To be solved until: 20.11.2019

Generation of Ultrahigh-Intensity Laser Pulses

Problem Set 4

1. Group velocity delay

The group delay dispersion (GDD) or D_2 is defined as

$$GDD = D_2 = \frac{x}{c} \cdot \frac{\partial^2(\omega \cdot n)}{\partial \omega^2}.$$

Instead of the GDD, many sources in the literature refer to the group velocity delay (GVD). The latter is defined as the GDD per unit length, i.e. GVD = GDD/x - typically measured in units of [fs^2/mm].

- What is the relation between GVD and group velocity v_g?
- Show that in terms of the wavelength, the GVD can be written as

$$\text{GVD} = \frac{\lambda^3}{2\pi c^2} \frac{d^2 n(\lambda)}{d\lambda^2}.$$

2. Dispersion in air

The refractive index of air is accurately described by a formula of the form

$$n(\lambda) = 1 + \frac{a_1}{b_1 - \lambda^{-2}} + \frac{a_2}{b_2 - \lambda^{-2}},$$

where $a_1 = 0.05792, b_1 = 238.0185, a_2 = 0.00168$ and $b_2 = 57.362$, each with units μm^{-2}.

- Plot the refractive index in the range from 0.2 to 1.4 micrometer. Note that this is not the original Sellmeier formula, but a variation. Does it contain any resonances? If so, at which wavelengths?
- Given a gaussian pulse with 800 nm central wavelength and a bandwidth limited duration of $\Delta\tau = 20$ fs, how much does the laser pulse stretch over 20 m of propagation in air due to group velocity dispersion?

3. Dispersion in a plasma

The dielectric function of a cold, non-relativistic plasma is

$$\epsilon(\lambda) = 1 - \frac{\lambda^2}{\lambda_p^2},$$

where λ_p is the plasma wavelength.

- Sketch the refractive index of the plasma in terms of λ/λ_p. Does the material show normal or anormal dispersion? What happens for $\lambda/\lambda_p < 1$?
- Calculate the group delay D_1 and the group velocity of light v_g in the plasma. Now sketch $v_g(\lambda)$. What do you notice?
- Calculate the group velocity delay (GVD). Does a plasma introduce positive or negative chirp?