LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

FAKULTÄT FÜR PHYSIK IM SOSE 2024
T M1/TV: ADVANCED STATISTICAL PHYSICS
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https://www2.physik.uni-muenchen.de/lehre/vorlesungen/sose_24/t_m1_advanced-statistical-physics/index.html

Sheet 8:

Hand-out: Friday, Jun. 07, 2024; Hand-in: Sunday, Jun. 16, 2024, 11:59 pm

Problem 1 Fluctuations around a tricritical point – **(solution: Central Exercise)**

The Hamiltonian

$$\beta\mathcal{H} = \int d^d\boldsymbol{x} \, \left(\frac{K}{2} (\nabla m)^2 + \frac{t}{2} m^2 + u m^4 + v m^6 \right), \tag{1}$$

with $u = 0$ and $v > 0$ describes a tricritial point.

(1.a) (**3 Points**) Calculate the heat capacity singularity as $t \to 0$ by the saddle point approximation.

(1.b) (**4 Points**) Include both longitudinal and transverse fluctuations by setting

$$\boldsymbol{m}(\boldsymbol{x}) = (\overline{m} + \phi_l(\boldsymbol{x}))\, \boldsymbol{e}_l + \sum_{\alpha=2}^{n} \phi_t^\alpha(\boldsymbol{x}) \boldsymbol{e}_\alpha, \tag{2}$$

and expanding $\beta\mathcal{H}$ to quadratic order in ϕ.

(1.c) (**3 Points**) Calculate the longitudinal and transverse correlation functions.

(1.d) (**3 Points**) Compute the first correction to the saddle point free energy from fluctuations.

(1.e) (**3 Points**) Find the fluctuation correction to the heat capacity.

(1.f) (**3 Points**) By comparing the results from (a) and (e) for $t < 0$ obtain a Ginzburg criterion, and the upper critical dimension, for the validity of mean-field theory at a tricritical point.

(1.g) (**3 Points**) A generalized multicritical point is described by replacing the term vm^6 in Eq.(1) with one of the form $v_{2n} m^{2n}$. Use simple power counting to find the upper critical dimension of this multicritical point.

Problem 2 Solvable model with a tricritical point – (solution: Tutorials)

Consider a 1D Ising chain with N spins, $\sigma_i = \pm 1$, and periodic boundary conditions. The chain also possesses an additional degree of freedom ϵ. Nonzero values of ϵ result in a dimerization of the chain, i.e. alternating bonds are strengthened (or weakened). The Hamiltonian for the system can be written as

$$\mathcal{H}(\{\sigma_j\}_j, \epsilon) = -\sum_{j=1}^{N} \left[1 - \epsilon(-1)^j\right] \sigma_j \sigma_{j+1} + N\omega\epsilon^2, \tag{3}$$

where ω is an additional parameter.

(2.a) (**2 Points**) Write down the partition function of the spin system with respect to $\{\sigma_j\}_j$ and ϵ.

(2.b) (**3 Points**) Use the transfer matrix method to rewrite the spin dependent part of the partition function in terms of two matrices T_{odd} and T_{even} corresponding to odd or even numbered lattice sites, respectively.

(2.c) (**3 Points**) Let $\lambda(\epsilon)$ be the largest eigenvalue of the transfer matrix $T = T_{\text{odd}} T_{\text{even}}$. Find an expression for the free energy per spin in terms of $\lambda(\epsilon)$ by the method of steepest descent (i.e. saddle point).

(2.d) (**3 Points**) Show that the largest eigenvalue of the transfer matrix is

$$\lambda(\epsilon) = 2\left[\cosh(2\beta) + \cosh(2\beta\epsilon)\right]. \tag{4}$$

(2.e) (**3 Points**) There will be no phase transition if $\omega > 0.25$. Show that if $\omega = 0.20$, the system will undergo a second-order phase transition to a dimerized state $\epsilon \neq 0$. Estimate the value of β at the transition.*

(2.f) (**3 Points**) Show that if $\omega = 0.24$, the system will undergo a first-order phase transition to a dimerized state. Estimate β at the transition (e.g. by plotting the free energy as a function of ϵ for a few temperatures).*

(2.g) (**3 Points**) Estimate the values of ω and β at the tricritical point.*

* These problems require computer algebra analysis.