TMP-TC2: Cosmology

Solutions to Problem Set 10 25 & 27 June 2024

1. MOND

The MONDified Newton's law reads

$$a_0\ f\left(\frac{a}{a_0}\right) = \frac{GM}{r^2}\ , \tag{1}$$

where the function $f(x)$ has the following properties :

$$f(x) \sim \begin{cases} x, & \text{for } x \gg 1 \\ x^2, & \text{for } x \ll 1\ . \end{cases}$$

1. Assuming that for large r the acceleration is smaller that a_0, we get

   $$f\left(\frac{a}{a_0}\right) \sim \left(\frac{a}{a_0}\right)^2\ ,$$

   therefore eq. (1) becomes

   $$a^2 = \frac{GMa_0}{r^2}\ .$$

   We know that for circular motion the acceleration a and circular velocity v are related by

   $$a = \frac{v^2}{r}\ ,$$

   which implies

   $$v = (GMa_0)^{1/4}\ . \tag{2}$$

   We observe that v does not depend on r if we accept this sort of modification to Newton's law.

2. We now want to estimate the value of a_0. We find from eq. (2) that it is given by

   $$a_0 = \frac{v^4}{GM}\ .$$

   Using $v \approx 200$ km s^{-1}, $M \approx 10^{11} M_\odot$ and $G = 4.302 \times 10^{-3} M_\odot^{-1}$ pc (km s)$^{-2}$, we find from the above

   $$a_0 \approx 1.2 \times 10^{-10}\ \text{m s}^{-2}\ ,$$

   which is roughly equal to cH_0, with H_0 the value of the Hubble parameter today.

2. Baryon Asymmetry of the Universe

As we have already seen in numerous occasions, decoupling happens when the expansion term and the reaction rate are roughly of the same order :

$$H(T_d) \sim \Gamma(T_d) \ . \tag{3}$$

The reaction rate is given by

$$\Gamma = \langle \sigma v \rangle n \ , \tag{4}$$

where n is the density of the target particles. Therefore, for anti-protons the targets are the protons. We know that the Hubble parameter is

$$H \approx 1.65 \sqrt{g_*} \frac{T^2}{M_{pl}} \ . \tag{5}$$

In the following we will ignore all numerical prefactors, because at the end we want to get just order of magnitudes. Hence, we have $H \sim \frac{T^2}{M_{pl}}$, $n = \eta n_\gamma$ and $\langle \sigma v \rangle \sim m_p^{-2}$ with m_p the mass of the proton.
Decoulpling happens at

$$\frac{T_d^2}{M_{pl}} \sim \eta m_p^{-2} T_d^3 \ , \tag{6}$$

which gives a decoupling temperature

$$T_d \sim \frac{m_p^2}{M_{pl}\eta} \sim \mathcal{O}(eV) \ . \tag{7}$$

Assuming that the processes are in equilibrium at decoupling, we can obtain an order of magnitude estimate for the particle density of the anti-protons :

$$n^{eq} = 2 \left(\frac{mT}{2\pi} \right)^{\frac{3}{2}} e^{-\frac{m-\mu}{T}} \ , \tag{8}$$

$$\bar{n}^{eq} = 2 \left(\frac{mT}{2\pi} \right)^{\frac{3}{2}} e^{-\frac{m+\mu}{T}} \ . \tag{9}$$

$$\tag{10}$$

We can eliminate the chemical potential by considering the product of the above

$$n^{eq} \bar{n}^{eq} \sim (m_p T_d)^3 e^{-\frac{2m_p}{T_d}} \ . \tag{11}$$

Using $n^{eq} = \eta n_\gamma \sim \eta T_d^3$, we find

$$\bar{n}^{eq} \sim \frac{m_p^3}{\eta} e^{-\frac{2m_p}{T_d}} \sim e^{-10^{11}} \ . \tag{12}$$

This number is so small that we can say that the universe contains practically no anti-protons.

3. Magnetic Monopole Problem

The energy density of monopoles today is given by

$$\epsilon_{\mathrm{M}}^0 = m_{\mathrm{M}} n_{\mathrm{M}}^0 \ , \tag{13}$$

with $m_{\mathrm{M}} \simeq 10^{17}$ GeV and n_{M}^0 the number density of monopoles today.

No annihilation or creation of monopoles took place, therefore,

$$n_{\mathrm{M}}^0 = n_{\mathrm{M}} R^3 = n_{\mathrm{M}} \left(\frac{T_0}{T_{\mathrm{GUT}}} \right)^3 \ , \tag{14}$$

where n_{M} is their number density at T_{GUT}. Since there is one monopole per Hubble patch,

$$n_{\mathrm{M}} = \frac{1}{r_H^3} = \left(\frac{T_{\mathrm{GUT}}^2}{M_{\mathrm{Pl}}} \right)^3 \ . \tag{15}$$

From the above it is easy to see that

$$n_{\mathrm{M}}^0 = \left(\frac{T_{\mathrm{GUT}}}{M_{\mathrm{Pl}}} \right)^3 T_0^3 \ , \tag{16}$$

which in turn results into

$$\epsilon_{\mathrm{M}}^0 = m_M \left(\frac{T_{\mathrm{GUT}}}{M_{\mathrm{Pl}}} \right)^3 T_0^3 \sim 10^{-14} \frac{g}{cm^3} \ . \tag{17}$$

This energy density is 16 orders of magnitude bigger than the critical energy density $\varepsilon_c \sim 10^{-30} \frac{g}{cm^3}$, so it is in complete disagreement with observations of our Universe today. This problem is called the magnetic monopole problem.
On the next sheet we will discuss inflation, which would solve the problem, because through an extreme expansion of the universe, the density of magnetic monopoles would decrease to appropriate levels. Besides inflation, there are also other solutions to this problem. If you are interested in it you can have a look on the Langacker-Pi mechanism, symmetry non-restoration or the monopole erasure by domain wall collisions.