LMU "Introduction to Physics of Neutrinos" Course 2023
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Homework 6

Problem 1

We have seen that the scalar sector of the Left-Right (LR) theory comprises

- a bidoublet Φ

$$\Phi = \begin{pmatrix} \tilde{\phi}_1 & \phi_2 \end{pmatrix} \,, \tag{1}$$

where ϕ_i, $i = 1, 2$, are two Standard Model Higgs doublets

$$\phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} \,, \quad \tilde{\phi}_i = i\sigma_2 \phi_i^* \,, \tag{2}$$

- two complex scalar fields Δ_L and Δ_R in the adjoint of $SU(2)_L$ and $SU(2)_R$, respectively.

1. Take the discrete LR symmetry to be parity, under which

$$\Phi \leftrightarrow \Phi^\dagger \,, \quad \Delta_{L,R} \leftrightarrow \Delta_{R,L} \,,$$

and write down the most general renormalizable potential involving all the fields.

Hint: You can have a look at eq. (9) of the paper "Higgs Sector of the Left-Right Symmetric Theory," by Maiezza, Senjanovic and Vasquez; it is posted on the Lecture Material tab of the website.

2. Show that when the bidoublet acquires its vacuum expectation value (vev), a small expectation value for Δ_L is also generated. Estimate its order of magnitude.

Hint: Recall that we showed that Δ_L gets a large mass proportional to v_R (see Table 1 in Maiezza et al.), so it cannot get a vev from the usual Higgs mechanism. Instead, it gets a vev from what we call a tadpole, a term linear in the field. Such a vev is necessarily small, which should be taken into account when estimating it.

Problem 2

In Homework 3 Problem 2, we discussed in details the weak decay(s) of a Majorana neutrino N. The purpose of the present exercise is to investigate what happens in the LR theory, with N having a gauge interaction with W_R.

- Compute the three body decay $N \to e_R + u_R + (d^c)_L$ from the W_R exchange. Assume the electron and the quarks to be massless.

  You can use the muon decay result to get this one. If you are brave and have some time to spare, compute this decay (or muon decay, the same) from scratch. This is a hard calculation for massive final states, but doable for the massless ones.

- Compare the above three body decay rate with the rate for $N \to W_L^+ + e_L$. Take $m_N = TeV$, $M_{W_R} = 100 M_{W_L}$ and $m_\nu = 0.1\, eV$.

- Is there a decay $N \to W_L^+ + e_R$?

  Hint: W_L and W_R can mix. Up to now we assumed that they are mass eigenstates, but in general that is not true. Try to find the source of the mixing (must come from the Higgs sector) and derive it.

  Compute then the decay $N \to W_L^+ + e_R$. Compare it with the three body decay computed above for $m_N = TeV$ and $m_\nu = 0.1\, eV$. Assume that the $W_L - W_R$ mixing is $\xi_{LR} \approx 10^{-4}$.