Sheet 03: Entropy II

Discussion: Thursday 01.06.2023

Exercise 1 Entropy of a Rubber Band

Following some empirical observations, we want to derive a plausible entropy function for a rubber band. At rest, the length of the rubber band is L_0. When stretched to a length $L_0 < L < L_{\rm el}$,it exerts a tension $\mathcal{T}$. We restrict our model to this regime, in which the force is proportional to the strain. Proceed as follows:

(a) Justify qualitatively the expression $E = cL_0T$ in a relevant temperature range.

(b) What corresponds to $-p\,\mathrm{d}V$ for the rubber band?

(c) $\mathcal{T} \propto L - L_0$. Determine $f(T)$ in

$$\mathcal{T}(T, L) = bf(T)\frac{L - L_0}{L_{\rm el} - L_0}\,. \tag{1}$$

Hint: use an identity for mixed second derivatives.

(d) Now compute $S(E, L)$.

Exercise 2 Free Expansion of a Gas

Consider an ideal Gas composed of N particles with energy E in an isolated container with volume V. The container is partitioned, such that at the beginning the gas can fill only the left half, while the other half is completely empty. The partition wall is removed and the gas can expand to the right half of the container. Determine the equilibrium state:

(a) using the entropy of an ideal gas.

(b) by means of our considerations about equilibrium states from chapter 9.

What kind of maxima do we have?

Exercise 3 Entropy and Disorder

Consider two ideal gases (gas 1 and gas 2) at temperature T, each of which is constituted by N particles, which are mixed in a small volume $V_{\rm init}$. Besides it, there are two volumes $V_{1,2} = 99V_{\rm init}$ which are separated from each other and also from $V_{\rm init}$. Now connect V_1 and V_2 with a semi-permeable membrane with $V_{\rm init}$, such that gas 1 can expand only in the volume V_1 and gase 2 can expand only in volume V_2. Determine the equilibrium state. By how much does the entropy increase? Would you argue that the disorder has increased?