TMP-TC2: Cosmology

Problem Set 10 27 & 29 June 2023

1 Flatness Problem

We have seen that the Friedmann equation can be written as

$$\Omega - 1 = \frac{k}{(aH)^2}, \tag{1}$$

where $\Omega = \Omega_\gamma + \Omega_m + \Omega_\Lambda$. Show that in order to have $|\Omega - 1| \approx 0.0003$ today, you must have $|\Omega - 1| \approx 10^{-8}$ at recombination, for radiation or matter-dominated universe. Why this observation is called the Flatness problem ?

2 Horizon Problem

Calculate the angle that contains one causally connected region in the CMB (at redshift $z = 1500$). You can assume a matter-dominated universe. You will observe that this angle is of the order of one. Why is this a problem ?

3 Phase Transitions and Bubble Nucleation

As you have seen in the lecture, the symmetry group of the Standard Model is $SU(3)\times SU(2)\times U(1)$. There are theories in which the symmetry group was larger at higher temperatures (for example Grand Unified Theories, the Left-Right symmetric theory, etc.). Whenever a symmetry is spontaneously broken, a phase transition occurs. In this problem, we aim to provide a brief overview of what may occur during these phase transitions.

1. (second-order phase transition) Consider a complex scalar field theory with potential

$$V(\phi) = m^2\phi^*\phi + \lambda(\phi^*\phi)^2 \ , \tag{2}$$

and assume the mass to be temperature dependent, i.e. take $m^2 = m(T)^2 = \mu^2(T^2 - T_C^2)$ with μ a dimensionless constant. Describe the behavior of the potential for $T = 0$, $T < T_C$, $T > T_C$, $T = T_C$.

2. (first order phase transition) Now take a $U(1)$ gauge theory with the following potential for the scalar field

$$V(\phi) = m^2(T)|\phi|^2 + \frac{m_0^2}{\nu\sigma^2}|\phi|^4 \ln\frac{|\phi|^2}{\sigma^2} \ , \tag{3}$$

with $m^2(T) = m_0^2 + \frac{1}{4}g^2T^2$. Here g is the gauge coupling, σ a parameter with the dimension of mass (the renormalization scale), and $\nu = \frac{16\pi^2 m_0^2}{3g^4\sigma^2}$.

The second term in the potential is generated radiatively (at one loop level) due to the interaction of the scalar field with the gauge boson. Describe this potential qualitatively. How does its shape change with decreasing temperature ?

3. As a simplified model for a first order phase transition we take the following model in one spatial dimension

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \lambda\phi^2(\phi^2 - v^2)^2 \tag{4}$$

   Find classical solutions to the field equations that describe domain walls that separate the symmetric phase from the broken phase. What is the energy of the domain wall ?
   Hint : First show that the static field equation is equivalent to the Bogomolny equation $\phi'(x) = \pm\sqrt{2V(\phi)}$ and then solve this equation for appropriate boundary conditions.

4. Now we add a second spatial dimension. With the domain wall solution, we can form a bubble of radius R :

$$\phi_{\text{bubble}} = \phi_{\text{DW}}(r - R) \tag{5}$$

   where inside the bubble the symmetry is broken and outside the symmetry is restored. This bubble will collapse. Use simple relativistic calculations in the thin wall approximation to find $R(t)$. **Hint :** Note that the energy of the domain wall that you calculated before is now an energy density σ_{DW}.

5. What term can you add to the potential to obtain a growing bubble ?

6. As we have seen it is possible to create bubbles that grow. These bubbles can arise in the early universe through thermal fluctuations and expand until they fill the entire universe with a broken vacuum. Looking ahead, do you have any thoughts on how we might be able to observe these phase transitions experimentally in the distant future ?

4 Magnetic Monopole Problem

Domain walls are not the only type of objects that can be created during phase transitions. In this problem, we will focus on the so-called magnetic monopoles.
During the phase transition, there were many causally disconnected regions. Each such region can obtain a different vacuum expectation value (in the case of a degenerate vacuum). Let us assume that there was one magnetic monopole created per causally disconnected region of radius $r_H \sim \frac{M_{\text{Pl}}}{T^2_{\text{GUT}}}$, with $T_{\text{GUT}} \sim 10^{16}$GeV the temperature at which the monopole of mass $m_M \sim 10^{17}$GeV was created.
Assuming that the monopole was not destroyed and also no more monopoles were created until now, what would be the contribution to the energy density today ? Why is this a problem ?