Sheet 09

Discussion: Thursday 11.07.2024

Exercise 1 Monte Carlo simulations

The Monte Carlo method aims at solving the problem of the effective statistical sampling of suitable observables by a reversible, ergodic Markov chain.

1. Describe the (Metropolis) Monte Carlo Algorithm as introduced in the lecture notes.
2. **Estimating π:**
   Write a code that computes $\pi = 3.1415\ldots$ by Monte Calo sampling.
   - How does the probability for a point to be found inside the circle look like?
   - Run the code multiple times for $N = 10^i$, $i = 1, 2, 3, \ldots$ random numbers.
   - How does the estimate for π improve with increasing N? Compute the deviation from the exact result and plot it on a log-log scale as a function of N. Which scaling do you get?
3. **Simulating the 1D Ising model:**
   The goal of this exercise is to simulate the one-dimensional Ising model

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} S_i^z S_j^z \tag{1}$$

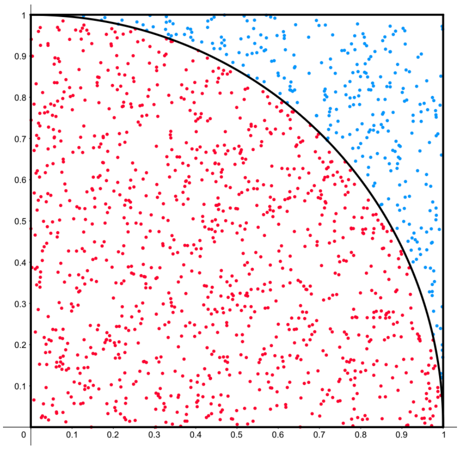

Abbildung 1: Figure from https://de.wikipedia.org/wiki/Monte-Carlo-Simulation

for periodic boundary conditions and temperatures $T \in [0.2, 5]$.

- How does the probability for accepting a new state look like?
- How does the acceptance rule arise from detailed balance?
- Compare your results to the analytical expression

$$E = \frac{e^{-J/T} - e^{J/T}}{e^{-J/T} + e^{J/T}}. \tag{2}$$