# TMP-TC2: Cosmology

**Problem Set 11** 2 & 4 July 2024

## 1. Flatness Problem

We have seen that the Friedmann equation can be written as

$$\Omega - 1 = \frac{k}{(aH)^2}, \tag{1}$$

where $\Omega = \Omega_\gamma + \Omega_m + \Omega_\Lambda$. Show that in order to have $|\Omega - 1| \approx 0.0003$ today, you must have $|\Omega - 1| \approx 10^{-8}$ at recombination, for radiation or matter-dominated universe. Why this observation is called the Flatness problem ?

## 2. Horizon Problem

Calculate the angle that contains one causally connected region in the CMB (at redshift $z = 1500$). You can assume a matter-dominated universe. You will observe that this angle is of the order of one. Why is this a problem ?

## 3. Equations of motion for a homogeneous scalar field in FLRW

Consider the homogeneous solutions of a scalar field $\phi$ described by the action

$$S[\phi] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] ,$$

which evolves in a flat FLRW universe with the metric $ds^2 = -dt^2 + a(t)^2\mathbf{dx}^2$. Derive the equations on motion :

$$\begin{cases} \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \\ H^2 = \dfrac{8\pi G}{3}\left(\dfrac{1}{2}\dot{\phi}^2 + V(\phi)\right) \end{cases} .$$

You can use results from Sheet 1.

## 4. Scalar field in FLRW spacetime

Consider a generic scalar field $\phi(t, \mathbf{x})$ that evolves in a flat FLRW spacetime with the metric $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$

1. Compare the energy-momentum tensor of the scalar field with that of a perfect fluid and determine the field's density $\rho$ and pressure $p$. Find $w \equiv p/\rho$.
2. Determine the condition for accelerated expansion.
3. Assume that the field is homogeneous. Starting from the continuity equation, derive its equation of motion.