Neutrino Physics Course

Lecture XXV

21/7/2023

LMU

Summer 2023

Domain Walls

Okun, Kobzarev, Zeldovich 1974

$\updownarrow$

SSB of a discrete symmetry

examples of discrete sym.:

- Parity (P) = LR symmetry
- Charge conjugation (C) = -"-
- CP = C×P = L→L sym.

↑ T.D. Lee '72

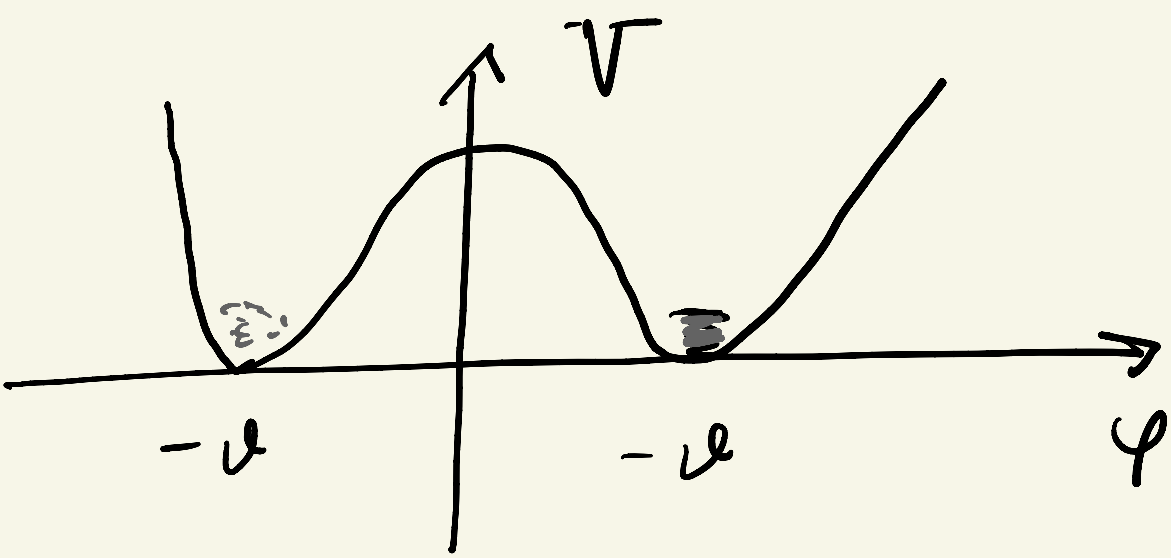
(~~CP~~ spont.)

generic ↓ discrete (D) sym.

$\varphi \in \mathbb{R}$

D: $\varphi \to -\varphi$ $(\mathbb{Z}_2)$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{\lambda}{2}\left(\varphi^2 - v^2\right)^2$$



$$\Rightarrow \mathcal{M}_0 = \{ \varphi_0 \therefore V(\varphi_0) = \min \}$$

$$= \{ \varphi_0 \therefore V = 0 \} =$$

$$= \{ \varphi_0^2 = v^2 \} = \{ \pm v \}$$

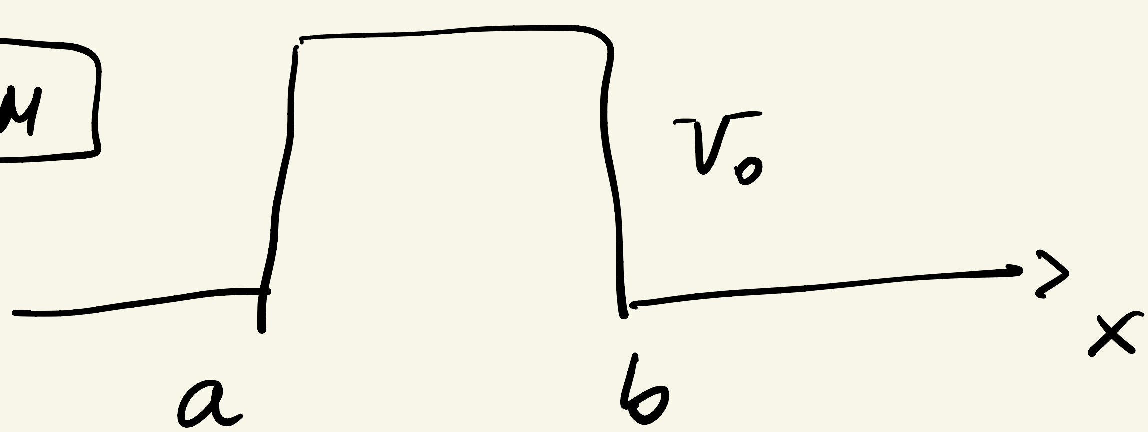
$$= \mathbb{Z}_2$$

but, in QM

$$\Downarrow$$

tunnel from $-v$ to v

$\boxed{QM}$



$$\Gamma_{WKB} \propto e^{-\int_a^b \sqrt{2V_0}\, dx} \neq 0$$

$$\Downarrow$$

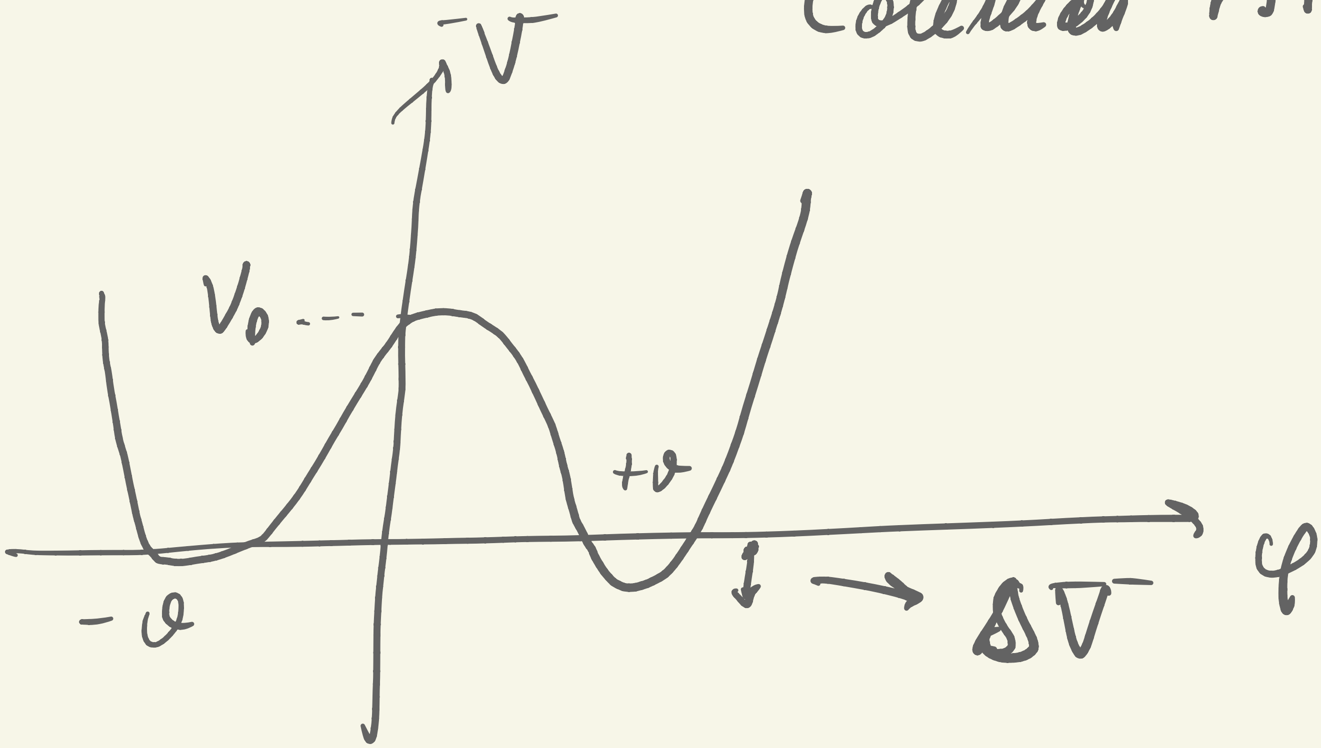
vacuum = ground state

= symm. superposition of two vacua

$\longrightarrow$ but, in QFT

NO tunneling between degenerate vacua

Coleman 1974-76



$\therefore \; \Delta V \ll V_0$

$\Downarrow$

$$\Gamma \propto e^{-(V_0/\Delta V)^?} \longrightarrow 0$$

$\Delta V \to 0$

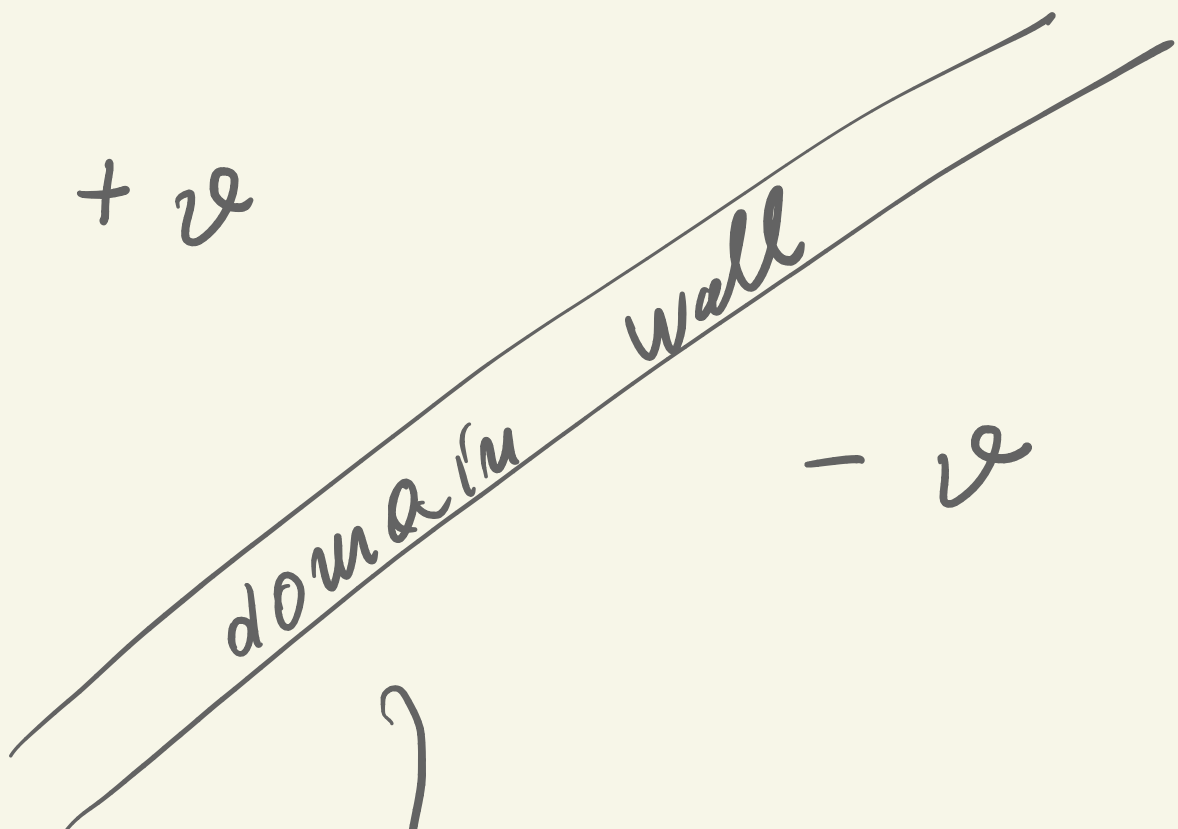
$\Delta V = 0 \longleftrightarrow$ our SSB of Z_2

$\Downarrow$

No tunneling

$$\Downarrow$$

$$\boxed{\text{SSB}}$$



$$\downarrow$$

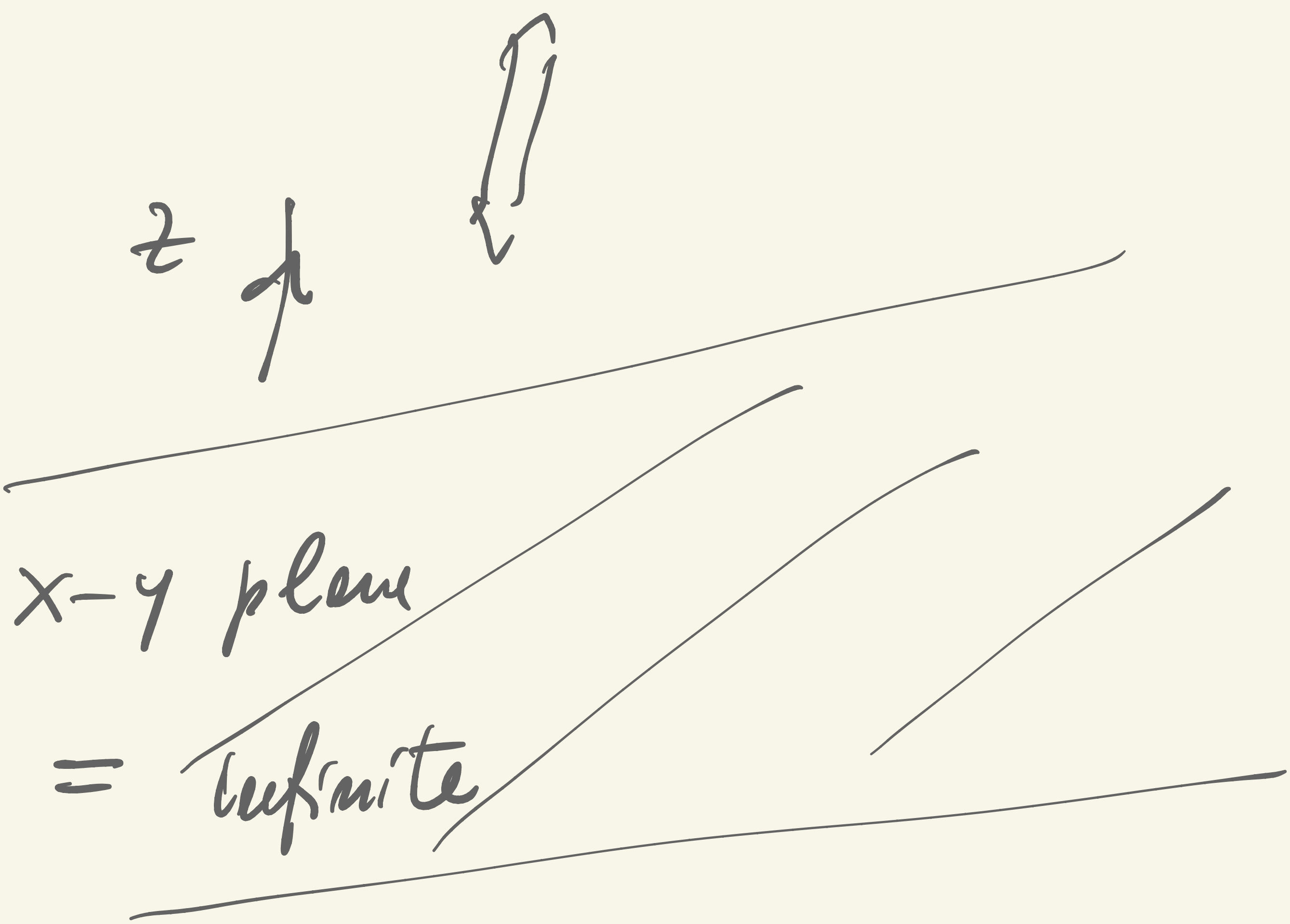
classical solution

static, finite energy

· $\underline{D = \mathbb{Z}_2}$ $\Longleftrightarrow$ $\mathcal{M}_0 = \{+v, -v\}$

coordinate z = free

$$\varphi_{cl}(z) \equiv \varphi_{dw}(z)$$



$$\Downarrow$$

$$E/S = \text{finite}$$

$$\| $$

$$(1) \quad E/S = \overbrace{\int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\left(\frac{d\varphi}{dz}\right)^2 + V \right]}$$

$$\underset{0}{\downarrow} \text{ at } \infty \qquad \underset{0}{\downarrow}$$

(infinite = very large

= size >> thickness of wall)

$$(2) \quad \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} = \frac{\partial \mathcal{L}}{\partial \phi}$$

$$\Downarrow$$

$$\Box \varphi = -\frac{\partial V}{\partial \varphi}.$$

$$\Downarrow$$

$$-\frac{d^2 \varphi_{cl}}{dz^2} = -\frac{\partial V}{\partial \varphi_{cl}} \Big/ \frac{d\varphi}{dz}$$

$$\Rightarrow \quad \frac{d}{dz}\left(\frac{1}{2}\left(\frac{d\varphi_{cl}}{dz}\right)^2\right) = \frac{dV}{dz}$$

$$\Downarrow$$

$$\frac{1}{2}\left(\frac{d\varphi_{cl}}{dz}\right)^2 = V(\varphi_{cl}) + \text{const.}$$

at ∞: $\quad 0 \qquad 0 \qquad 0$

$$\Downarrow$$

$$\boxed{\frac{d\varphi_{cl}}{dz} = \pm\sqrt{2V}}$$

$$\frac{d\varphi_{cl}}{dz} = \pm\sqrt{\lambda}\,(\varphi_{cl}^2 - v^2)$$

$$\boxed{\frac{d\varphi_{cl}}{\varphi_{cl}^2 - v^2} = \pm\sqrt{\lambda}\,dz}$$

$$\Updownarrow$$

$$\boxed{\varphi_{cl}(z) = \pm\, v \tanh \sqrt{\lambda}\, v z}$$

$\textcircled{+}$ = domain wall

$-$ = anti $-$ " $-$

$$\tanh x = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

$\longrightarrow 1, \quad x \to \infty$

$-1, \quad x \to -\infty$

$0 \to 0, \quad x \to 0$

$\varphi_{cl}(z) \longrightarrow v, \quad z \to +\infty$

$\varphi_{cl}(z) \to -v, \quad z \to -\infty$

$\mathcal{M}_\infty = \{+\infty, -\infty\} = \mathbb{Z}_2$

$\varphi_{cl}(z)$ = map from $\mathcal{M}_\infty$ to $\mathcal{M}_0$

(i) <u>trivial map</u>

φ_0 = vacuum state

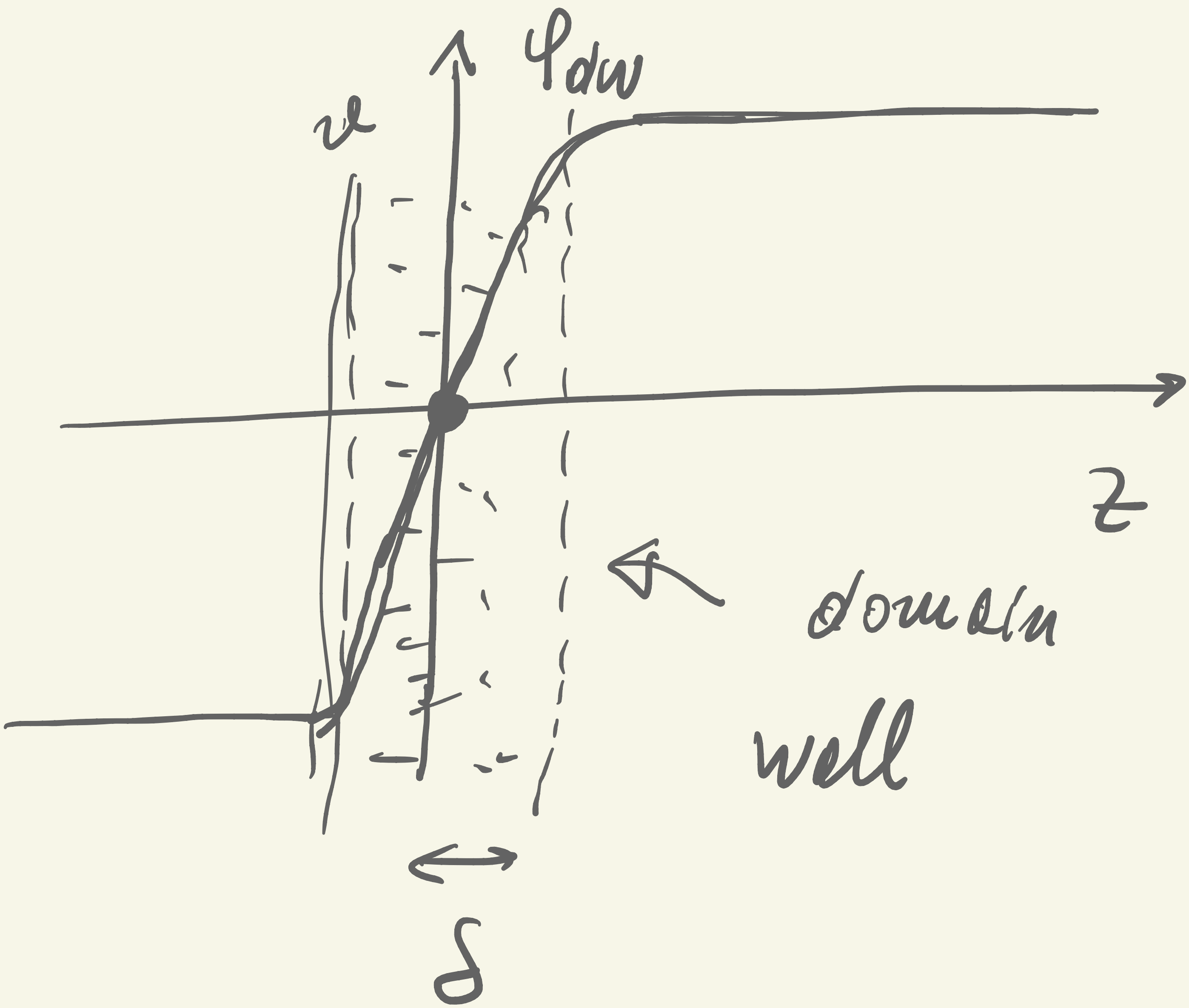
$= +v \ (-v)$

(ii) <u>non-trivial map</u>

$$\boxed{\varphi_{dw} = v \tanh \sqrt{\lambda}\, z v}$$

$+\infty \longrightarrow +v$

$-\infty \longrightarrow -v$



$$\boxed{\delta = \frac{1}{\sqrt{\lambda}\, v} \simeq 1/v}$$

$$\Uparrow$$

$$\boxed{\varphi_{dw}(z) = v \tanh \delta z}$$

<u>LRSM</u> $\quad v_{LR} > \text{TeV}$

$$\text{GeV}^{-1} \simeq 10^{-14}\,\text{cm}$$

$$\boxed{\delta \simeq \frac{1}{v_{LR}} \leq 10^{-3}\,\text{GeV}^{-1} \leq 10^{-17}\,\text{cm}}$$

$$E/_S = \int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\left(\frac{d\varphi_{dw}}{dz}\right)^2 + V\right]$$

$$= \int_{-\infty}^{+\infty} dz\, 2V = \int_{-\infty}^{+\infty} \frac{dz}{d\varphi}\, d\varphi\, 2V$$

$$= \int_{-v}^{+v} d\varphi \sqrt{2V} \simeq v^3 \neq 0$$

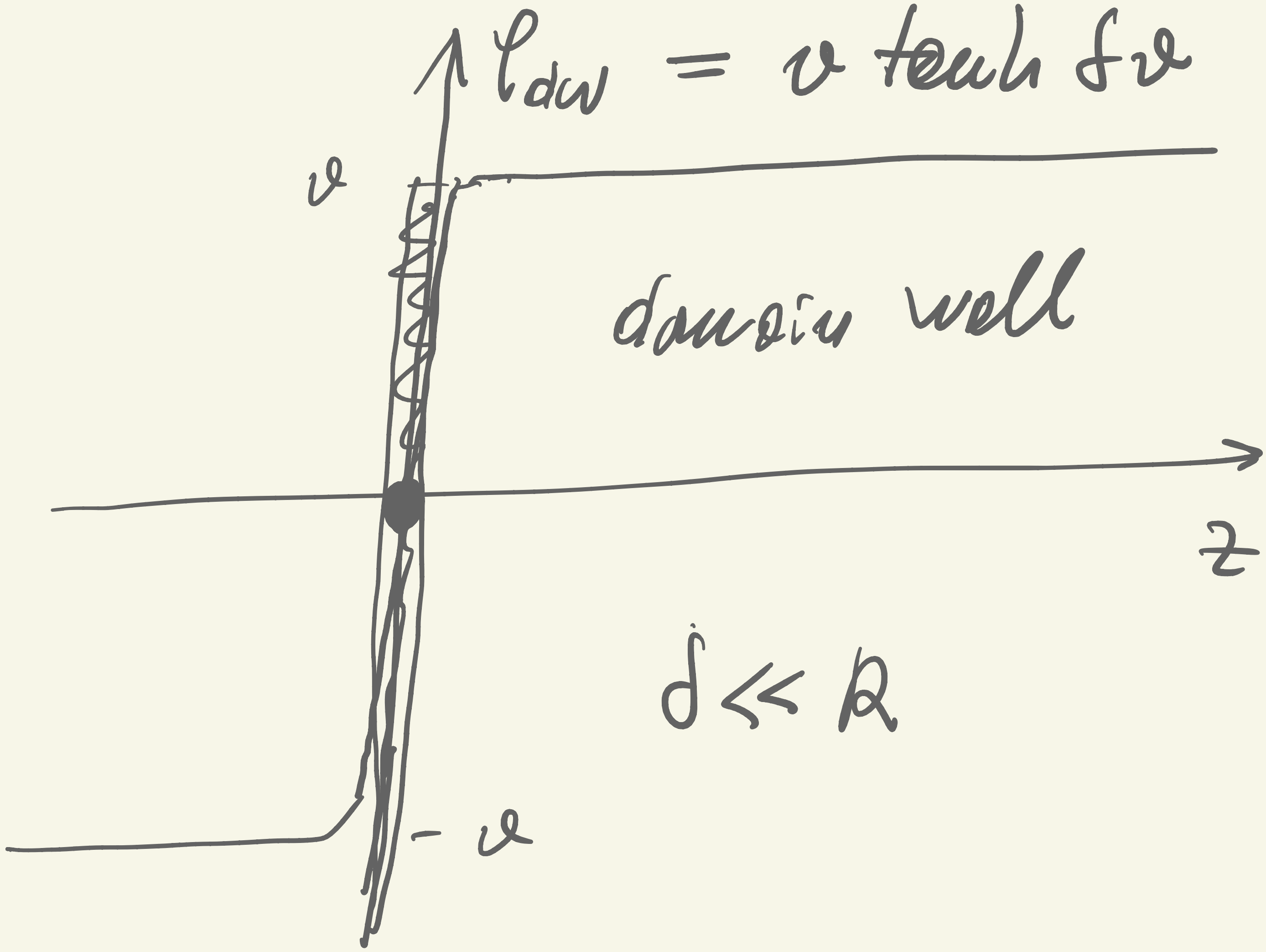
$$v_{LR} = \langle \Delta_R \rangle = M_R$$

$$M_R \simeq M_{W_R},\ M_{Z_R}$$

$$M_{W_R} = g \langle \Delta_R \rangle$$

$$\langle \Delta_R \rangle = M_{W_R}/g \gtrsim \frac{4\,\text{TeV}}{0.5}$$

$$\boxed{\langle A_n \rangle \gtrsim 10\ \mathrm{TeV}}$$



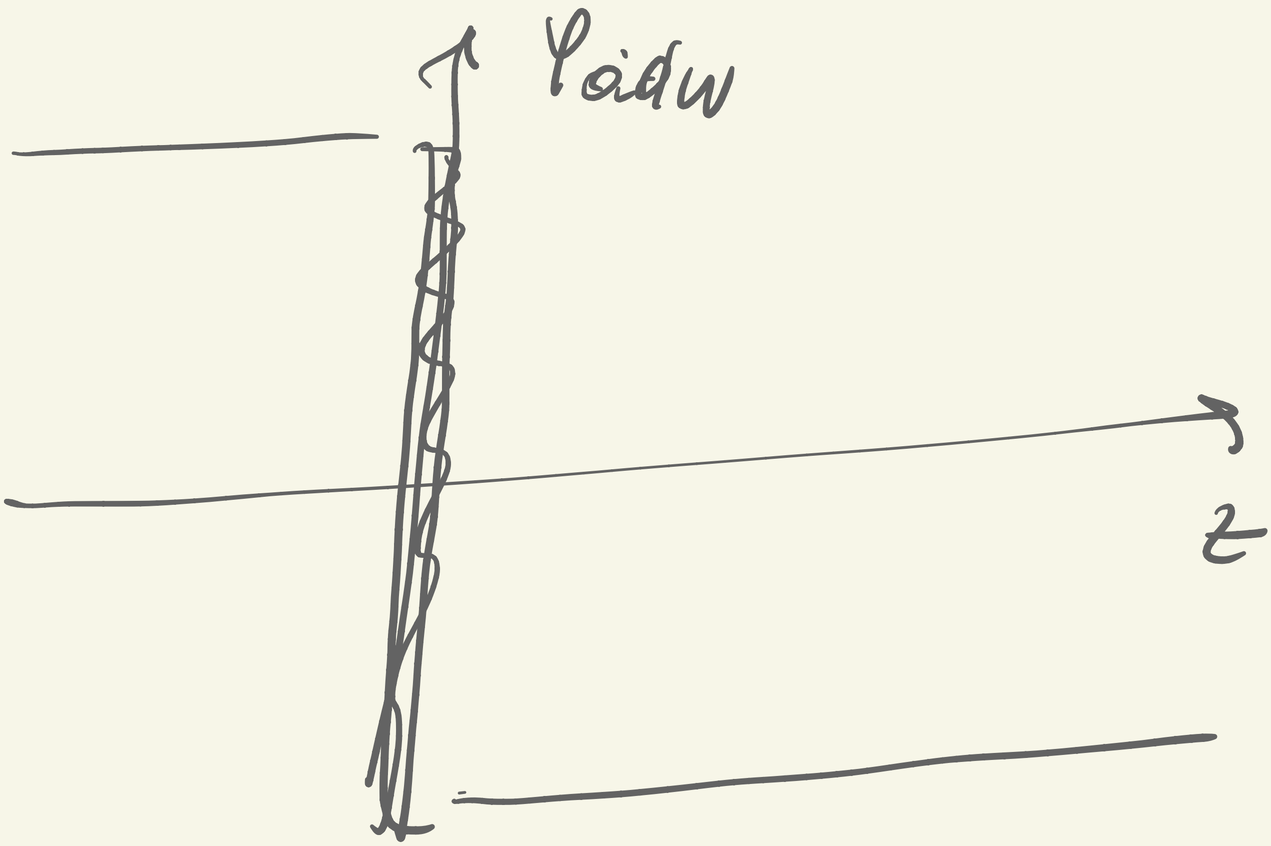
$$E/_S = \text{infinite} \iff (S = R \times R)$$

$$R \gg \delta$$

$$\boxed{\text{by def.}: \quad R \gg \text{cm} \quad \text{macroscopic objects}}$$

anti-domain wall

$$\varphi_{adw} = -v \tanh \delta v$$



$$E_{dw}/_S \simeq v^3$$

$$E_{vac}/_S = 0$$

$dw \to \varphi_0$?

possible?

NO!

$(z;t)$

$$j^\mu = \varepsilon^{\mu\nu}\partial_\nu \varphi$$

$$(\varepsilon^{\mu\nu} = -\varepsilon^{\nu\mu})$$

$$\Rightarrow \partial_\mu j^\mu = \varepsilon^{\mu\nu}\partial_\mu\partial_\nu\varphi = 0$$

$$j^0 = \partial_z \varphi = \frac{\partial \varphi}{\partial z}$$

$$\Rightarrow \quad Q_{dw} = \int_{-\infty}^{+\infty} dz \, j^0 (dw) =$$

$$= \int_{-\infty}^{+\infty} dz \, \frac{d\varphi_{cl}(z)}{dz}$$

$$= \varphi_{cl}(+\infty) - \varphi_{cl}(-\infty)$$

$$= 2v$$

in short:

• $Q\,(\text{vacuum}) = \varphi_0(\infty) - \varphi_0(-\infty)$

$= 0$

• $Q\,(dw) = 2v$

• $Q\,(\bar{a}dw) = -2v$

$\Downarrow$

$\boxed{\text{single dw (}\bar{a}\text{dw) = stable}}$

$\downarrow$ large domain
wall in universe

(large = horizon)

$$E_{dw} = {}^{E}/_{S} \cdot S_u$$

$$\simeq {}^{E}/_{S} \cdot R_U^2 \, , \quad \boxed{R_U \simeq 10^{28}\,\text{cm}}$$

$$\Downarrow$$

$$\boxed{E_{dw} \simeq v^3 R_U^2} \qquad \boxed{u = \text{universe}}$$

$$\Downarrow$$

$$\boxed{\varepsilon_{dw} = \frac{E_{dw}}{V_u} \simeq \frac{v^3}{R_U}}$$ energy density

(stars)

$\varepsilon_u \simeq 10\, \varepsilon_{\text{baryons}}$ | energy density of universe

$$\varepsilon_u \simeq 10\, n_B\, GeV$$

$$n_B \simeq 10^{-9} n_\gamma \simeq 10^{-9}\, T_0^3$$

$$T_0 \simeq 10^{-4}\, eV \simeq 10^{-13}\, GeV$$

$$\varepsilon_u \simeq 10^{-8}\, 10^{-39}\, GeV^3\, GeV$$

$$\boxed{\varepsilon_u \simeq 10^{-47}\, GeV^4}$$

$$\cdot\; v = v_{LR} = \langle \Delta_R \rangle \gtrsim 10^4\, GeV$$

$$\Downarrow$$

$$\varepsilon_{dw} \gtrsim 10^{12}\,\mathrm{GeV}^3 \Big/ 10^{28}\,\mathrm{cm}$$

$$\left(\begin{array}{l} \mathrm{GeV}^{-1} = 10^{-14}\,\mathrm{cm} \\ \Rightarrow \dfrac{1}{\mathrm{cm}} = 10^{-14}\,\mathrm{GeV} \end{array} \right)$$

$$\Downarrow$$

$$\varepsilon_{dw} \gtrsim 10^{+12} \cdot 10^{-28} \cdot 10^{-14}\,\mathrm{GeV}^4$$

$$\boxed{\varepsilon_{dw}(LR) \gtrsim 10^{-30}\,\mathrm{GeV}^4}$$

$\boxed{\text{Domain Wall Problem}}$

Solutions

(i) dw did not get formed

$\Leftarrow$ symmetry non-restoration at high T

Weinberg, '74

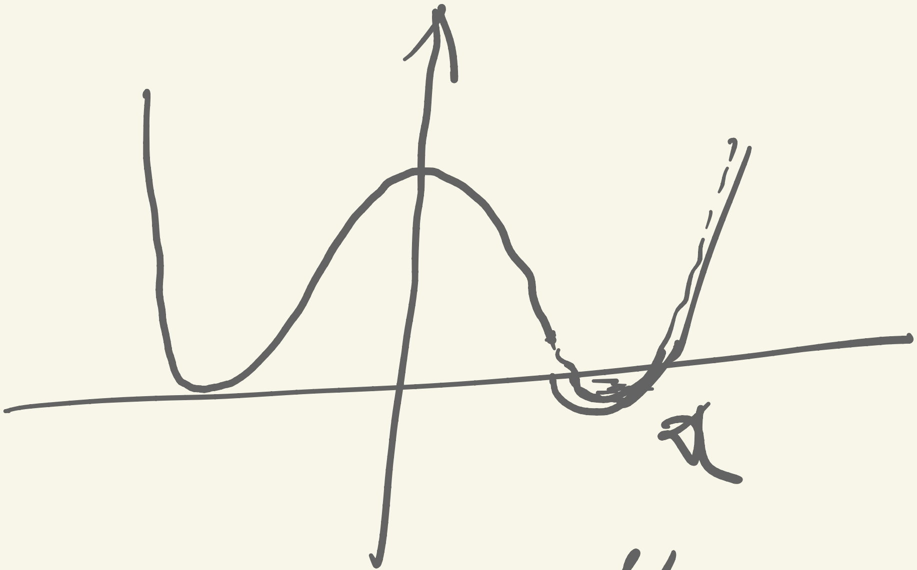
Mohapatra, G.S. '79

$\boxed{\text{Dvali, G.S '94}}$

Dvali, Melfo, G.S '95

(ii) inflation

(iii) small bias



Vilenkin '80s

1/Λpe

Rai; G.S.

G.S. Beyond. pdf

to be posted

gravity from a dw

$$\nabla^2 V_{gr} \propto G_N M \quad (\text{Newton})$$

$$\nabla^2 V_{gr} \propto f(T_{\mu\nu})$$

$$T_{\mu\nu}(dw) = \begin{cases} T_{00} = \rho_{dw} > 0 \\ T_x{}^x = T_y{}^y = -\rho_{dw} \\ T_z{}^z = 0 \end{cases}$$

$$\Rightarrow \nabla^2 V_{gr} \propto G_N \left(T_{00} - T_x{}^x - T_y{}^y\right)$$

$$\propto (-)\, G_N \rho_{dw}$$

Dark energy

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

$$\boxed{+ \Lambda g_{\mu\nu} (?)}$$

cosmological constant

$$\Lambda g_{\mu\nu} \leftarrow T_{\mu\nu}$$

$$\boxed{T_{\mu\nu} (\text{vacuum}) \propto g_{\mu\nu}}$$

vacuum energy $\Leftrightarrow$ cosmological constant Λ

$\Updownarrow$

acceleration of far galaxies