TMP-TC2: Cosmology

Problem Set 9 18 & 20 June 2024

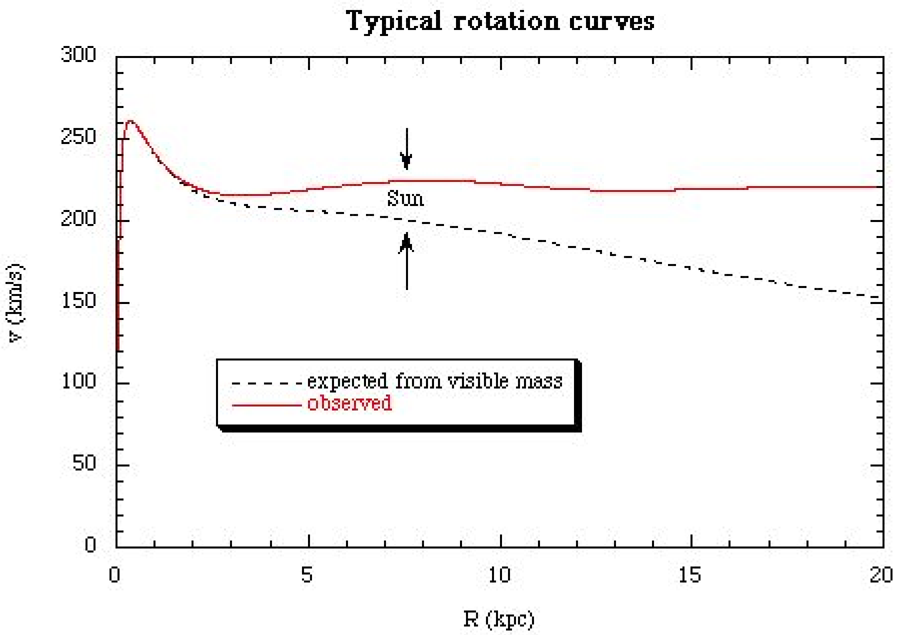
1. Rotation curves of galaxies

Consider the rotation of a small object around an isolated galaxy. Assuming for simplicity that the galaxy is spherically symmetric, how does the velocity of the object depend on its distance to the galaxy's centre, according to Newton's law ?

Consider two cases :

1. The density of the galaxy is concentrated around its centre, i.e. $\rho = \text{const.}$ for $r < r_{core}$.
2. The density of the galaxy decreases according to a power law $\rho \propto r^{-\alpha}$, $\alpha > 0$.

FIGURE 1 – Comparison of the rotation curves between the expectation based on visible matter and the observations



3. The first assumption is consistent with what we would expect from the mass density profile of visible matter in galaxies. However, as you can see in Figure 1, the actual observed curve becomes flat at distances $r \gtrsim r_{core}$, that is, the velocity no longer depends on r. What is the value of α that fits flat rotation curves at these distances ?

2. Weakly Interacting Massive Particles

Let us discuss the possibility that dark matter comprises some stable massive particles X with mass M. Their stability implies that they can only be produced and annihilated in pairs through reactions $SM \to X\bar{X}, X\bar{X} \to SM$, where SM stands for Standard Model particles. Assuming that in the early Universe these particles were in equilibrium with other species, one can estimate the allowed region for the annihilation cross-section $\sigma_{ann} = \sigma_0/v$, where v is the velocity of the (non-relativistic) particles. To this end, one needs to find their abundance Ω_X today as a function of σ_0 and compare it with the known abundance $\Omega_{DM} = 0.25$ of dark matter.

1. Let X be in equilibrium with cosmic plasma at $T > T_f$, where $T_f \ll M_X$, and let $n_X = n_{\bar{X}}$. Assuming that decoupling takes place at radiation domination, find T_f.
2. Find the concentration $n_X\left(t_f\right)$ of X particles at the moment of decoupling and today's concentration $n_X\left(t_0\right)$.
3. Find Ω_X. Using the inequality $\Omega_X \leqslant \Omega_{DM}$, put the bound on σ_0. Indication : As a crude estimate, one may use the relation $\sigma_0 \sim M_X^{-2}$.

3. Primordial Black Holes as Dark Matter Candidates

There exist many different processes in the Early Universe that open the possibility that black holes could have formed in the first instants of the Universe's history. Once inflation ends, rare events or perturbations can become large enough to lead to the formation of enough black holes (but hopefully not too many !). These are referred to as Primordial Black Holes (PBHs) and their masses can range from a few grams to several orders of magnitude above Solar mass. For this reason, they can have masses much below those expected from black holes forming from the collapse of a star.

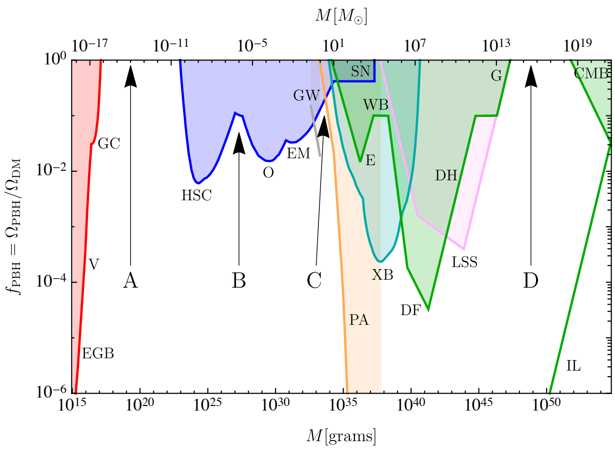


FIGURE 2 – Constraints on f(M) from evaporation (red), lensing (blue), gravitational waves (GW) (gray), dynamical effects (green), accretion (light blue), CMB distortions (orange) and large-scale structure (purple). Source : `arxiv.org/abs/2006.02838`

PBHs present an alternative to particle-like candidates for Dark Matter. We can define the fraction of DM density in the form of PBHs as

$$f_{\mathrm{PBH}} = \frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{DM}}}\,. \tag{1}$$

Figure 2 shows the current bounds on $f_{\rm PBH}$ in terms of the initial mass M of the PBH. One can see that there are only two minor windows of mass ranges that allow for PBH to constitute all of DM.

1. What properties of black holes make them a suitable candidate for Dark Matter ?
2. From Figure2, what values of the initial mass M are still allowed for PBHs to constitute all of DM ?
3. In order for $f_{PBH} = 1$ today, what should be the value of Ω_{PBH} at the end of inflation ? You may assume that matter-radiation equality occured at around $t_{eq} \sim 50000$yr.
4. If one assumes that a Black Hole evaporates completely (according to the standard Hawking semiclassical process), its lifetime is given by

$$t_H = \frac{5120\pi\hbar M^3}{M_P^4}\,, \tag{2}$$

   where M_P is the Planck mass. What is the mass of the lightest PBH that would still be present today ? How does this translate into one of the bounds of Figure 2 ?
5. However, there are convincing reasons (e.g. the so-called "memory burden" phenomenon that can stabilize a BH and therefore extend its lifetime) why we cannot actually expect that the Hawking-like evaporation can be trusted after the BH's half-decay time. Let us assume that the above lifetime is extended as

$$t_H \Longrightarrow \tilde{t}_H = t_H \frac{M^2}{M_P^2}\,. \tag{3}$$

   What is the new evaporation bound ?