TMP-TC2: Cosmology

Problem Set 9 20 & 22 June 2023

1. Fourth neutrino

In this problem we will consider that there is a fourth neutrino which is massive and interacts weakly as the others. The goal is to find available intervals for its mass by using constraints from cosmology.

1. Find the following relationship
$$n_{\nu_4}(T^*)\langle\sigma v\rangle = 1.65\sqrt{g^*}\frac{T^{*2}}{M_{Pl}} \tag{1}$$
where n_{ν_4} is the particles density of the neutrinos, σ describes the cross-section of neutrino anti-neutrino scattering, v is the velocity of the neutrinos and T^* denotes the decoupling temperature.
Hint : You can assume that $n_{\nu_4} = n_{\bar{\nu}_4}$.
2. The energy density of the new neutrino should not exceed the density of dark matter. Using this fact, find
$$2m_{\nu_4}n_{\nu_4}(T^*)\frac{g^*(T_0)}{g^*(T^*)}\left(\frac{T_0}{T^*}\right)^3 \leq \Omega_{DM}\rho_c \tag{2}$$
3. Assume that the fourth neutrino is very light so that you can use the relativistic limit at decoupling. Use the results of the first two parts and the thermal average product $\langle\sigma v\rangle \sim G_F^2 T^{*2}$ to find an upper bound for the mass m_{ν_4}.
4. Now assume that the neutrino is very heavy so that you need to use the non-relativistic limit at decoupling. What is the lower bound for the mass m_{ν_4} ? You can take $\langle\sigma v\rangle \sim G_F^2 m_{\nu_4}^2$.

2. Baryon Asymmetry of the Universe

At temperatures $T \gg m_p$ the universe had an excess of protons of the order of
$$\frac{n_p - n_{\bar{p}}}{n_p + n_{\bar{p}}} = 10^{-10}$$
This number changes due to annihilation and pair creation of protons and anti-protons until decoupling happens. Estimate the decoupling temperature and give a comment on the existence of anti-protons today.

3. Recombination

In this exercise we want to find the recombination temperature, i.e. the temperature when the electrons and protons recombined in hydrogen atoms.

1. Consider protons, electrons hydrogen atoms and photons held in equilibrium. Derive the Saha equilibrium condition, i.e. write the hydrogen atoms density n_H as a function of n_p and n_e.
2. Determine the temperature for which the number of protons is equal to the number of hydrogen atoms.
   You can use the fact that the baryon to photon number density $\eta \equiv \frac{n_B}{n_\gamma} \approx 10^{-10}$ was conserved in the history of the universe and that the plasma is electrically neutral.