TMP-TC2: Cosmology

Problem Set 8 13 & 15 June 2023

Big Bang Nucleosynthesis

At $T \sim 1$MeV, most of the matter in the Universe is in the form of relativistic photons, electrons, positrons and neutrinos. However, there is still a small fraction in the form of baryons, which can be in the form of either neutrons, protons or heavier nuclei. These will convert into each other through weak nuclear reactions leading to the synthesis of the lighter elements, Hydrogen, Helium, Lithium and Berylium, in the Early Universe. This is known as the Big Bang Nucleosynthesis (BBN).

In principle, to determine the ratios of the final products of this process, one has to to solve a system of Boltzman equations for each of the elements present simultaneously, which has been done numerically to great accuracy. However, the purpose of this exercise is to gain a deeper understanding of the complex processes involved in BBN and to analytically estimate the ratio

$$\frac{4n_{\mathrm{He}}}{n_{\mathrm{H}}} = \frac{1}{4}, \tag{1}$$

between the final mass fraction of Helium and Hydrogen, assuming that no heavier elements than Helium are produced.

Part A : Neutron-to-Proton Ratio

We will start by trying to find the neutron-to-proton ratio $\frac{n_n}{n_p}$ when the universe is in chemical equilibrium.
You may make use of the following numbers : neutron mass $m_n = 939.57$ MeV, proton mass $m_p = 938.27$ MeV and baryon-to-photon ratio $\eta \sim 10^{-9}$.

1. Through which processes can protons and neutrons convert into each other ?
2. To consider these processes in chemical equilibrium, we require an estimate of μ_e. Show that $\mu_e \ll T$, in the ultra-relativistic limit.
   Hint : You can use the fact that the universe is electrically neutral and the proton number is of the same order as the baryon number, i.e. $n_{e^-} - n_{e^+} \sim n_B$.
3. Use the non-relativistic limit to show that at the temperature $T \sim 10 - 30$keV the ratio $\frac{\mu_e}{T}$ is of order one.
4. Now, for $T \gg 30$keV, and assuming that $\mu_\nu \ll T$, show that in equilibrium

   $$\left(\frac{n_n}{n_p}\right)_{\mathrm{eq}} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-\frac{Q}{T}}, \tag{2}$$

   where $Q = m_n - m_p \sim 1.3$ MeV.
   This result tells us that, for $T \gg Q$, we have as many neutrons as protons.

Part B : Neutron Freeze-out and Decay

After the universe cooled down to around $T \sim 0.8$ MeV, neutrinos decoupled and weak interactions ceased to operate.

1. Calculate the freeze-out abundance of neutrons, defined as

$$X_n^{\text{freeze-out}} = X_n(T \sim 0.8\text{MeV}) = \frac{n_n}{n_n + n_p}. \tag{3}$$

2. For $T \sim 0.2$MeV, the age of the universe becomes comparable to the neutron lifetime $\tau_n \approx 886.7s$. Assuming no further interactions, what is $X_n(t)$ after neutron freeze-out ?

Part C : Deuterium Bottleneck and Helium Production

So far we have completely ignored the fact that a neutron and a proton can form Deuterium through the process

$$n + p \leftrightarrow D + \gamma.$$

Once enough Deuterium is produced, i.e. when the number of deuterium nuclei becomes comparable to the number of protons, Helium can form through

$$\begin{aligned} &D + p \leftrightarrow {}^3He + \gamma, \\ &D + {}^3He \leftrightarrow {}^4He + p. \end{aligned}$$

You may use the following numbers : Deuterium mass $m_D = 1875.61$ MeV and Helium mass $m_{^4He} = 3699.11$ MeV.

1. Show that in equilibrium

$$\left(\frac{n_D}{n_p}\right)_{\text{eq}} \approx \frac{3}{4} n_n^{\text{eq}} \left(\frac{4\pi}{m_p T}\right)^{\frac{3}{2}} e^{\frac{B_D}{T}} \tag{4}$$

   where $B_D = m_n + m_p - m_D$ is the binding energy of Deuterium. Estimate the order of magnitude of this ratio for $T \gg B_D$. Is it a problem that we neglected Deuterium production in Parts A and B ?

2. At which temperature does the nucleosynthesis of ^{4}He start ?
3. Take $g_* = 3.38$ and calculate the age of the universe when nucleosynthesis started.
4. What is the reason that Helium is produced almost immediately after Deuterium ?
5. In the end, all neutrons form ^{4}He-nuclei and we are left with mostly only Helium and Hydrogen. What is the value of the ratio $\frac{n_{\text{He}}}{n_{\text{H}}}$?

Part D : ^{4}He-Abundance

The Helium-4 abundance by weight is defined as

$$X_{^4He} = \frac{4n_{^4He}}{n_B}. \tag{5}$$

1. How does the Helium abundance $X_{^4He}$ change if we change the number of relativistic degrees of freedom at nucleosynthesis ?

   Hint : You may use the fact that the reactions under consideration decouple at a temperature very close to the neutrino decoupling temperature, that is,

   $$\frac{T^{*2}\sqrt{g^*}}{M_P} \sim G_F^2 T^{*5} \ . \tag{6}$$

2. What does this say about the presence of additional neutrinos ?
3. Estimate the change in the abundance of ^{4}He that would be caused by increasing $m_n - m_p$ by 10 % and by decreasing the neutron lifetime τ_n also by 10 %. You can assume for this that BBN lasted for around 20mins.