To be solved until: 04.12.2019

Generation and Applications of Ultrahigh-Intensity Laser Pulses

Problem Set 6

1. Misaligned grating compressor

Consider the above grating compressor with a line density of 1480 mm^{-1} and an angle of 20.2 degree between both gratings. The laser has a central wavelength of 800 nm and a spectral bandwidth of $\Delta\lambda = 50$ nm (gaussian spectrum).

- Calculate the incident angle on the first grating.
- Assuming that the laser is focused using an f/20 parabola, with an aperture $d_{FWHM} = 5$ cm, how much does the pulse elongate in focus due to the angular chirp if the compressor is misaligned by an angle of 0.1 mrad?

Hint: Check chapter 2.10 of the lecture notes online.

2. Ray Transfer Matrix Analysis

In many cases, the propagation of light in a resonator or other optical systems can be approximated using the paraxial approximation, i.e. all rays are assumed to be at a small distance (r) from the optical axis, with a small angle (θ) and axial symmetry. A useful formalism to analyze the propagation of light in the paraxial approximation is the transfer-matrix method. Here, a single light ray is expressed by its position r and angle θ

$$\boldsymbol{x} = \begin{pmatrix} r \\ \theta \end{pmatrix}.$$

Starting from an initial state $\boldsymbol{x}_1$, a linear optical element will transform the light ray to a state

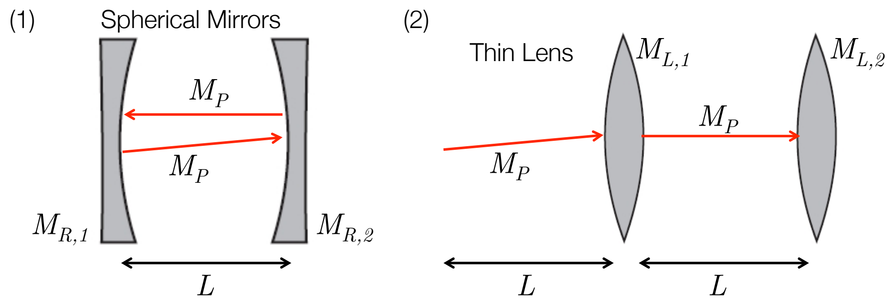
$$\boldsymbol{x}_2 = \boldsymbol{M} \cdot \boldsymbol{x}_1,$$

where

$$\boldsymbol{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

is the so-called ray transfer matrix. Accordingly, the transport of a beam through a number of optical electrons can be calculated by multiplying the individual transfer matrices $\boldsymbol{x}_n = (\boldsymbol{M_n} \cdots \boldsymbol{M_3} \cdot \boldsymbol{M_2} \cdot \boldsymbol{M_1})\, \boldsymbol{x}_1$.

- Find the transfer matrices for (a) free propagation of light, (b) a thin lens and (c) a spherical mirror.
- Calculate the ray transfer matrices for a single roundtrip inside a laser cavity (1) and single trip in a double lens system (2).
- Compare both results.



see next page for exercise 3

3. Stability of a laser resonator

In order to be stable, we require that a cavity will return to its initial condition at some point, i.e.

$$\boldsymbol{x}_n = \boldsymbol{M}^n \cdot \boldsymbol{x}_0 \stackrel{!}{=} \boldsymbol{x}_0$$

This is an eigenvalue problem

$$M^n \cdot \vec{r}_0 = \lambda^n \cdot \vec{r}_0$$

and we are trying to find eigenvalues with $|\lambda| = 1$.

- Calculate the eigenvalues $\lambda_{1,2}$ for a general ray transfer matrix

$$\boldsymbol{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

- Calculate $\lambda_{1,2}$ explicitly for the ray transfer matrix of a laser resonator of length L, consisting of two spherical mirrors with curvature $R_{1/2}$ (cf. exercise 2).
- Use the result to derive the stability condition

$$0 \leq \left(1 - \frac{L}{R_1}\right) \cdot \left(1 - \frac{L}{R_2}\right) \leq 1.$$