Neutrino Physics Course

Lecture $\overline{\underline{XVII}}$

23/6/2023

LMU

Summer 2021

LRSM: (even) more on SSB

- **first stage:** LR scale!

$G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{BL}$

$$G_{LR} \xrightarrow{\langle \Delta_R \rangle} SU(2)_L \times U(1)$$

$$V = -\underline{\mu_\Delta}^2 \left(Tr\, \Delta_L^\dagger \Delta_L + Tr\, \Delta_R^\dagger \Delta_R \right)$$

$$+ \frac{\lambda_1}{4} \left[\left(Tr\, \Delta_L^\dagger \Delta_L \right)^2 + \left(Tr\, \Delta_R^\dagger \Delta_R \right)^2 \right]$$

$$+ \frac{\lambda_2}{2} \left[Tr\, \Delta_L^2\, Tr (\Delta_L^\dagger)^2 + L \leftrightarrow R \right]$$

$$+ \frac{\lambda_3}{2} \quad Tr\, \Delta_L^\dagger \Delta_L \; Tr\, \Delta_R^\dagger \Delta_R$$

$$+\frac{\lambda_4}{2}\left(\mathrm{Tr}\,\Delta_L^2\,\mathrm{Tr}\,\Delta_R^{+2}+L\leftrightarrow R\right)$$

$\boxed{\text{Maiezza et al '16}}$

$$\langle\Delta_L\rangle=0,\quad\langle\Delta_R\rangle=\begin{pmatrix}0&0\\v_R&0\end{pmatrix}$$

$$\Delta_L=\begin{pmatrix}\delta_L^{+}&\delta_L^{++}\\\delta_L^{0}&-\delta_L^{+}\end{pmatrix}$$

$$\Delta_R=\begin{pmatrix}\delta_R^{+}&\delta_R^{++}\\v_R+H_R+iG_R&-\delta_R^{+}\end{pmatrix}$$

eaten by W_R^+

eaten by Z_R

$(\lambda_1 > 0,\ \lambda_2 > 0)$

$$\boxed{\begin{aligned} m_{H_R}^2 &= 2\lambda_1 v_R^2 \\ m_{\delta_R^{++}}^2 &\propto \lambda_2 v_R^2 \end{aligned}}$$

Δ_R sector

$$\Downarrow$$

Check *

$$\boxed{m_{\delta_L^0}^2 = m_{\delta_L^+}^2 = m_{\delta_L^{++}}^2 \propto (\lambda_1 - \lambda_3)\, v_R^2}$$

$$\Updownarrow$$

$$\boxed{m_{\Delta_L}^2 \propto (\lambda_1 - \lambda_3)\, v_R^2}$$

(>0)

$$\Uparrow$$

explanation

$$v_L = 0, \quad v_R \neq 0$$

$$\Updownarrow$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\downarrow \qquad \frac{Y}{2} = T_{3R} + \frac{B-L}{2}$$

$$SU(2)_L \times U(1)_Y$$

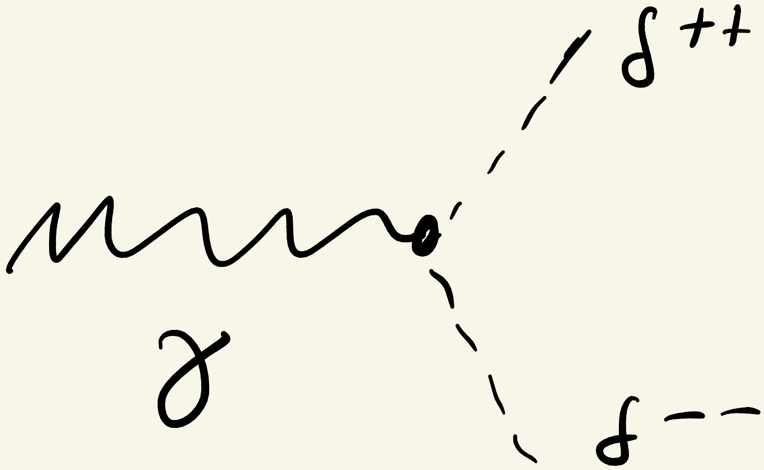
unbroken !!!

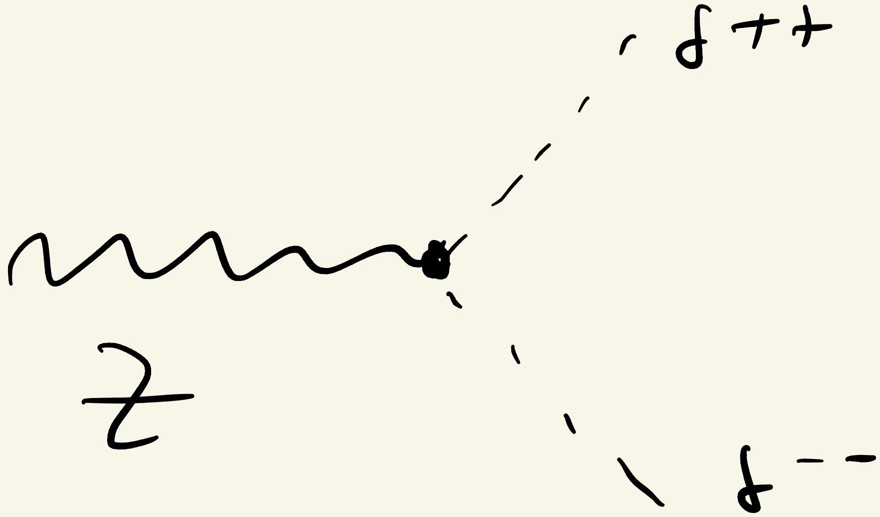
$$\left(\text{LHC}: \quad m_{\delta_L^{++}},\ m_{\delta_R^{++}} \gtrsim 400\,\text{GeV} \right)$$

$$\Downarrow \quad \text{theory}$$

$$/\!/ \quad m_{\delta_L^{+}} \gtrsim 400\ \text{GeV}$$

$m_{\delta_L^0} \gtrsim 400 \text{ GeV}$





$$\delta^{--} \to \begin{matrix} e + \mu \\ e + e \\ \mu + \mu \end{matrix}$$

• <u>next stage</u>: weak scale

$$SU(2)_L \times U(1)_Y \xrightarrow{\langle\phi\rangle} U(1)_{em}$$

SM Higgs doublet

$$\boxed{\phi \subseteq \Phi \longrightarrow U_L \, \Phi \, U_R^\dagger} \qquad \tilde{\phi} \equiv i\sigma_2 \phi^*$$

$$\Phi = \begin{pmatrix} \tilde{\phi}_1 & \phi_2 \end{pmatrix} = \begin{pmatrix} \varphi_1^{0*} & \varphi_2^{+} \\ -\varphi_1^{-} & \varphi_2^{0} \end{pmatrix}$$

$\boxed{\text{let's study } \Phi}$

Φ = 2x2 matrix

$\hookrightarrow U_L \, \Phi \, V_R^\dagger$

reminder: $H \to U H U^\dagger \quad (H = H^\dagger)$

$H \to$ diagonalize

$$\boxed{\begin{gathered} M \text{ (general } n\times n) \\ \Downarrow \\ U_L \, M \, V_R^\dagger = \text{diagonal} \end{gathered}}$$

$\Downarrow$

$$\Phi \longrightarrow \begin{pmatrix} v_1 e^{i\alpha_1} & 0 \\ 0 & v_2 e^{i\alpha_2} \end{pmatrix}$$

still!

$$U_L = \begin{pmatrix} e^{i\alpha_L} & 0 \\ 0 & e^{-i\alpha_L} \end{pmatrix}$$

$$U_R = \begin{pmatrix} e^{i\alpha_R} & 0 \\ 0 & e^{-i\alpha_R} \end{pmatrix}$$

$$\Downarrow$$

$$\Phi \longrightarrow \begin{pmatrix} v_1\, e^{i(\alpha_L - \alpha_R + \alpha_1)} & 0 \\ 0 & v_2\, e^{-i(\alpha_L - \alpha_R - \alpha_2)} \end{pmatrix}$$

$$\therefore \quad \alpha_L - \alpha_R = -\alpha_1$$

$\Downarrow$

$$\Phi \to \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix}$$

(basis)

$\Downarrow$

$\boxed{\text{3 invariants}}$

$\boxed{\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2}$

(i) $\mathrm{Tr}\, \Phi^\dagger \Phi$!

$\Phi \to U_L \Phi U_R^\dagger$

$\Phi^\dagger \to U_R \Phi^\dagger U_L^\dagger$

(ii) $\mathrm{Tr}\, \Phi^\dagger \Phi \Phi^\dagger \Phi$

$\tilde{\Phi} \to U_L \tilde{\Phi} U_R^\dagger$

(iii) $\mathrm{Tr}\, \tilde{\Phi}^{+} \Phi$, (iv) $\mathrm{Tr}\, \Phi^{+} \tilde{\Phi}$

(v) $\det \Phi$, (vi) $(\det \Phi)^{*}$

$\Big\downarrow$ 3 independant

$$\boxed{\text{(a)}\ \mathrm{Tr}\, \Phi^{+}\Phi,\quad \det \Phi,\quad \det \Phi^{*}}$$

4

$$\boxed{\text{(b)}\ \mathrm{Tr}\, \Phi^{+}\Phi,\quad \mathrm{Tr}\, \tilde{\Phi}^{+}\Phi,\quad \mathrm{Tr}\, \Phi^{+}\tilde{\Phi}}$$

$$\text{(a)} \Longleftrightarrow \text{(b)} \Longleftarrow \quad \mathrm{Tr}\, \tilde{\Phi}^{+}\Phi = 2 \det \Phi$$

$$V_\Phi = -\frac{\mu_\phi^2}{2}\, \mathrm{Tr}\, \Phi^\dagger \Phi - \frac{\bar{\mu}_\phi^2}{2}\, \mathrm{Tr}\, \tilde{\Phi}^\dagger \Phi \quad (+h.c.)$$

$$+ \frac{\lambda_\phi}{4} \left(\mathrm{Tr}\, \Phi^\dagger \Phi\right)^2 \leftarrow \text{trivial inv.}$$

$$+ \frac{\lambda_\phi'}{4}\, \mathrm{Tr}\, \Phi^\dagger \Phi \left(\mathrm{Tr}\, \tilde{\Phi}^\dagger \Phi + h.c.\right)$$

$$+ \frac{\lambda_\phi''}{4}\, \mathrm{Tr}\, \tilde{\Phi}^\dagger \Phi \; \mathrm{Tr}\, \Phi^\dagger \tilde{\Phi}$$

$$+ \frac{\lambda_\phi'''}{4}\, \mathrm{Tr}\, \Phi^\dagger \Phi\, \tilde{\Phi}^\dagger \Phi$$

$$\boxed{\tilde{\Phi}^\dagger \Phi = \frac{1}{2}\left(\mathrm{Tr}\, \tilde{\Phi}^\dagger \Phi\right) \mathbb{1}}$$

(verif) $\mathrm{Tr}\ \Phi^\dagger \Phi\, \tilde{\Phi}^\dagger \Phi =$

$$= \frac{1}{2}\ \underbrace{\mathrm{Tr}\, \Phi^\dagger \Phi}_{\text{inv.}}\ \underbrace{\mathrm{Tr}\, \tilde{\Phi}^\dagger \Phi}_{\text{inv.}}$$

<u>example</u> adjoint of $SU(n)$

$$\Sigma \to U \Sigma U^\dagger$$

$$\left(\Sigma^\dagger = \Sigma, \quad \mathrm{Tr}\, \Sigma = 0\right)$$

$$\Downarrow$$

$\Sigma \to$ diagonal

(a) $\underline{SU(2)} \Rightarrow \Sigma_{2\times 2} \to \mathrm{diag}\,(\varphi, -\varphi)$

$\updownarrow$

1 invariant $= \mathrm{Tr}\,\Sigma^2$

(b) $SU(3) \Rightarrow \Sigma_{3\times 3} \to \mathrm{diag}\,(\varphi_1, \varphi_2, -(\varphi_1+\varphi_2))$

2 invariants $= \mathrm{Tr}\,\Sigma^2;\ \mathrm{Tr}\,\Sigma^3$

$$\mathrm{Tr}\,\Sigma^4 = \varphi_1^4 + \varphi_2^4 + (\varphi_1+\varphi_2)^4$$

$$= \varphi_1^4 + \varphi_2^4 + (\varphi_1^4 + 4\varphi_1^3\varphi_2 + 6\varphi_1^2\varphi_2^2$$

$$+ 4\varphi_1\varphi_2^3 + \varphi_2^4)$$

$$= 2\varphi_1^4 + 2\varphi_2^4 + 4\varphi_1^3\varphi_2 + 6\varphi_1^2\varphi_2^2$$

$$+ 4\varphi_1\varphi_2^3$$

i. $\mathrm{Tr}\,\Sigma^2 = \varphi_1^2 + \varphi_2^2 + (\varphi_1+\varphi_2)^2$

$$= 2\varphi_1^2 + 2\varphi_2^2 + 2\varphi_1\varphi_2$$

$$\Rightarrow \boxed{2(\varphi_1^2 + \varphi_2^2 + \varphi_1\varphi_2) = \mathrm{Tr}\,\Sigma^2}$$

ii. $\mathrm{Tr}\,\Sigma^3 = \varphi_1^3 + \varphi_3^2 - (\varphi_1+\varphi_2)^3$

$$= \cancel{\varphi_1^3} \cancel{+\varphi_2^3} \cancel{-\varphi_1^3} \cancel{-\varphi_2^3} - 3\varphi_1^2\varphi_2 - 3\varphi_1\varphi_2^2$$

$$\Rightarrow \boxed{-3\varphi_1\varphi_2(\varphi_1+\varphi_2) = \mathrm{Tr}\,\Sigma^3}$$

$$\mathrm{Tr}\,\Sigma^4 = 2\varphi_1^4 + 2\varphi_2^4 + 2\varphi_1^3\varphi_2 + 2\varphi_1\varphi_2^3$$

$$+ 6\varphi_1^2\varphi_2^2 + 2\varphi_1^3\varphi_2 + \cdots$$

$$= 2(\varphi_1^3 + \varphi_2^3)(\varphi_1 + \varphi_2) + \cdots$$

$$\Downarrow$$

$$\boxed{\mathrm{Tr}\,\Sigma^4 = f(\mathrm{Tr}\,\Sigma^2) + f(\mathrm{Tr}\,\Sigma^3)}$$

$$\cancel{\mathrm{Tr}\,\Sigma^2 \cdot \mathrm{Tr}\,\Sigma^3}$$

$$\boxed{\mathrm{Tr}\,\Sigma^4 = \frac{1}{2}(\mathrm{Tr}\,\Sigma^2)^2}$$

$\mathrm{Tr}\,\Sigma^5 \propto \mathrm{Tr}\,\Sigma^3\,\mathrm{Tr}\,\Sigma^2$ Claim!

(c) $\underline{SU(4)}$

$$\Sigma \to \mathrm{diag}\left(\varphi_1, \varphi_2, \varphi_3, -(\varphi_1+\varphi_2+\varphi_3)\right)$$

$\Downarrow$ invariants (3)

$$\left(\mathrm{Tr}\,\Sigma^2, \ \mathrm{Tr}\,\Sigma^3, \mathrm{Tr}\,\Sigma^4\right)$$

$\Downarrow$

$$\mathrm{Tr}\,\Sigma^5 \propto \mathrm{Tr}\,\Sigma^2\,\mathrm{Tr}\,\Sigma^3$$

(d) $\boxed{SU(5)} \supseteq SU(3)\times SU(2)\times U(1)$

$$\Sigma \to \mathrm{diag}\left(\varphi_1 , \varphi_2 \ \varphi_3 \ \varphi_4 \ -(\textstyle\sum_{i=1..4} \varphi_i)\right)$$

4 inv. $\Downarrow$

$$\mathrm{Tr}\,\Sigma^2,\ \mathrm{Tr}\,\Sigma^3,\ \mathrm{Tr}\,\Sigma^4,\ \mathrm{Tr}\,\Sigma^5$$

$$\Downarrow$$

$$\mathrm{Tr}\,\Sigma^6 = a(\mathrm{Tr}\,\Sigma^2)^2 + b(\mathrm{Tr}\,\Sigma^3)^2 + c\,\mathrm{Tr}\,\Sigma^2\,\mathrm{Tr}\,\Sigma^4 + \cdots$$

(Δ, Φ) system

$$V = V_\Delta + V_\Phi + \underbrace{V_{\Delta,\Phi}}_{\text{new}}$$

$$\downarrow \text{SSB}$$

/ used $SU(2)_R$ $(+\lambda_2 > 0)$ ...

$$\langle \Delta \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

I cannot use $SU(2)_R$ anymore

$$\Phi \to U_L \, \Phi \, U_R^\dagger$$

no freedom anymore

$$\Downarrow$$

$\langle \Phi \rangle \neq$ diag (cannot be argued)

$$\Phi = \begin{pmatrix} \varphi_1^{0*} & \varphi_2^{+} \\ -\varphi_1^{-} & \varphi_2^{0} \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix}}$$

in order that $\Delta Q_{em} = 0$

$$V_{\Phi\Delta} = \alpha_1 \, \mathrm{Tr}\, \Phi^{+}\Phi \left(\mathrm{Tr}\, \Delta_L^{+}\Delta_L + L \leftrightarrow R\right)$$

$$+ \alpha_2 \left(\mathrm{Tr}\, \tilde{\Phi}^{+}\Phi \; (h.c.)\right) \left(- \;\;'' \;\; - \right)$$

$$+ \alpha_3 \left(\mathrm{Tr}\, \Delta_R^{+} \, \Phi^{+}\Phi \, \Delta_R + R \to L\right)$$

$$+\alpha_4 \, \mathrm{Tr}\, \Delta_R^+ \tilde{\Phi}^+ \Phi \, \Delta_R$$

$$\|$$

$$\frac{1}{2}\alpha_4 \, \mathrm{Tr}\, \Delta_R^+ \Delta_R \, \mathrm{Tr}\, \tilde{\Phi}^+ \Phi \quad (\alpha_2)$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix}$$

$$M_W^2 = \frac{g^2}{4}\left(v_1^2 + v_2^2\right)$$

$$M_Z^2 \cos^2\theta_W = M_W^2$$

$\cancel{\exists}$ bottom line

$\phi_1, \quad \phi_2$ $\qquad (\alpha = 0)$

$\tan\beta \equiv v_2/v_1$

$$h = \cos\beta \, \phi_1 + \sin\beta \, \phi_2$$

$$H = -\sin\beta \, \phi_1 + \cos\beta \, \phi_2$$

$$\langle h \rangle = \sqrt{v_1^2 + v_2^2}$$

$$\langle H \rangle = 0$$

$$\Rightarrow \boxed{\begin{array}{l} m_h^2 \propto M_W^2 \\ m_H^2 \propto M_A^2 \end{array}}$$

To be proven (stay tuned!)