Generation and Applications of Ultrahigh-Intensity Laser Pulses

Problem Set 2

Time-Bandwidth Product and Peak Intensity

1. Time-Bandwidth-Product I

The duration of a pulse and its frequency bandwidth are related via the Fourier Transform from time to frequency. The relation is called the Time-Bandwidth-Product and it defines the shortest pulse duration you can reach with a given spectral shape. Calculate the product $(\Delta t \cdot \Delta \nu)$ of the FWHM duration Δt and the FWHM frequency bandwidth $\Delta \nu$ for a Gaussian pulse with

$$E(t) = E_0 \cdot \exp\left(-\frac{t^2}{2\sigma_t^2}\right)$$

Calculate the same for the FWHM of the intensity I(t), i.e. $\Delta_I t \cdot \Delta_I \nu$.

2. Time-Bandwidth-Product II

Calculate the minimum possible duration for the following pulse parameteres

- A central wavelength of $\lambda_0 = 1000 \text{ nm}$ and a gaussian spectrum of $\Delta \lambda = 4 \text{ nm}$ bandwidth.
- A central wavelength of $\lambda_0 = 800 \,\mathrm{nm}$ and a gaussian spectrum of $\Delta \lambda = 60 \,\mathrm{nm}$ bandwidth.

3. Field Strength at Focus

Consider a Titanium:Sapphire laser that generates light pulses with a gaussian shape, 20 fs duration at full width at half maximum (FWHM) and 60 J energy per pulse. The laser pulses have a central wavelength of $\lambda = 800$ nm.

- Assuming a gaussian beam profile with a waist $w_0 = 20 \,\mu\text{m}$, which peak intensities are reached in focus?
- What are the peak electric fields?
- Calculate the Rayleigh length of the beam.