TMP-TC2: Cosmology

Problem Set 7 6, 7, 8 June 2023

1. Proton gas

We consider a universe dominated by radiation containing photons (γ), protons (p), antiprotons ($\bar{p}$) and pions (π). We consider the reaction $p\bar{p} \leftrightarrow n\pi$, $n \geq 2$, with cross section $\sigma = m_\pi^{-2}$, where m_π is the pion's mass. Consider first the case of a neutral gas, i.e. $n_p = n_{\bar{p}}$.

1. Find the decoupling temperature T_d assuming that protons are non-relativistic. *Indication :* Use the age-temperature relation that we found on a previous sheet.
2. Find the ratio between the proton and photon densities at time t_d.
3. Calculate the density of photons and use the ratio of the previous point to find the current density of residual protons.
4. Assuming that the universe has an excess of protons $\frac{n_p - n_{\bar{p}}}{n_p + n_{\bar{p}}} = 10^{-10}$ for $T \gg m_p$, find the present ratio ($T \ll m_p$) between proton and photon densities.

2. Neutrino Decoupling

The cosmic neutrino background and the cosmic microwave background are both background radiations from the early Universe. While the cosmic microwave background is made up of photons, the cosmic neutrino background is composed of neutrinos and offers a window into the Universe's first few seconds of existence. In this problem we will estimate at which temperature neutrinos decouple and what the cosmic neutrino background temperature is.

1. The leading process that keeps neutrinos in equilibrium comes from weak interactions, by the exchange of the W boson in the processes

$$e^+ + e^- \rightleftarrows v_e + \bar{v}_e, \quad e^\pm + v_e \to e^\pm + v_e, \quad e^\pm + \bar{v}_e \to e^\pm + \bar{v}_e \ . \tag{1}$$

.

We estimate them using Fermi theory. The relevant diagrams are given in figure 1 ; note that every vertex $\sim \frac{\alpha_W}{M_W^2}$ (can you argue why ?).

Estimate qualitatively the temperature of neutrino decoupling. You can assume the universe was radiation dominated at the time.

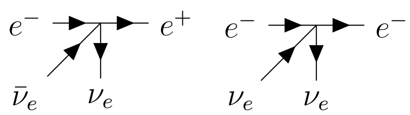

FIGURE 1 – Neutrinos weak interactions

Indication: You don't need to precisely calculate the diagrams, try to estimate the cross-section instead.

2. Before the temperature of decoupling, which particles contribute to the plasma ?
3. Using the fact that the entropy stays constant, calculate the neutrino background temperature.

3. Decoupling and concentration

Consider a neutral particle s with spin 0 in thermal equilibrium in the Universe. Suppose that the particle decouples at temperature $T_d = 150$ MeV and has a mass $m_s = 100$ eV.

1. Which particles contribute to the plasma before the temperature T_d ?
2. Estimate the relationship between the temperature T_s of the particles s and the temperature of photons T_γ at the moment of freezing of e^+e^- annihilation (~ 10 keV).
3. Estimate the density n_s of these particles today and their cosmological abundance Ω_s.
4. Is the existence of these particles consistent with cosmological observations ?

4. Investigating the kinetic equation numerically

Let us analyse the electron-positron annihilation process ($e^+e^- \to \gamma\gamma$) in more detail. The kinetic equation for the total density n of electrons and positrons reads as follows,

$$\frac{\partial n}{\partial t} + 3Hn = -\Gamma(n - n_{eq}), \tag{2}$$

where H is the Hubble parameter and Γ is the rate of the reaction $e^+e^- \to \gamma\gamma$. In two regimes (when $T \gg m$ and $T \ll m$, where m is the mass of an electron) analytical approximations to the solution of equation (2) can be found. To find n in general, however, one has to use numerical techniques.

1. Before you use numerical methods, find analytical expressions for n_{eq} in the limits $T \ll m$ and $T \gg m$.
2. As a next step toward the solution of Eq.(2), find numerically the equilibrium particle density n_{eq}. Check that in the limits $T \ll m$ and $T \gg m$, n_{eq} is well approximated by the analytical expressions found before.
3. Calculate the rate Γ and check the limits $T \ll m$ and $T \gg m$.
   Indications : Γ is written in the form

   $$\Gamma = \langle \sigma f_{eq} v \rangle. \tag{3}$$

   In the expression above, σ is a total cross section of the reaction $e^+e^- \to \gamma\gamma$, which is given by

   $$\sigma = \frac{\pi e^4}{m^2}\frac{1-v^2}{4v}\left(\frac{3-v^4}{v}\log\frac{1+v}{1-v} - 2(2-v^2)\right), \tag{4}$$

and v is the velocity of particles,

$$v = \frac{\sqrt{E^2 - m^2}}{E}. \tag{5}$$

4. Plug n_{eq} and Γ into Eq.(2) and solve the resulting equation numerically.
   Indication : Consider a radiation-dominated Universe.