Exercises on General Relativity TVI TMP-TC1

Problem set 6

Exercise 1 – Maxwell theory

(i) Consider the Maxwell action functional

$$S[A] = -\tfrac{1}{4}\int_{M_4} F^2(A)\ . \tag{1}$$

Write down this action explicitly in spherical spatial coordinates.

(ii) Show that in Maxwell theory charges with the same sign repel each other. In order to do this, take an external point source J_μ with charge Q as previously done for Fierz Pauli. Calculate the resulting electric field using (1) with an extra term $J_\mu A^\mu$. Couple a point mass with charge q and 4-velocity u to this electric field with $q u_\mu A^\mu$ and discuss weather the external source attracts or repels your point mass depending on the sign of q.

Using your results, recall why a vector theory is not a suitable theory to describe gravity.

Exercise 2 – First step to a gauge-invariant self-interaction

(i) Rewrite the Fierz-Pauli action it in the following useful form by finding M:

$$S_{FP}[h] = (2G_N)^{-1}\int_{M_4} \partial_\alpha h_{\mu\nu} M^{\alpha\mu\nu\beta\rho\sigma}\partial_\beta h_{\rho\sigma}. \tag{2}$$

(ii) Derive the energy-momentum tensor (EMT) T_{FP} of h. Is this EMT gauge invariant?

(iii) In order to couple the h field to itself, we introduce a source term $S_I[h]$ to the Fierz-Pauli action,

$$S_I[h] = \int_{M_4} h_{\mu\nu} T^{\mu\nu}_{FP}\ . \tag{3}$$

How does this change the EMT of the theory $\Big(\text{EMT}(S_{FP}) = \text{EMT}(S_{FP}{+}S_I) \text{ or } \text{EMT}(S_I) = 0\ \Big)$?

(iv) Recall why this is a problem in order to find a self-consistent completion of the Fierz-Pauli theory.

(Hint: You do not need to find the complete expression for the gauge transformation or the new contribution to the EMT, but it is sufficient to show that at least one of the many terms does not cancel.)

Exercise 3 – Expansion of the measure function

Expand, to second order in $\frac{1}{\zeta}$, the integral measure function $\sqrt{-\det(g)}$ by using the expansion $g \to \eta + \frac{h}{\zeta}$, where η is the Minkowski metric and h a small fluctuation around it. The mass dimension $[\zeta]$ is equal to $[h]$.

General information

The lecture takes place on Monday at 10:00-12:00 and on Wednesday at 10:00-12:00 in A348.

Presentation of solutions:
Thursday at 08:00 - 10:00 in A 348

There are two tutorials:
Monday at 12:00 - 14:00 in A 249
Friday at 14:00 - 16:00 in A 348

The webpage for the lecture and exercises can be found at

`www.physik.uni-muenchen.de/lehre/vorlesungen/wise_19_20/tvi_tc1_gr/index.html`