right) \langle \Phi \rangle A_\mu^a A^{\mu b}$$

$$= \frac{g^2}{4} \boxed{A_\mu^a A^\mu_a} \langle \Phi^+ \rangle \langle \Phi \rangle$$

$$\|$$

$$\langle \Phi^+ \Phi \rangle$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L + \cdots$$

$$\boxed{\begin{pmatrix} t' \\ b' \end{pmatrix}_L \; ; \; t'_R, b'_R}$$

$$(\bar{t}' \; \bar{b}')_L \quad \Phi \quad b'_R$$

$$Y'_L \, (2) \qquad\qquad Y'_R$$

$$\boxed{\begin{pmatrix} N \\ E \end{pmatrix}_L \quad \cdots}$$

$\psi'(e) = ?$

$$\langle \bar{q}_L q_R \rangle = \Lambda_{QCD}^3$$

break chiral symmetry

$$q_L \to e^{i\alpha} q_L, \quad q_R \to q_R$$

$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$, q_R = singlet

$\Longleftrightarrow$ Higgs

$SU(5)$:

building

$\underline{SU(3) \times SU(2)} = \boxed{SU(5)}$

strong + weak

$SU(5) = G_{min}$

$r(SU(5)) = 4 \Rightarrow$

$U(1)$ for free

$\Downarrow$

quantization of charge

SU(5) $\qquad U^+ U = 1$

$\qquad\qquad\qquad\qquad \det U = 1$

$$F \rightarrow U F$$

$$U = e^{iH} \qquad H = H^+$$

$$\mathrm{Tr}\, H = 0$$

$$H = \sum_i T_i \theta_i \qquad i = 1, \dots, 24$$

$$[T_i, T_j] = i f_{ijk} T_k$$

$T_{1,2} = \frac{\sigma^{1,2}}{2}$ in SU(2)

place them in all possible directions

$$\Downarrow$$

$$\underset{(2)}{T_1} = \frac{1}{2}\begin{pmatrix} 0 & 1(i) & 0 & 0 & 0 \\ (-i)1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T_4 = \begin{pmatrix} 0 & 0 & 1(i) & \\ 0 & & & \\ (-i)1 & & & \cdots \end{pmatrix}$$

$$\begin{aligned} 12,\ 13,\ 14,\ 15 &= 4\times 2 \\ 23,\ 24,\ 25 &= 3\times 2 \\ 34,\ 35 &= 2\times 2 \\ 45 &= 2 \\ \hline & \boxed{20} \end{aligned}$$

$+ \boxed{\text{Cartan}}$

$$\left\{\begin{pmatrix} \times \\ \times \\ \times \\ \hline \bullet \\ \bullet \end{pmatrix}\right. \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} SU(3)_C \\ \longrightarrow SU(2)_W \end{matrix}$$

$$T_3 = T_3^{\,c} = \frac{1}{2}\begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$$

$$T_8 = T_8^{\,c} = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & -2 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$$

$$\boxed{\operatorname{Tr} T_i T_j = \frac{1}{2}\delta_{ij}}$$

$$\begin{matrix} T_3^{\,W} \\ (T_{22}) \end{matrix} = \frac{1}{2}\left(\begin{array}{ccc|cc} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ \hline & & & 1 & \\ & & & & -1 \end{array}\right)$$

$$T_{24} \perp T_3^c,\ T_8^c,\ T_3^W$$

$$\Downarrow$$

$$T_{24} = \frac{1}{\sqrt{60}} \left(\begin{array}{ccccc} 2 & & & & \\ & 2 & & & \left.\right\} \text{color} \\ & & 2 & & \\ \hdashline & & & -3 & \\ \text{weak} \left\{\right. & & & & -3 \end{array} \right)$$

$$\Downarrow$$

$$[Y, \text{color}] = 0$$

$$[Y, \text{weak}] = 0$$

$$\boxed{Q_{em} = \sum_{\text{Cartan}} c_i T_i}$$

$$\Rightarrow Y = \sum_{\text{Cartan}} c_i' T_i'$$

$$\boxed{Y \propto T_{24}}$$

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fermions in SM = ?

$$(6) \; \begin{pmatrix} u \\ d \end{pmatrix}_L^{\alpha} \qquad u_L^{c\;\alpha}, \; d_L^{c\;\alpha} \; (6)$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_L^{c}$$

$$\boxed{\begin{array}{c} 12 \text{ quarks} + 3 \text{ lepton} \\ = 15 \end{array}}$$

SU(5)

$F = 5$ $\quad F_i \to U_{ij} F_j$

$5_i \to U_{ij} 5_j$

$$\underbrace{5_i \times 5_j}_{R_{ij}} \to U_{ik} U_{jl} 5_k \times 5_l$$

$$= U_{ik} R_{kl} U^T_{lj}$$

$$5 \times 5 = 25 \ (?)$$

- $S_{ij} = S_{ji} \Rightarrow$

$$S \to U S U^T$$

$S = S^T$ is preserved

- $A_{ij} = -A_{ji}$ $\quad A^T = -A$

(preserved)

$$\boxed{S = S^T \;;\quad \frac{5\cdot 6}{2} = 15}$$

$$A = -A^T : \quad \frac{5\cdot 4}{2} = 10$$

$$5 = \left(\frac{\boxed{\begin{matrix} x \\ x \\ x \end{matrix}}}{\text{vect}} \right) \begin{matrix} \big\} \text{colw} \\ \\ \leftarrow + \end{matrix}$$

$$5 = (3c, 1w) + (1c, 2w)$$

$$5 \times 5 = \left[(\underline{3c, 1w}) + (1c, 2w) \right]$$

$$\textcircled{S} \longleftarrow \quad \left[(\underline{3c, 1w}) + (1c, 2w) \right]$$

$$= (6_c + 3^{*}c, \underline{1}w) + \text{- - - -}$$

Ⓐ

colr sextet

S (15 of SU(5)) =

$= (6_c, 1_w)$

NO such particles

- we tried: 15 SM fermions

$= 15_F$ of SU(5)

FAILS

- $\underset{(F)}{15 = 5} + \underset{(A)}{10}$?? ??

$SU(N) \longleftrightarrow SO(N)$

Pauli $\sigma_{1,2}$ +

\+ Cartan

"real"

$T_a^* = -S\, T_a\, S^\dagger$

$SS^\dagger = 1$

$\Downarrow$

$T_r\{T_a, T_b\} T_c = 0$

$\Rightarrow \boxed{A_{abc} = 0}$

$$\boxed{\begin{array}{l} SU(5) = \\ = \text{anomalies} \end{array}}$$

~~$15_F \Rightarrow$ Anomaly~~ !

$$\boxed{\underbrace{5 + 10\ ?}_{\text{anomaly cancels?}}}$$

$$\boxed{\text{Why unification?}}$$

Fermi ⟶ Glashow ---

effective ⟶ messengers

$(\bar{p}\, d)(\bar{e}\, \nu)$