# Neutrino Physics Course

## Lecture XIV

13/6/2023

LMU

Summer 2023

# Neutrino mass and Lepton Number Violation (LNV)

$\nu_L$, $N_L = C\bar{\nu}_R^T \propto \nu_R^*$

$$\Downarrow \qquad \begin{array}{c|cc} & \nu^0 & N^0 \\ \hline \nu^0 & 0 & \underline{M}_D^T \\ N^0 & \underline{M}_D & \underline{M}_N \end{array}$$

$$\underline{M}_{\nu N} = \begin{array}{c} \nu^0 \\ N^0 \end{array} \begin{pmatrix} 0 & \underline{M}_D^T \\ \underline{M}_D & \underline{M}_N \end{pmatrix}$$

$\nu^0$, $N^0$ = weak eigenstates

$\neq$ physical states

$\nu, N$ = mass eigenstates

= physical states

$$\begin{pmatrix} \nu \\ N \end{pmatrix} = U \begin{pmatrix} \nu \\ N \end{pmatrix}^0$$

$$UU^+ = U^+U = 1$$

$$\Updownarrow$$

$$\mathcal{L}_{kin} = i\bar{f}\gamma^\mu \partial_\mu f$$

$$\rightarrow i\bar{f}U^+U\gamma^\mu\partial_\mu f = \text{inv.}$$

$$\Leftrightarrow \quad \begin{pmatrix} \nu \\ N \end{pmatrix} \longrightarrow U \begin{pmatrix} \nu \\ N \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & \theta^{+} \\ -\theta & 1 \end{pmatrix} \quad (\text{up to } \theta^2)$$

$$\theta = \frac{1}{M_N} M_D$$

$$U^{T} M_{\nu N} U \simeq \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{M_\nu = - M_D^{T} \frac{1}{M_N} M_D}$$

seesaw

$$M_N^T = M_N \Rightarrow M_\nu^T = M_\nu$$

<u>natural</u> $\quad M_D \sim M_\ell$

$$\Downarrow$$

$$M_\nu \ll M_\ell$$

$$(M_N \simeq M_R \gg M_W)$$

$\Downarrow$ central (essence)

$$\boxed{\Delta L = 2}$$

· <u>1 gen.</u>

$$\underbrace{\nu_L^T C \nu_L \, m_\nu^M}_{\Delta L = 2}$$

$$\Leftrightarrow \quad \nu_M = \nu_L + C\bar{\nu}_L^T$$

$$= \nu_L + (\alpha \nu_L^*)$$

$$\downarrow \qquad\qquad \downarrow$$

$p$ (50%) $\quad \bar{p}$ (50%)

$\Delta L = 2$

processes

(i) $\Delta L = 2$ in $N$ decays
(direct)

(ii) $\Delta L = 2$ low energy =

$= 0\nu 2\beta$ (indirect)
(neutrinoless double beta)

1935
Goepert-Mayer

$$^{76}Ge \not\to {}^{76}Ar + e + \bar{\nu}_e$$

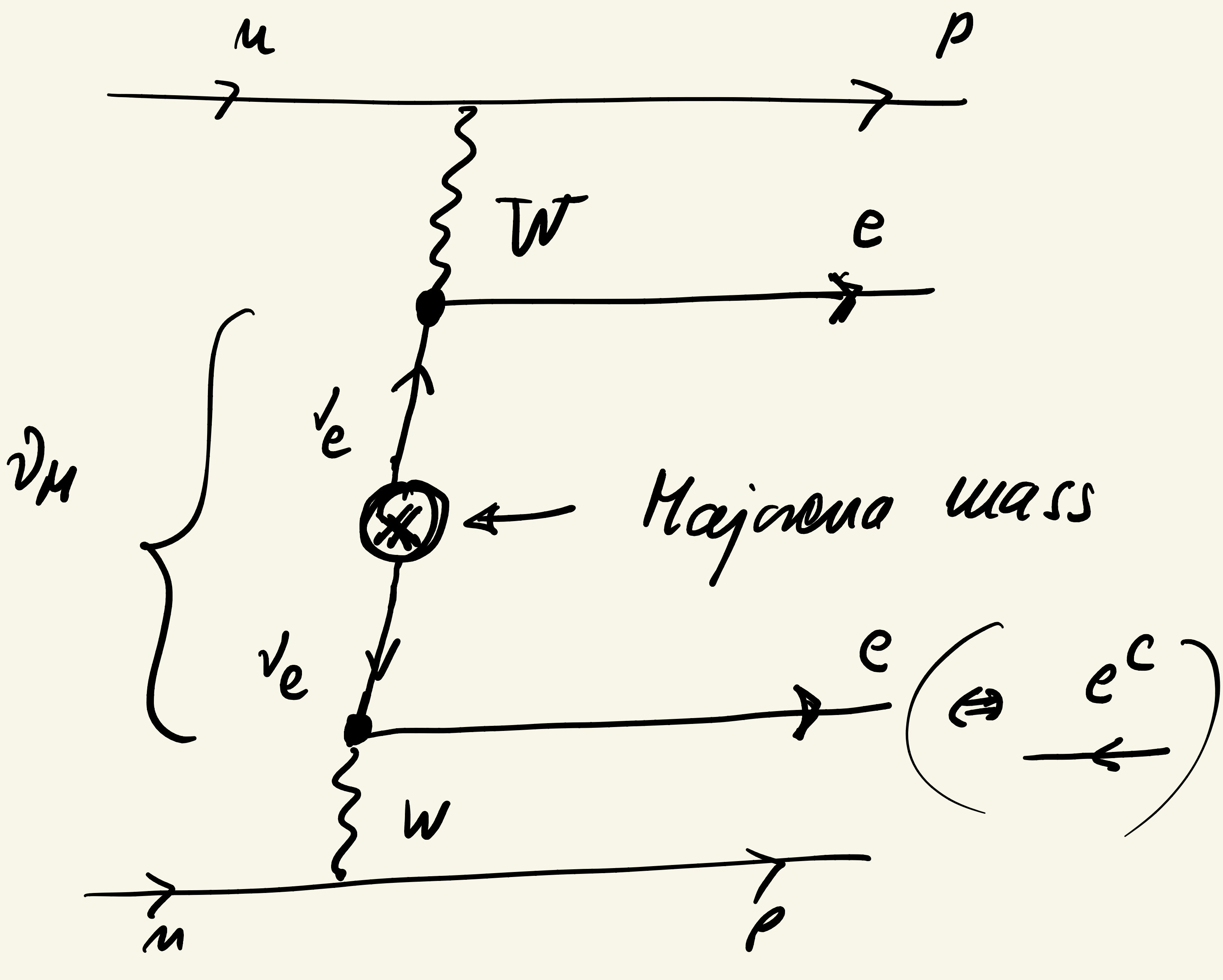
$$m_{Ar} \gtrsim m_{Ge}$$

$\Downarrow$

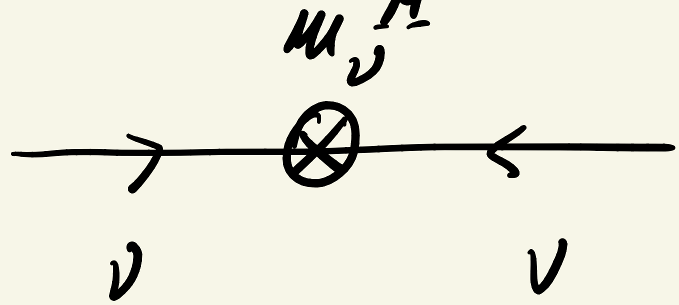
$$Ge^{76} \longrightarrow Se^{6} + p + \bar{p} + e + e + \bar{\nu}_e + \bar{\nu}_e$$

(double beta)

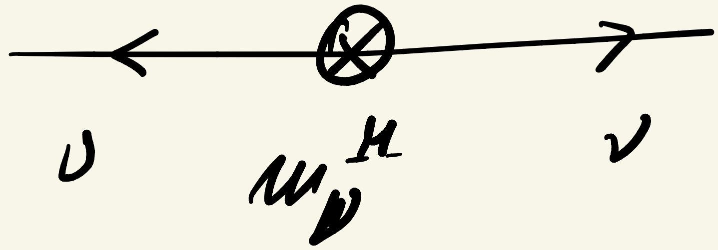
$$\tau_{2\beta} \simeq 10^{21}\ yr$$



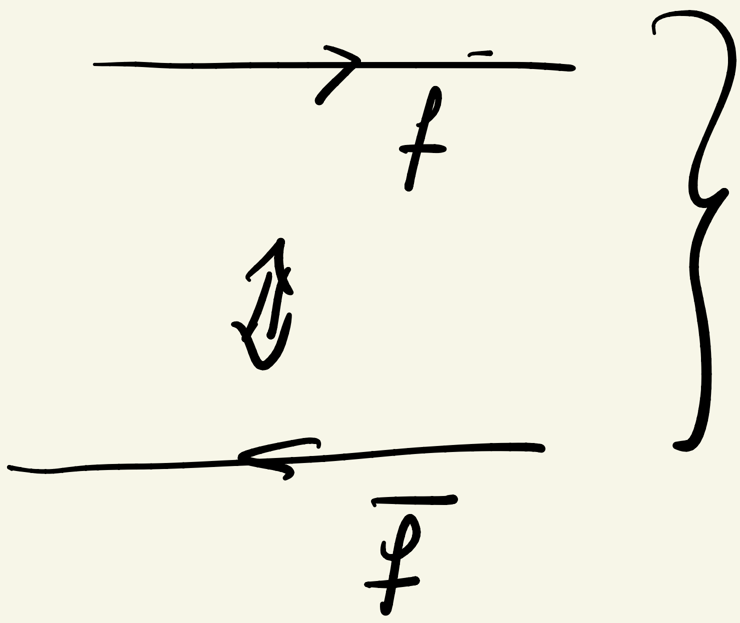
$m_\nu^M \; \nu_L^T C \; \nu_L$




$m_\nu^M \; \nu_L^+ C^+ \; \nu_L^*$




Fermionen :

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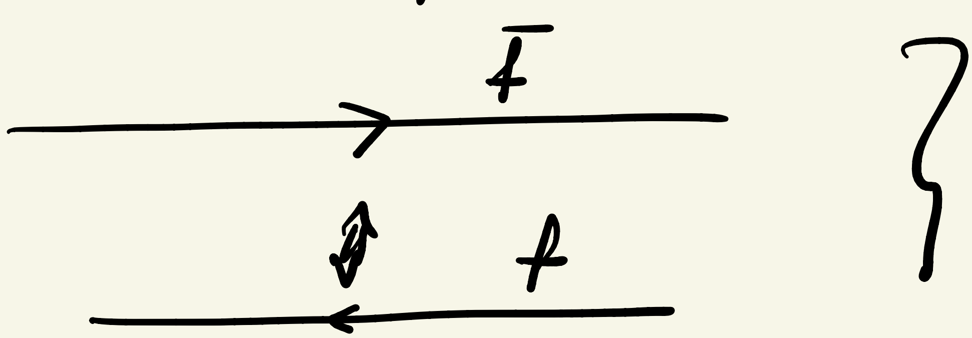


$\cdot\ f_L \Rightarrow (f^c)_R = C\,\overline{f_L}^T$

To $\overline{(\nu^c)_R}\,\gamma^\mu (e^c)_R \propto \overline{e_L}\,\gamma^\mu \nu_L$

$(C \equiv i\,\gamma_2\,\gamma_0)$

Proof:

$$\overline{(\nu^c)_R}\,\gamma^\mu (e^c)_R = \overline{C\,\overline{\nu_L}^T}\,\gamma^\mu\, C\,\overline{e_L}^T$$

$$= \left(C\,(\nu_L^+\gamma^0)^T\right)^+ \gamma^0 \gamma^\mu\, C\,\overline{e_L}^T$$

$$= \left(C\,\gamma_0\,\nu_L^*\right)^+ \gamma^0 \gamma^\mu\, C\,\overline{e_L}^T$$

$$= \nu_L^T\,\gamma_0\,C^+\,\gamma^0\,\gamma^\nu\, C\,\overline{e_L}^T$$

$$= \nu_L^T\,(-C^+)\,\gamma_0^2\,\gamma^\nu\, C\,\overline{e_L}^T$$

$$= \nu_L^T \underbrace{(-C^T \gamma^\mu C)} \bar{e}_L^T$$

$$= \nu_L^T \; \gamma_\mu^T e_L^T = -\bar{e}_L \gamma_\mu \nu_L$$

$$\boxed{\text{Q. E. D.}}$$

- recall:

$$f_M = f_L + C \bar{f}_L^T$$

$$\Rightarrow \mathcal{L}_M = i \bar{f}_M \gamma^\mu \partial_\mu f - m_M \bar{f}_M f_M$$

$$\Downarrow$$

$$S^M(f_M) = \frac{1}{\not{p} - m_M}$$

$\Downarrow$ electron part

$$A^{0\nu 2\beta}_{\nu_M} \propto \overline{e_L}\, \gamma^\mu \frac{1}{\not{p} - m_\nu^M} \gamma^\nu (e^c)_R$$

$$= \bar{e}\, \gamma_\mu L \frac{\not{p} + m_\nu^M}{p^2 - (m_\nu^M)^2} \gamma_\nu R\, e^c$$

$$= \bar{e}\, \gamma_\mu \frac{\not{p} R + m_\nu^M L}{p^2 - (m_\nu^M)^2} \gamma_\nu R\, e^c$$

$$= \bar{e}\, \gamma^\mu \frac{\cancel{\not{p} \gamma_\nu L} + m_\nu^M \gamma_\nu R}{p^2 - (m_\nu^M)^2} R\, e^c$$

$$L \cdot R = 0 \qquad R \cdot R = R$$

$$= \bar{e}\,\gamma^{\mu}\,\frac{m_{\nu}{}^{M}}{p^2}\,\gamma^{\nu} R\, e^{c}$$

$\Updownarrow$

diagram for $0\nu 2\beta$

$$\boxed{\begin{cases} p \simeq 100\ \mathrm{MeV} \\ m_{\nu} \le \mathrm{eV} \end{cases}}$$

$\Downarrow$

$$A^{0\nu 2\beta}_{\nu_M}\ \bar{p}\,\bar{p}\,\bar{e}\,\bar{e}\,u\,u \cdots$$

$$A^{0\nu 2\beta}_{\nu_M} \simeq G_F^2 \frac{m_\nu^M}{p^2}$$

$$\simeq 10^{-10} (\mathrm{GeV})^{-4} \frac{10^{-10}\,\mathrm{GeV}}{(10^{-1}\,\mathrm{GeV})^2}$$

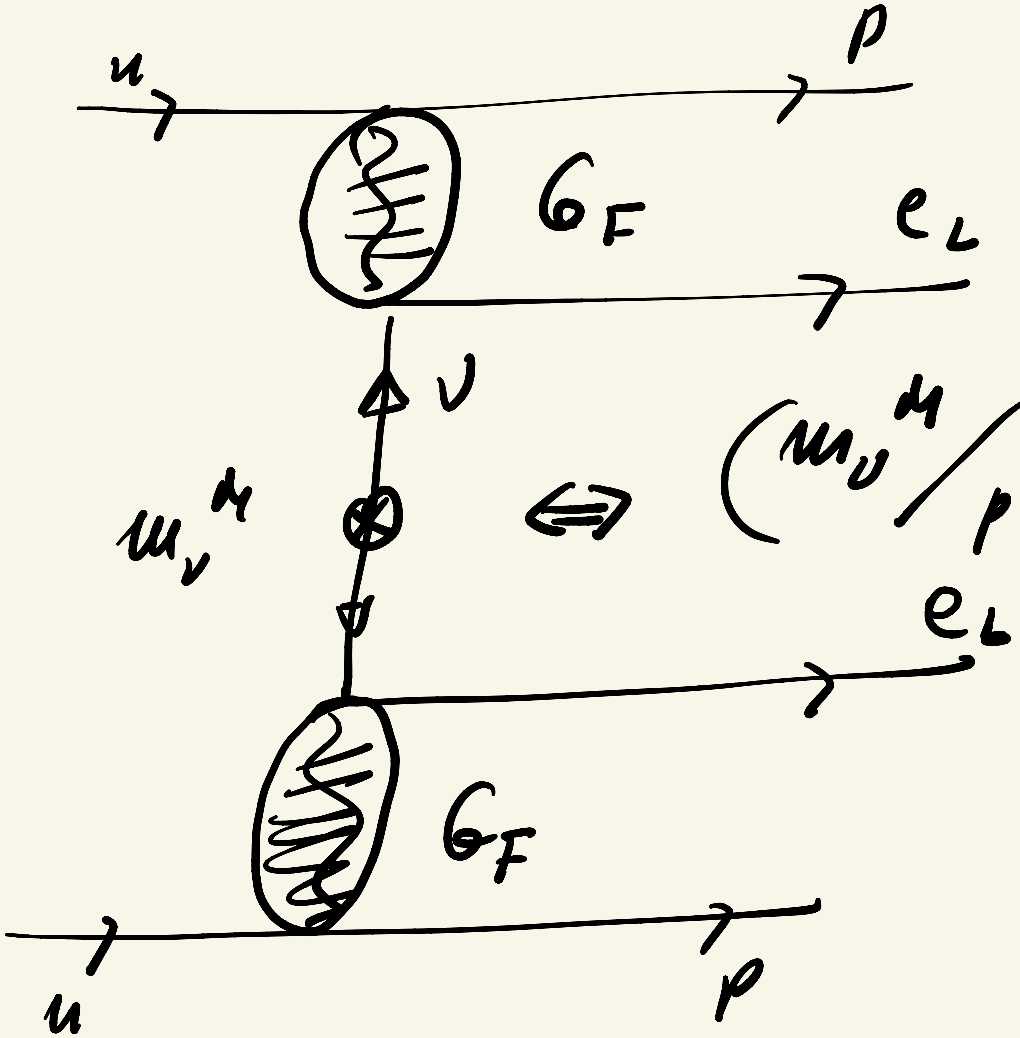
$$\simeq 10^{-18}\,\mathrm{GeV}^{-5}$$

$$\tau_{0\nu 2\beta}\ (\mathrm{exp}) \gtrsim 10^{26}\ \mathrm{yr}$$

GERDA 2022

$$\Downarrow$$

$$\boxed{m_\nu^M \leq 0.3\,\mathrm{eV}} \quad (\mathrm{exp})$$



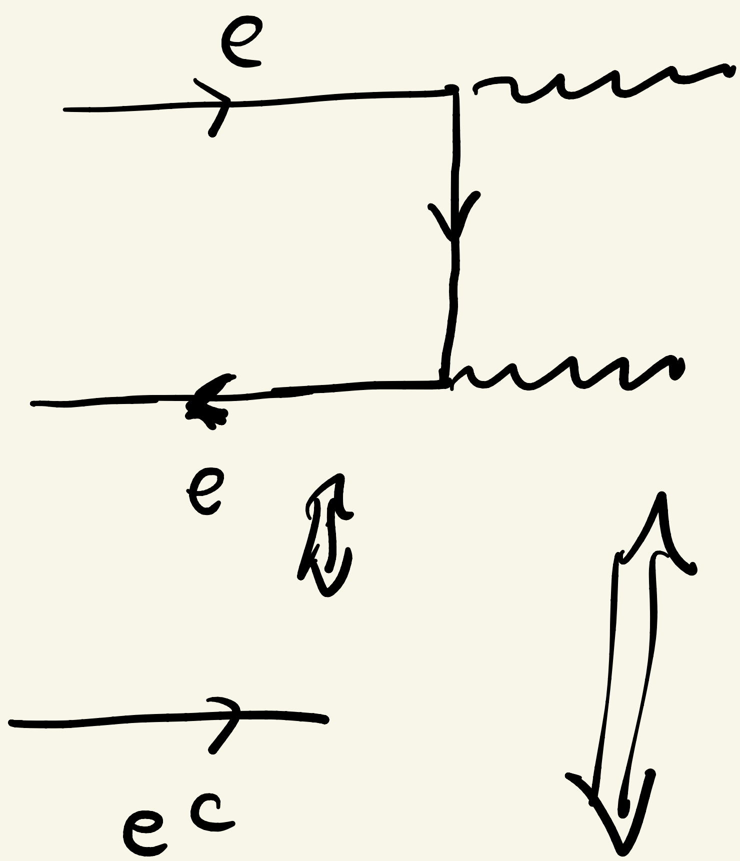

$$\Longleftrightarrow \left( m_\nu^M / p^2 \right) \quad \boxed{\Delta L = 2}$$


$$\left( \nu_L^T C \nu_L \right)^\dagger m_\nu^M \qquad\qquad \bar{\nu}_M \nu_M m_\nu^M$$

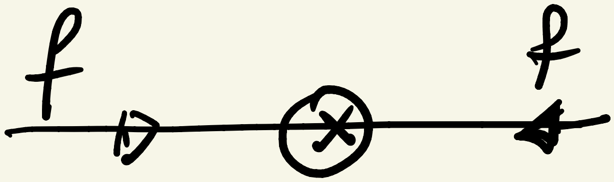
$$\nu_M = \nu_L + C \bar{\nu}_L^T \quad \text{but}$$

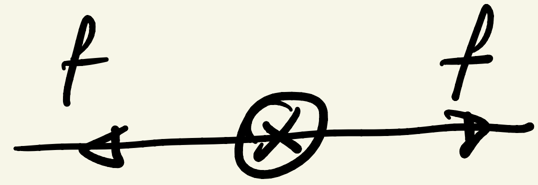
$$\boxed{\nu_\mu = \nu_\mu^c}$$

analogy with QED




$$e + e^c \longleftrightarrow 2\gamma$$




$m_M$ $m_M$

$$m_M \left( f^T C f + f^\dagger C^\dagger f^* \right)$$

$$\boxed{\begin{aligned} f_M &= f + f^c \\ &= f + C \bar{f}^T \end{aligned}}$$

$$\Updownarrow$$

$$\boxed{(f_M)^c = f_M}$$

Majorana anti-particle

= Majorana particle

Neutrino Majorana mass

$\Longleftrightarrow$ new physics

$\Downarrow$

new physics contribution

to $0\nu 2\beta$

$\Downarrow$

$$A^{0\nu 2\beta} \quad \underbrace{\bar{p}\bar{p}\bar{e}\bar{e}uu}_{d=9}$$

$$A^{0\nu 2\beta}_{\nu_M} \lesssim 10^{-18}\,\text{GeV}^{-5}$$

$\Updownarrow$ new physics

$$\frac{1}{\Lambda^5_{0\nu 2\beta}} \quad \bar{p}\bar{p}\bar{e}\bar{e}uu$$

$$\Lambda^5_{0\nu 2\beta} \gtrsim 10^{18}\,\text{GeV}^5$$

$\Downarrow$

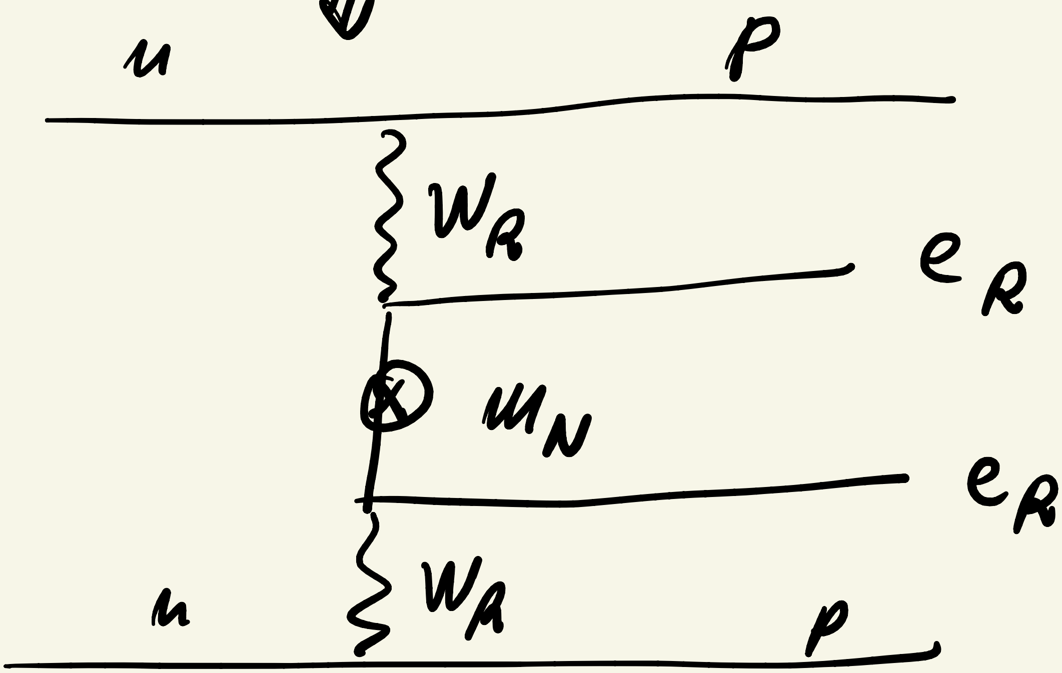
$$\boxed{\Lambda_{0\nu2\beta} \gtrsim 3\ \mathrm{TeV}}$$

tailor-made for LHC

<u>example</u>

LRSM

$\Downarrow$



$$A_{N_M}^{0\nu2\beta} \simeq \left(\frac{1}{M_R^4}\right)\frac{m_N}{\cancel{p}^2 - m_N^2}$$

$$\simeq G_F^2\left(\frac{M_L}{M_R}\right)^4\frac{1}{m_N}$$

$(p \ll m_N)$

$$\boxed{M_R \gtrsim 4\,\mathrm{TeV}, \quad m_N = ?}$$

$\Downarrow$

$0\nu2\beta \Longrightarrow$ e polarization

$\Longrightarrow$ at least one $e \neq e_L$

new physics behind $0\nu2\beta$

$\Lambda_{0\nu2\beta} \simeq 3$ TeV

LHC ?

but new collider!

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loops $\Longleftrightarrow$ concentrate

on processes $= 0$ (tree)

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$\boxed{\text{QFT}}$

Couplings $= f(E)$

all !