

CONDENSED MATTER MANY-BODY-

PHYSICS & FIELD THEORY I

TMP-TA3

LMU SoSe 2023

Prof. F. Grusdt



ORGANIZATION:

This is the second course taught in the series

- THP-TA1: Theor. Cond. Mat. Phys.
- THP-TA3: Cond. Mat. Many-Body-Phys. & FieldThy. I
- THP-TA4: Cond. Mat. Many-Body-Phys. & FieldThy. II

I will assume the following topics are known from previous theoretical physics courses:

- * basics of classical mechanics
- * basics of electromagnetism
- * basics of quantum mechanics
- * basic (quantum) theory of solids (band-theory, point-group symmetries, phonons, basic transport theory, ...)
- * basics of statistical physics

Basic organization:

- We will have 2 lectures / week:

MO 12¹⁵ - 13⁴⁵, A348

WED 14¹⁵ - 15⁴⁵, A449

- We will have problem sets: 1 sheet / week

⇒ no grading, but solutions; first set on FR, Apr. 21

- We will have tutorials held by Mattia Moroder

⇒ First tutorial on: TUE, April 25 (1st solutions on May 2)

- We will have a final exam on: ~ early Aug - TBC

- The lecture is worth 9 ECTS

0) CONTENTS:

Goal: The goal of this lecture is to learn the basic tools for describing quantum many-body systems. Such systems involve so many degrees of freedom that exact descriptions are almost always entirely impossible. Hence here are two main goals we can hope to achieve:

i) Develop a precise mathematical formalism to define well-posed physical/mathematical problems.
I.e. learn to see your enemy!

=> This will be done at different levels of detail and mathematical rigor!

ii) Develop tools to actually solve a problem. Typically this requires studying very special models, or strong (and often poorly controlled) mathematical simplifications.

Ultimately, we only understand a rather limited set of physical problems, but many situations can be related to a known limit.

⇒ This leads to the idea of universality in (quantum or classical) many-body physics - namely that, independent of certain details that we need to specify, a characteristic type of collective behavior is observed.

Outline of the lecture:

1) Introduction

Many-body quantum systems, condensed matter, quantum simulation, emergent phenomena, open problems in the field

2) Second quantization & applications

Formalism, Weakly interacting bosons & Bogoliubov theory, Hubbard models: Mott insulators & Superfluids, 1D systems: Jordan-Wigner transformation, BCS theory of superconductivity: Mean-field theory

3) Green's functions & functional integrals

Green's functions & path integral representation

Coherent states & Grassman variables

Quantum-classical mapping

4) Feynman's diagrammatic perturbation theory

Formalism & basic derivation: $T=0$

Examples: Lindhard function & Hartree-Fock energy

Screened Coulomb interactions: RPA and large N

Finite temperature formalism

5) Landau Fermi-liquid theory

Weak interactions: Fermi liquid

Random phase approximation (RPA) & response functions (part 1)

Collective excitations

6) Linear response theory

Formalism & application to Kubo formula

Fluctuation-dissipation theorem

Spectroscopic probes

7) BCS theory & charged superfluids

Hubbard stratonovich transformation: derivation of
Ginzburg-Landau theory

Anderson-Higgs mechanism

8) Quantum Magnetism

Fermi-Hubbard model: phenomenology

Heisenberg model: mean-field & spin-wave theory

1D Heisenberg chain: a quantum spin liquid

Field theory: non-linear σ -model

Literature:

- P. Coleman: "Introduction to Many-Body Physics"
- E. Fradkin: "Field Theories of Condensed Matter Physics"
- A. Auerbach: "Interacting Electrons & Quantum Magnetism"
- S. Sachdev: "Quantum Phase Transitions"
- X.-G. Wen: "Quantum Field Theory of Many-Body Systems"
- A. Altland, B. Simons: "Condensed Matter Field Theory"
- E. Fradkin: "Quantum Field Theory"

Condensed Matter Many-Body Physics & Field Theory I

TMP - TA3, SoSe 2023

Prof. Fabian Grusdt
LMU München

CONTENTS:

- I) Introduction
 - I.1) Basic terminology & quantum many-body systems
 - I.2) Quantum fields & condensed matter
 - I.3) Modern research directions
- II) Second Quantization & Applications
 - II.1) Formalism: 2nd quantization
 - II.2) Weakly interacting bosons & Bogoliubov theory
 - II.3) Hubbard model: Mott insulators & superfluids
 - II.3.1) Fermi-Hubbard model
 - II.3.2) Bose-Hubbard model
 - II.4) Phase diagram of interacting Bosons
 - II.4.1) Recap: Classical phase transitions
 - II.4.2) Quantum phase transitions — a quick overview
 - II.4.3) Quantum phase transitions in the 3D Bose-Hubbard model
 - II.4.4) General theorems of spontaneous symmetry breaking
 - II.4.5) Phase diagram: Bose-Hubbard model in 1D, 2D
 - II.5) 1D systems: Jordan-Wigner transformation
 - II.5.1) The Jordan-Wigner mapping
 - II.5.2) Solution of the 1D quantum XY-model
 - II.6) BCS-theory of superconductivity
 - II.6.1) Short history
 - II.6.2) The Cooper-pair wavefunction
 - II.6.3) BCS mean-field theory
- III) Green's functions & Functional integrals
 - III.1) Green's functions
 - III.1.1) Harmonic oscillator & free boson Green's function
 - III.1.2) Free fermion Green's function & Grassman numbers
 - III.1.3) Spectral decomposition
 - III.2) Path-integral formulation of many-body systems
 - III.2.1) Quick review: Feynman's path integral

- III.2.2) Coherent states
 - III.2.3) Bosonic path integral
 - III.2.4) Fermionic path integral
 - III.2.5) Matsubara frequencies and summations
 - III.2.6) Basics of Hubbard-Stratonovich decoupling
 - III.3) Quantum-classical mapping
 - III.3.1) General concept
 - III.3.2) Example: Transverse-Field Ising model
 - III.3.3) Limitations: Berry-phase terms
 - III.3.4) Finite temperature: classical theory
 - III.4) Example: Hubbard models
 - III.4.1) Bose-Hubbard model / weakly interacting bosons
 - III.4.2) Fermi-Hubbard model
- IV) Feynman's diagrammatic perturbation theory
 - VI.1) Feynman Rules & Feynman diagrams
 - VI.1.1) Real-space Feynman rules & examples
 - VI.1.2) Formal derivation of Feynman rules (sketch)
 - VI.1.3) Linked-cluster theorem
 - VI.1.4) Momentum-space Feynman rules
 - VI.1.5) Relation to the ground state energy
 - VI.1.6) The self-energy & Dyson equation
 - VI.1.7) Hartree-Fock self-energy
 - VI.1.8) Response functions
 - VI.2) Applications: RPA and screening of the Coulomb interaction
 - VI.2.1) The large-N electron gas
 - VI.2.2) Screened interactions & plasma mode
 - VI.2.3) Bardeen-Pines interaction
 - VI.3) Finite temperatures
- V) Landau Fermi-Liquid theory
 - IV.1) Basic concepts & Landau theory
 - IV.1.1) Adiabatic concept & Gell-Mann — Low theorem
 - IV.1.2) The quasiparticle concept
 - IV.1.3) Neutral Fermi liquid & Landau parameters
 - IV.2) Other aspects of Fermi-liquids
 - IV.2.1)* Charged Fermi liquids: Landau — Silin theory
 - IV.2.2) Inelastic quasiparticle scattering
 - IV.2.3) Microscopic aspects
 - IV.2.4) Luttinger's theorem
 - IV.3) Random-phase approximation (RPA) & Collective modes
 - IV.3.1)* Response function: Equations-of-motion (EOM) approach to the RPA
 - IV.3.2) Collective excitations: Zero sound & Stoner instability
 - IV.3.3) Spin- & charge correlations from RPA
 - IV.3.4)* Limitations of RPA & 2-particle self-consistent theory

VI) Linear response theory

VII.1) Response functions & Fluctuation-dissipation theorem

VII.2) Calculation of response functions

VII.3) Electron transport & Kubo formula

VII.3.1) Kubo formula

VII.3.2) Drude conductivity

VII) BCS-theory & charged superfluids

VIII.1) Path-integral formulation of BCS theory

VIII.1.1) The simplified BCS Hamiltonian

VIII.1.2) Saddle-point mean-field solution

VIII.1.3) Nambu-Gor'kov Green's function

VIII.2) Ginzburg-Landau theory of superconductivity & Anderson-Higgs mechanism

VIII.2.1) Ginzburg-Landau functional and gauge invariance

VIII.2.2) Meissner effect

VIII.2.3) Anderson-Higgs mechanism

VIII) Quantum Magnetism

V.1) Magnetism in the Fermi-Hubbard model

V.1.1) Mean-field theory

V.1.2) Ferromagnetic mean-field solution

V.1.3) Antiferromagnetic mean-field solution

V.1.4)* Another perspective on the mean-field state: RPA

V.1.5) The Heisenberg model: Strong coupling limit

V.1.6) Summary: Half-filling phase diagram of the 2D Fermi-Hubbard model

V.2) Field-theory of the Heisenberg antiferromagnet: Non-linear sigma model

V.2.1) Non-linear sigma model Lagrangian

V.2.2) Beyond the non-linear sigma model: Topological term

V.3) Variational wavefunctions & frustrated magnets

V.3.1) J1-J2 model & Majumdar-Gosh state

V.3.2) Anderson's RVB states & Gutzwiller projection