String Theory I

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Exercise sheet 9

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Exercise 1: Polarization tensor

Consider the closed string state

$$|\phi\rangle = \xi_{\mu\nu}\, \alpha^{\mu}_{-1}\, \tilde{\alpha}^{\nu}_{-1}\, |0;k\rangle \ , \tag{1.1}$$

where $\xi_{\mu\nu}$ denotes the polarization tensor.

(a) What do the Virasoro constraints on physical states (for $A = -1$),

$$(L_0 - 1)|\phi\rangle = (\tilde{L}_0 - 1)|\phi\rangle = L_m|\phi\rangle = \tilde{L}_m|\phi\rangle = 0 \quad , \quad m > 0 \ , \tag{1.2}$$

imply for k and $\xi_{\mu\nu}$? Show first that if a state is annihilated by L_1 and L_2 it is annihilated by all L_n with $n \geq 1$.

(b) Show the presence of the gauge equivalence

$$\xi_{\mu\nu} \cong \xi_{\mu\nu} + a_\mu k_\nu + k_\mu b_\nu \quad , \quad a \cdot k = b \cdot k = 0 \tag{1.3}$$

by finding the relevant null states, i.e. physical states which are spurious (i.e. orthogonal to all physical states).

Exercise 2: A ghost at the second excited level

Consider the open string state (with NN boundary conditions in all directions)

$$|\phi_k\rangle = \Big(c_1 \alpha_{-1} \cdot \alpha_{-1} + c_2\, \alpha_0 \cdot \alpha_{-2} + c_3 (\alpha_0 \cdot \alpha_{-1})^2\Big)|0;k\rangle \ . \tag{2.1}$$

Determine the relations between c_1, c_2 and c_3 which follow when demanding $|\phi_k\rangle$ to be physical (use $A = -1$, i.e. $(L_0 - 1)|\phi_k\rangle = 0$). Then compute

$$\langle \phi_k | \phi_{k'} \rangle = \frac{2c_1^2}{25}(D-1)(26-D)\delta^D(k-k') \ . \tag{2.2}$$

Exercise 3: Field equations of the Kalb-Ramond field $B_{\mu\nu}$

Here we examine the field theory of a massless antisymmetric tensor gauge field $B_{\mu\nu} = -B_{\nu\mu}$. This gauge field is a tensor analog of the Maxwell gauge field A_μ. In Maxwell theory one defines the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. For $B_{\mu\nu}$ one defines a field strength $H_{\mu\nu\rho}$ according to

$$H_{\mu\nu\rho} \equiv \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} \ . \tag{3.1}$$

(a) Show that $H_{\mu\nu\rho}$ is totally antisymmetric.

(b) Prove that $H_{\mu\nu\rho}$ is invariant under the gauge transformations

$$\delta B_{\mu\nu} = \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu \ . \tag{3.2}$$

(c) The above gauge transformations have a special feature: the gauge parameters themselves have a gauge invariance. Show that ϵ'_μ given by

$$\epsilon'_\mu = \epsilon_\mu + \partial_\mu \lambda \tag{3.3}$$

generates the same gauge transformation as ϵ_μ.

(d) Consider the spacetime action

$$S = \int d^D x \left(-\frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) . \tag{3.4}$$

Find the field equation for $B_{\mu\nu}$.

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