Prof. Dr. Viatcheslav Mukhanov | Winter term 2019/2020

Exercises on Quantum Mechanics II (TM1/TV)

Problem set 12, discussed January 20 - January 24, 2020

Exercise 68 - Central Tutorial

In this exercise we want to discuss quantum teleportation. For this, suppose Alice holds an unknown spin-$\frac{1}{2}$ state $|\,\phi\rangle = a\,|+\rangle + b\,|-\rangle$, which she would like to transmit to Bob. However, they can only communicate their measurement results classically, i.e. for example via e-mail. Suppose that Alice and Bob in addition are each in possession of one member of a pair in an entangled spin-$\frac{1}{2}$ system, which is in the singlet state $|\,\Psi\rangle$.

(i) Suppose that Alice measures the two spin particles she holds in the so-called *Bell basis*, which consists of the four states

$$\begin{aligned} \left|\,\Phi^{\pm}\right\rangle &:= \frac{1}{\sqrt{2}}\left[|\,{+}{+}\rangle \pm |\,{-}{-}\rangle\right], \\ \left|\,\Psi^{\pm}\right\rangle &:= \frac{1}{\sqrt{2}}\left[|\,{+}{-}\rangle \pm |\,{-}{+}\rangle\right], \end{aligned} \tag{1}$$

where, for example $|\,{+}{-}\rangle := |\,+\rangle \otimes |\,-\rangle$. Write the complete state $|\,\phi\rangle \otimes |\,\Psi\rangle$ of the three particles by using the Bell basis for Alice's particles. Determine the probabilities of the outcome of Alice's measurement.

(ii) After Alice has done her measurement, what is the state of Bob's particle? What can Alice and Bob do in order to turn Bob's particle into the state $|\,\phi\rangle$?

Exercise 69

Consider a spin-$\frac{1}{2}$ atom in a Stern-Gerlach experiment. From the Schrödinger equation, derive differential equations describing the behaviour of the atom when passing through a constant magnetic field in the z direction, active in a region $a < x < b$.
Hint: Separate the total wave function into

$$\psi(x, z, t) = \psi_x(x, t)\psi_z(z, t), \tag{2}$$

and consider how the action of the potential in the Hamiltonian depends on the spin.

Exercise 70

Show that for a successful Stern-Gerlach experiment the following condition has to be satisfied,

$$\frac{\mu}{(2\hbar)^{1/2}}\left(\frac{\partial H}{\partial z}\right)\frac{b^{3/2}m}{p_{0x}^{\;3/2}} > 1\ . \tag{3}$$

(The detector shall be at $x = b$.)

Exercise 71

Let $\hat{H}_{\text{int}} = \hat{S} \cdot \hat{X}$ be the interaction term between system S and device A (where $\hat{S}$ and $\hat{A}$ are the operators corresponding to the observables in which we are interested within the system and device respectively) and $\hat{X}$ is the canonical conjugate operator to $\hat{A}$, $[\hat{X}, \hat{A}] = -i\hbar$. Prove that

$$[\hat{A}, e^{-\frac{i}{\hbar}\hat{H}\triangle t}] = e^{-\frac{i}{\hbar}\hat{H}\triangle t}\hat{S}\triangle t\ . \tag{4}$$

General information

The *lecture* takes place on:
Monday at 10:00 - 12:00 c.t. in B 052 (Theresienstraße 37)
Friday at 10:00 - 12:00 c.t. in B 052 (Theresienstraße 37)
The *central tutorial* takes place on Monday at 12:00 - 14:00 c.t. in B 139 (Theresienstraße 37)
The *webpage* for the lecture and exercises can be found at

`https://www.physik.uni-muenchen.de/lehre/vorlesungen/wise_19_20/T_M1_TV_-Quantum-Mechanics-II`