

Black Holes and their Thermodynamics<sup>1</sup>  
**Problem Sheet 7**

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Tutorials take place on Mondays, 2-4pm (c.t.) in room A 449, Theresienstr. 37. Please hand in your solutions at the next tutorial on December 2, 2019 or send them in a single pdf in an email before the tutorial.

The typo in eq. (2) is corrected in this version.

**Exercise 1** Starting from Kruskal-Szekeres coordinates, construct the maximally extended Penrose diagram of the Schwarzschild spacetime. Show that the singularities  $r = 0$  are mapped onto horizontal lines. Determine where the horizon and the spacelike, null and timelike infinities are located in the Penrose diagram.

**Exercise 2** A Reissner-Nördstrom black hole is called extremal when its charge equals its mass. Write down the metric for an extremal RN black hole in the usual  $(t, r, \theta, \phi)$  coordinates. Let  $\epsilon$  be a constant, transform the metric into the coordinates defined by

$$\tilde{t} = \epsilon t, \quad \tilde{r} = \frac{r - M}{\epsilon} \quad (1)$$

and take the limit  $\epsilon \rightarrow 0$ . The result is the near-horizon metric of an extremal black hole.

**Exercise 3** The Robinson-Bertotti metric is

$$ds^2 = -\lambda^2 dt^2 + M^2 \lambda^{-2} d\lambda^2 + M^2 d\Omega^2 \quad (2)$$

This is the product  $AdS_2 \times S^2$  where  $AdS_2$  denotes 2-dim anti-de Sitter spacetime. By replacing the time coordinate  $t$  by one of the radial null coordinates  $u = t + M/\lambda$ ,  $v = t - M/\lambda$  show that the singularity at  $\lambda = 0$  is merely a coordinate singularity. By introducing the new coordinates  $(U, V)$ , defined by  $u = \tan(U/2)$ ,  $v = -\cot(V/2)$ , obtain the maximal analytic extension of the RB metric and deduce its Penrose diagram (more precisely: deduce the Penrose diagram of the  $AdS_2$  part of the RB metric). Is this spacetime globally hyperbolic?

<sup>1</sup> [www.physik.uni-muenchen.de/lehre/vorlesungen/wise\\_19\\_20/bh-info\\_wise\\_2019\\_20](http://www.physik.uni-muenchen.de/lehre/vorlesungen/wise_19_20/bh-info_wise_2019_20)

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