LMU GUT Course

Fall 2020

Lecture I

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unif. particle interactions

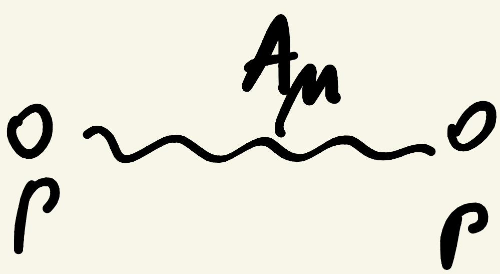
<u>but</u> gravity

$$V_{gr} \simeq G_N \frac{m_p^2}{r}$$

$$\boxed{\hbar = c = 1}$$

$$G_N = 10^{-38} \frac{1}{m_p^2}$$

$$m_p = \mathrm{GeV}$$



$$V_{em} \simeq \frac{\alpha}{r} \simeq 100/r$$

$$\frac{\text{gravity}}{\text{em}} \simeq 10^{-36}$$

$$M_\odot = 10^{60} m_p$$

weak + em + strong

unif (SM) grand unification

- Spontaneous Symm. Breaking
- SU(2) group theory
- SM gauge picture

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Standard Model

must

Schwinger – Glashow
attempt to unify em + weak

SU(2) gauge theory

'50 $\rightarrow$ QED em

$U(1)_{em}$ gauge theory

$$\mathcal{L}_{QED} = i\bar{f}\gamma^\mu D_\mu f - m_f \bar{f} f - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \quad (1)$$

$$D_\mu = \partial_\mu - ieQA_\mu \quad (2)$$

$$Q \equiv Q_{em} \therefore Qf = q_f f \quad (3)$$

$$q_e = -1, \; q_\nu = 0$$

$n = udd \quad p = uud \quad q_u = 2/3, \; q_d = -1/3$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (4)$$

$$F_{0i} = E_i, \quad F_{ij} = \varepsilon_{ijk} B_k$$

anti-sym.

$$f \to e^{i\alpha(x) Q} f$$

$$A_\mu \to A_\mu - \frac{1}{e} \partial_\mu \alpha(x) \quad (5)$$

$$\Updownarrow$$

$$A_\mu Q \to U A_\mu Q U^+ + \frac{i}{e} U \partial_\mu U^+$$

$$U = e^{i\alpha(x) Q} \quad (6)$$

$\alpha = \text{const.}$ → U(1) global sym.

$$\Rightarrow \quad \partial_\mu j^\mu_{em} = 0 \therefore j^\mu_{em} = \bar{f} \gamma^\mu Q f \quad (7)$$

$\boxed{\alpha = \alpha(x)} \Longleftrightarrow \boxed{\exists \text{ messenger}}$

A_μ = photon $\left(m_A = 0 \right)$

$\boxed{\cdot\ m_A \le 10^{-14}\, m_e \le 10^{-20}\, eV}$

$\cdot\ m_\nu \le 1\, eV \Rightarrow m_\nu \overset{?}{=} 0$ wrong

$\rightarrow$ direct limit from the derivation of $1/r$

$\uparrow$

$V(r) \propto \frac{1}{r} e^{-m_A r}$
$(m_A \neq 0)$

limit on m_A

m_γ limit from galactic $\vec{B}$

Adelberger, Dvali

$QA = 0 \Longrightarrow$

photon mass is OK with conserved charge

$$\boxed{\begin{gathered} e \longrightarrow \nu + \gamma \\ \tau_e \gtrsim 10^{26-29}\ \text{yr} \end{gathered}}$$

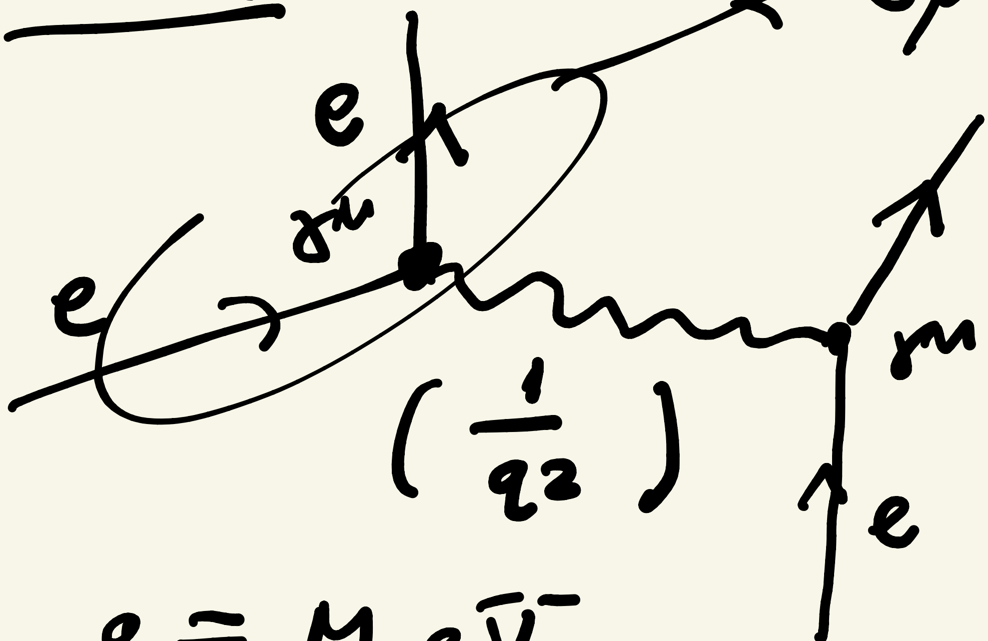
$\tau_p \gtrsim 10^{34}$ yr proton decay

$p \to e^+ + \gamma$

50's   weak int.

— effective theory

QED



q = exchanged momentum

$q = MeV$

$$\boxed{H_{eff}^{em}(QED) = \frac{\alpha}{q^2} J_\mu^{em} J^\mu_{em}}$$

$$H_{int} = e A_\mu \bar{f} \gamma^\mu Q f$$

$$\boxed{\begin{array}{l}\alpha \equiv e^2/4\pi \\ \alpha = \alpha(q)\end{array}}$$

$$\alpha = \frac{1}{137}$$

Fermi '34

Marshak, Sudarshan

$$\boxed{\mathcal{H}^{w}_{eff} = \frac{G_F}{\sqrt{2}} J^{\mu}_{w} J^{w}_{\mu}} \quad (8)$$

"V − A"

'57

'54 Yang - Mills

gauge embly of U(1)

SU(2)

$$n \to p + e + \bar{\nu}_e \quad (\Delta Q = 1)$$

$$(d \to u + e + \bar{\nu}_e)$$

$$\mu \to e + \nu_\mu + \bar{\nu}_e$$

$$J_\mu^W = \left[\bar{u}\gamma^\mu O d + \bar{\nu}\gamma^\mu O' e\right]$$

$$O = O' = (1+\gamma_5)$$

$$\{\gamma_5, \gamma_\mu\} = 0 \qquad (9)$$

SU(2) gauge

$$\mathcal{L}_{SU(2)} = i\bar{f}\gamma^\mu D_\mu f - m_f \bar{f} f$$

$$- \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} \qquad (10)$$

invariant under SU(2):

$\boxed{2\times 2}$

$$U^+ U = U U^+ = 1 \qquad (11)$$

$$\det U = 1 \qquad (12)$$

$$f \longrightarrow U f \qquad f = \begin{pmatrix} u \\ d \end{pmatrix}$$

$(11) \Rightarrow U = e^{iH} \qquad H = H^{+}$

$(12) \Rightarrow \quad \mathrm{Tr}\, H = 0$

$$H = \theta_a T_a \qquad \begin{array}{l} T_a^{+} = T_a \\ \mathrm{Tr}\, T_a = 0 \end{array}$$

$a = 1,2,3$

Euler angles

$$[T_a, T_b] = i\, \epsilon_{abc} T_c$$

(12)

$\boxed{\text{algebras}}$

$\boxed{\begin{array}{c} SO(3) \Longleftrightarrow SU(2) \\ \text{same algebra} \end{array}}$

$$T_a \equiv \frac{\sigma_a}{2} \qquad \text{(Pauli matrices)}$$

$$\boxed{\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

T_3 "charge"

cartan sub-algebra

$$C = \{[T_\alpha, T_\beta] = 0\}$$

diagonal

$$\boxed{C_{SU(2)} = \{T_3\}}$$

"spin ½" $\quad T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

"spin 1" $\quad T_3 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}$

$$\boxed{\begin{gathered} T_3 |t_3\rangle = t_3 |t_3\rangle \\ \Rightarrow t_3 = \pm n \frac{1}{2} \end{gathered}}$$

U(1) = Abelian

$T^{\pm} = T_1 \pm i T_2$

SU(bigger) = = non-Abelian

$T_3 (T_{\pm} |m\rangle) = (m \pm 1)(T_{\pm} |m\rangle)$

$T_3 |m\rangle = m |m\rangle$

$[T_3, T_{\pm}] = \pm T_{\pm}$ (13)

$\boxed{\text{Schwinger – Glashow '57 – '60}}$

$\boxed{\text{gauge weak int. theory}}$

$$\mathcal{L}_{SU(2)} = i \bar{f} \gamma^{\mu} D_{\mu} f - m_f \bar{f} f - \frac{1}{4} F_{\mu\nu}{}^{a} F^{\mu\nu}{}_{a}$$

(14) ≠ (10)

$\Downarrow$

$$D_\mu = \partial_\mu - i g\, T_a A_\mu^a \qquad a = 1,2,3$$

$$T_a A_\mu^a \longrightarrow U\, T_a A_\mu^a\, U^\dagger + \\ + \frac{i}{g}\, U \partial_\mu U^\dagger \quad (15)$$

$$f \longrightarrow U f$$

$$T^+ |m\rangle \propto |m+1\rangle$$

$$f = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$T^\pm \qquad T^+ d = u$$

$$T^- u = d$$

$$T_1 A_\mu^1 + T_2 A_\mu^2 \propto$$

$$\propto T_+ W_\mu^+ + T_- W_\mu^-$$

$$\boxed{W^\pm = \frac{A_1 \mp i A_2}{\sqrt{2}}} \qquad (16)$$

↓ weak int. gauge boson

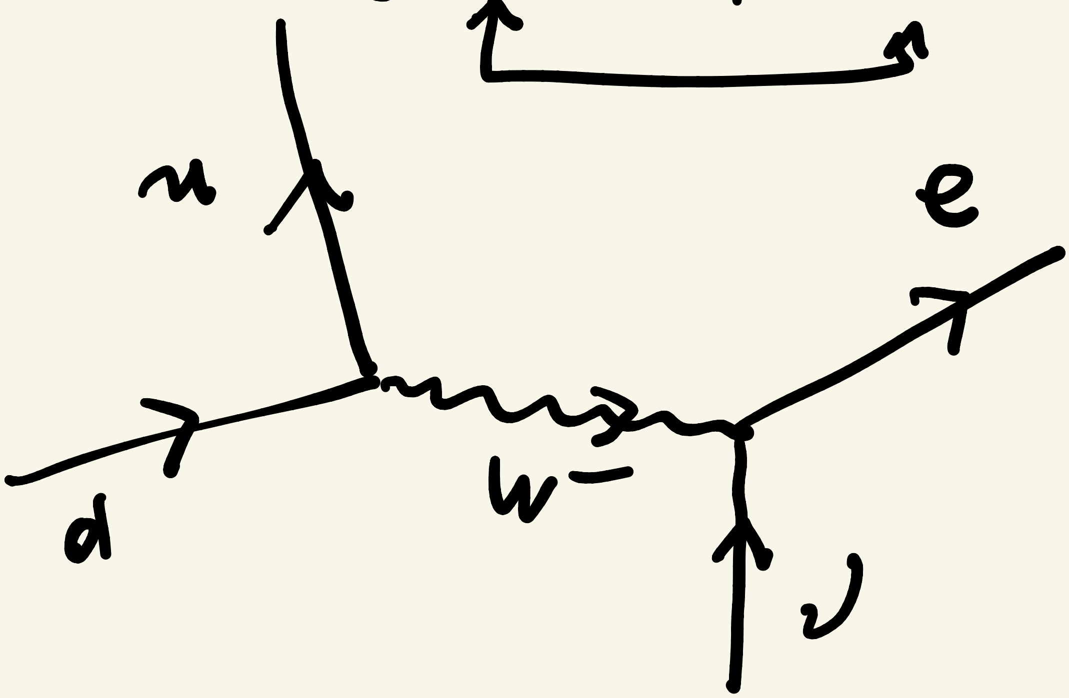
'83 at CERN

$M_W = 80$ GeV

$$\bar f \gamma^\mu D_\mu f \to \bar f \gamma^\mu g \,(A_1 T_1 + A_2 T_2)_\mu f$$

$$= g \bar f \gamma^\mu (W^+ T_+ + W^- T_-)_\mu f$$

$$= g\, \bar u \gamma^\mu W_\mu^+ d + h.c. \qquad (17)$$



$$\bar f = f^\dagger \gamma^0$$

Feynman: p enters $\Longleftrightarrow$ $\bar{p}$ leaving

<u>unity</u>: em + weak

$$\left[\begin{array}{l} Q_{em} = \text{diagonal} \\ Q_{em}\, f = q_{em}\, f \end{array}\right]$$

$$Q_{em} = T_3 \; (\times c)$$

$$\Rightarrow \boxed{q_{em} \propto n\, q_{min}}$$

$$q_{min} = c\,\tfrac{1}{2}$$

- charge quantization

$\boxed{1/10^{20} \text{ precision}}$

$$\left.\begin{aligned} q_\nu &= 0, \quad q_e = -1 \\ q_d &= -1/3 \\ q_e &= 3\, q_d \\ q_u &= -2\, q_d \\ q_W &= \pm 3\, q_d \end{aligned}\right\}$$

"great"

- <u>failure</u>: <u>not</u> explain charges

"beautiful theory killed by ugly facts of nature"

???

MS $\boxed{V-A}$

Marshak
Sudarshan

$$g\, W_\mu^+ \,\bar{u}\, \gamma_\mu \underbrace{\frac{1+\gamma_5}{2}\, d}_{}$$

breaks P (!)

$$e\, A_\mu\, \bar{u}\, \gamma^\mu\, 2/3\, u$$

$$\psi_L = \frac{1+\gamma_5}{2}\psi \equiv L\psi$$

$$\psi_R = \frac{1-\gamma_5}{2}\psi \equiv R\psi$$

$$L^2 = L,\; R^2 = R,\; LR = 0$$

P: $\psi_L \longleftrightarrow \psi_R$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \; (u_L \leftrightarrow u_R) \; \psi_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$\boxed{\begin{aligned} \text{weak} &= L \\ \text{em} &= L+R \end{aligned}}$$

$$\boxed{SU(2)_L \times U_Y(1)} \quad \boxed{\cancel{\tan\theta_W} = g'/g}$$

$g \qquad g'$

$$[T_a, Y] = 0 \qquad a = 1, 2, 3 \qquad (20)$$

$$\boxed{e = g \sin\theta_W}$$

$\parallel$

arbitrary

$$Q_{em} = T_3 + \frac{Y}{2} \qquad (21)$$

$$\underbrace{\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L}_{SU(2)\text{ doublets}}$$

$W \qquad W$

weak int

u_R, d_R
e_R

$\boxed{Q_{f_L} = Q_{f_R}}$

$$\boxed{\frac{Y}{2} = Q_{em} - T_3}$$

exp

group

$\boxed{\text{why is } Q_{em} = u\,?}$

$\boxed{\text{why is } Y \text{ quantized?}}$

$\boxed{\text{why is } Q_e = 3Q_d\,?}$

Weinberg
'67 Higgs ← Glashow '61
(SM)

'68 Salam

'70s

't Hooft ⇒ theory

strong int. = SU(3)c

QCD gauge theory

Quantum Chromo Dynamics

SU(2) = ew unification
SU(3)

$(3)^{\dagger} + (8) = 1$

$SU(2) \times SU(3) \leq G$

$G_{min} = SU(4) \quad (15)$

$SU(3): \; U_3 U_3^{\dagger} = 1 \qquad U = e^{i\theta_i T_i}$

$\det U_3 = 1$

$a = 1 \ldots$

$T_i : (3 \times 3) \qquad T_i = T_i^{\dagger}$

$\mathrm{Tr}\, T_i = 0$

$\Rightarrow 8$

$\boxed{\text{rank} = \#\text{ of Cartan elements}}$

$r(SU(2)) = 1 \qquad r(SU(3)) = 2$

$\downarrow \qquad \downarrow$

$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \qquad \Bigg| \qquad \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$SU(2)$ $\qquad\qquad\qquad\qquad$ $SU(3)$

$$r(SU(4)) = 3$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -2 & \\ & & & 0 \end{pmatrix}, \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$$

~~<u>$SU(4)$</u>~~

$$\cancel{\begin{pmatrix} \cdot \\ \vdots \\ \hline \cdot \end{pmatrix} \begin{matrix} \}\ SU(3) \\ \leftarrow SU(2) \end{matrix}}$$

$$\cancel{E = ma^2}$$

$$\cancel{E = mb^2}$$

$$\Downarrow$$

$$E = mc^2$$

Gauge

$$\left(\begin{array}{l} \left.\begin{array}{l}\cdot\ \nu \\ \cdot\ u \\ \cdot\ d\end{array}\right\} \text{gluons} \quad SU(3)_c \\ \hline \begin{pmatrix}\nu \\ e\end{pmatrix} W \quad SU(2) \end{array}\right)$$

$\boxed{SU(5)}$ $\quad r(SU(5)) =$

$SU(3)$ color

$$\begin{pmatrix}1&&&&\\&-1&&&\\&&0&&\\&&&0&\\&&&&0\end{pmatrix}, \quad \begin{pmatrix}1&&&&\\&1&&&\\&&-2&&\\&&&0&\\&&&&0\end{pmatrix}$$

$$\begin{pmatrix}0&&&&\\&0&&&\\&&0&&\\&&&1&\\&&&&-1\end{pmatrix}, \quad \begin{pmatrix}1&&&&\\&1&&&\\&&1&&\\&&&-3/2&\\&&&&-3/2\end{pmatrix}$$

weak

$$r(SU(5)) = 4$$

$\Downarrow$

$\boxed{Y = \text{quantized}}$

charge quantization

∃ magnetic monopoles

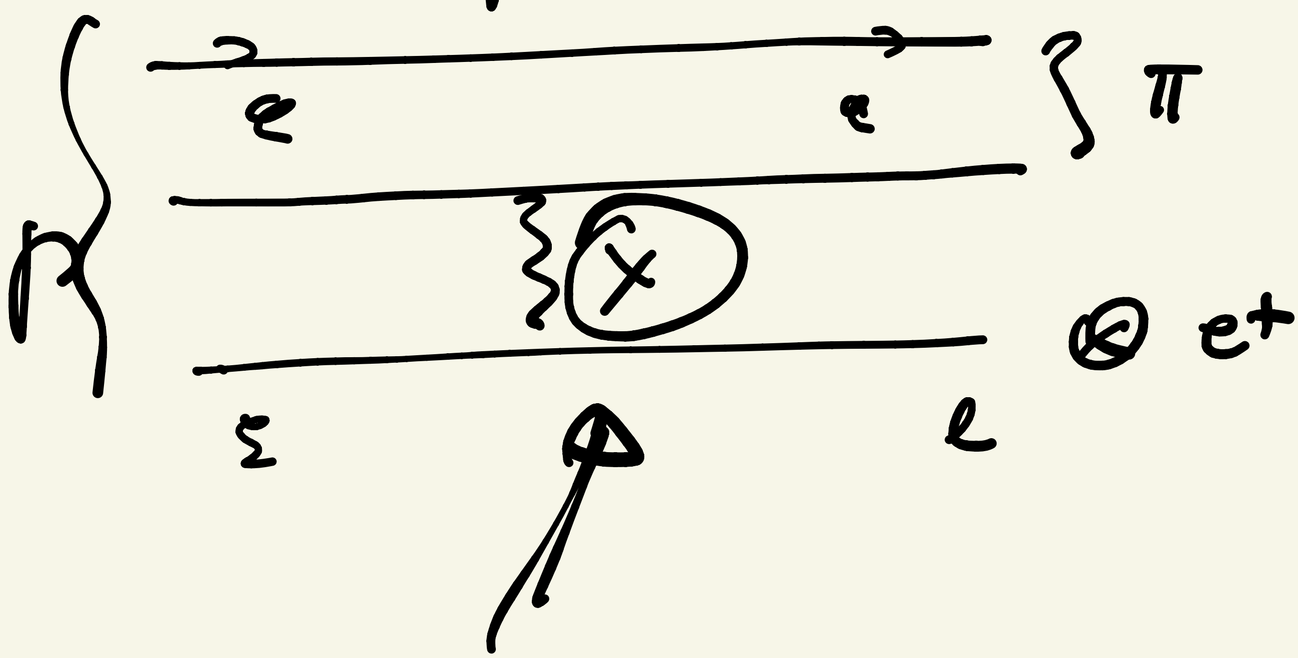
proton decay

new messages : X

$q \to \ell, \quad q \to q$

$\Downarrow$ proton decay

? $p \to e^+ + \pi^0$



$$\tau\,(\text{weak}) \propto M_W^{+4}$$

$$\Gamma\,(\text{weak}) \propto M_W^{-4}$$

$$\left(G_F = \frac{g^2}{M_W^2}\right)$$

$$\boxed{\begin{array}{l} M_X \simeq 10^{16}\,\text{GeV} \\ \Rightarrow \tau_p \simeq 10^{34} - 10^{35}\,\text{yr} \end{array}}$$

β decay breaking nature

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main task of GOT