Prof. Dr. Viatcheslav Mukhanov Winter term 2019/2020

Exercises on Quantum Mechanics II (TM1/TV)

Problem set 13, discussed January 27 - January 31, 2020

EPR paradox vs Bell's inequality - Central Tutorial

Exercise 72 - Central Tutorial

One way to understand the ability of quantum mechanics to protect from information detection is from the fact that non-orthogonal states cannot be perfectly distinguished.

(i) In the course of a quantum key distribution protocol, suppose that Alice randomly chooses one of the following two states and transmits it to Bob:

$$|\,\phi_0\rangle = \frac{1}{\sqrt{2}}(|\,0\rangle + |\,1\rangle), \qquad \text{or} \qquad |\,\phi_1\rangle = \frac{1}{\sqrt{2}}(|\,0\rangle + i\,|\,1\rangle). \tag{1}$$

Eve intercepts the qubit and performs a measurement to identify the state. The measurement consists of the orthogonal states $|\,\psi_0\rangle$ and $|\,\psi_1\rangle$, and Eve guesses the transmitted state was $|\,\phi_0\rangle$ when she obtains the outcome $|\,\psi_0\rangle$, and so forth. What is the probability that Eve correctly guesses the state, averaged over Alice's choice of the state for a given measurement? What is the optimal measurement Eve should make, and what is the resulting optimal guessing probability?

(ii) Now suppose Alice randomly chooses between two states $|\,\phi_0\rangle$ and $|\,\phi_1\rangle$ separated by an angle θ on the Bloch sphere. What is the measurement which optimizes the guessing probability? What is the resulting probability of correctly identifying the state, expressed in terms of θ? In terms of the states? *Hint:* Review the main results from exercise 60 and think about how to represent the measurement probability $P = |\langle a\,|\,b\rangle|^2$ in terms of Block sphere vectors.

Exercise 73

Consider a system consisting of two coils as in the lecture, a superconducting coil with induction I and current $J = \dot{q}$ interacting with a second coil with current $j(t)$ flowing in it. The latter is free to rotate, where the angle about the rotation axis is denoted by φ and has moment of inertia T. The Lagrangian of the system is given by

$$L = \frac{1}{2}I\dot{q}^2 + \frac{1}{2}T\dot{\varphi}^2 - M_0\varphi\dot{q}j(t), \tag{2}$$

where in the interaction term, the mutual inductance is approximated by $M(\varphi) \approx M_0\varphi$, with $M_0 = const.$ Verify that the Hamiltonian is given by

$$H = \frac{p_q^2}{2I} + \frac{p_\varphi^2}{2T} + \frac{Ig^2(t)\varphi^2}{2} + g(t)\varphi p_q\ , \tag{3}$$

where $g(t) = \frac{M_0}{I}j(t)$. Check that the condition for a non-demolition measurement is satisfied.

Exercise 74

Let p_q in Exercise 73 be given by $p_q = -\frac{p_\varphi(t_f) - p_\varphi(t_i)}{g_0\triangle t} - Ig_0\varphi(t_i)$. Why is the uncertainty $\triangle p_q$ bounded by

$$\triangle p_q \geq \sqrt{\frac{\triangle p_\varphi^2}{g_0^2\triangle t^2} + I^2 g_0^2 \triangle\varphi_i^2} \quad ? \tag{4}$$

Exercise 75

Let $\hat{\tau} = m\frac{\hat{x}}{\hat{p}} \equiv \frac{m}{2}(\hat{x}\hat{p}^{-1} + \hat{p}^{-1}\hat{x})$ be the "time operator" of a quantum clock. Check that the uncertainty relation $[\hat{\tau}, \hat{E}] = i\hbar$ is valid, where the "energy operator" $\hat{E}$ is given by $\hat{p}^2/2m$. Shortly interpret the result compared to the violation of the energy-time uncertainty principle in the lecture.

General information

The *lecture* takes place on:
Monday at 10:00 - 12:00 c.t. in B 052 (Theresienstraße 37)
Friday at 10:00 - 12:00 c.t. in B 052 (Theresienstraße 37)
The *central tutorial* takes place on Monday at 12:00 - 14:00 c.t. in B 139 (Theresienstraße 37)
The *webpage* for the lecture and exercises can be found at

`https://www.physik.uni-muenchen.de/lehre/vorlesungen/wise_19_20/T_M1_TV_-Quantum-Mechanics-II`