Generation and Applications of Ultrahigh-Intensity Laser Pulses

Problem Set 1

Basic Concepts

1. Fourier-Transforms

Fourier-Transforms are an important tool in advanced optics. Derive the Fourier-Transforms

$$\hat{f}(k) = \mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x)e^{-ikx}dx$$

for the following analytical functions:

- Cosine function $f(x) = \cos(x)$

- Top hat function $f(x) = \begin{cases} 1 & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$

2. Influence of Phase on Beat

- Show how the phase ϕ affects the carrier and envelope of the combined signal $I = I_1 + I_2$ for

$$I_1 = \cos(\omega_1 t + \phi_1)$$

$$I_2 = \cos(\omega_2 t + \phi_2)$$

- What happens when you consider more than two frequencies, i.e. $I(t) = \sum I_i(t, \omega_i, \phi_i)$?

3. Full width at half maximum

Arbitrary peaked signals are often characterized by the full width at half maximum (FWHM), which is the width of a curve measured between those points which are half the maximum amplitude.

- Derive the relation between the FWHM and σ_x for a Gaussian distribution

$$f(x) = f_0 \times \exp[-(x - x_0)^2/(2\sigma_x^2)]$$

- Calculate the FWHM for a sinc function

$$f(x) = \frac{\sin(x)}{x}$$