Neutrino Physics Course

Lecture XIII

9/6/2023

LMU

Summer 2023

LRSM and neutrino mass:

the seesow

- $\nu_L, \nu_R \longrightarrow N_L = C\bar{\nu}_R{}^T \propto \nu_R^*$

$$\Downarrow$$

$$\mathcal{L}_{mass}(\nu) = \bar{\nu}_R\, m_D\, \nu_L + h.c.$$

$$+ m_{\nu_R} \frac{1}{2} \nu_R{}^T C \nu_R + h.c.$$

$$\parallel$$

$$N_L{}^T C N_L$$

$$= N_L{}^T C\, m_D\, \nu_L + h.c.$$

$$+\frac{1}{2} N_L^T C N_L \underbrace{m_{\nu_R}^*}_{\equiv m_N} + h.c.$$

where we used:

$$N_L^T C \nu_L = \bar{\nu}_R C^T C \nu_L \overset{?}{=} -\bar{\nu}_R \nu_L$$

$$(C^T C = -1)$$

$$\Downarrow$$

$$\mathcal{L}_{mass}(\nu) = \frac{1}{2} m_D \left[N_L^T C \nu_L + N_L^T C \nu_L \right] + h.c.$$

$$+ \frac{1}{2} m_N N_L^T C N_L + h.c.$$

$$= \frac{1}{2} m_D \left[N_L^T C \nu_L + (-1) \nu_L^T C^T N_L \right]$$

$$+\frac{1}{2}\, m_N\, N_L^T C N_L \quad + \text{h.c.}$$

$$= \frac{1}{2}\Big[m_D \left(N_L^T C \nu_L + \nu_L^T C N_L \right)$$

$$+ m_N\, N_L^T C N_L \Big] \quad + \text{h.c.}$$

$$\boxed{m_D, m_N \in \mathbb{R}} \quad \Downarrow$$

$$\underline{M}_{\nu N} = \begin{array}{c c} & \begin{array}{cc} \nu_L & N_L \end{array} \\ \begin{array}{c} \nu_L \\ N_L \end{array} & \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \end{array}$$

$$m_N = y_\Delta\, v_R \equiv y_D\, \underline{M}_R$$

$$m_D = y_\Phi\, v_{SM} \equiv y_\Phi\, \underline{M}_W$$

exp. $\quad M_R \gg M_W$

$(M_R \gtrsim 4\,\mathrm{TeV} \Leftarrow \mathrm{LHC})$

$$\Downarrow$$

$$\boxed{m_N \gg m_D}$$

see-saw

$M_{\nu N}$

$$\mathrm{Tr}\, M_{\nu N} = m_N = m_L + m_H$$

light and heavy

$$\det M_{\nu N} = -m_D^2$$

$$\Downarrow$$

$$m_N \doteq m_H + m_L \simeq m_H$$

(a) $m_H \simeq m_N$

(b) $m_L \, m_H = - m_D^2$

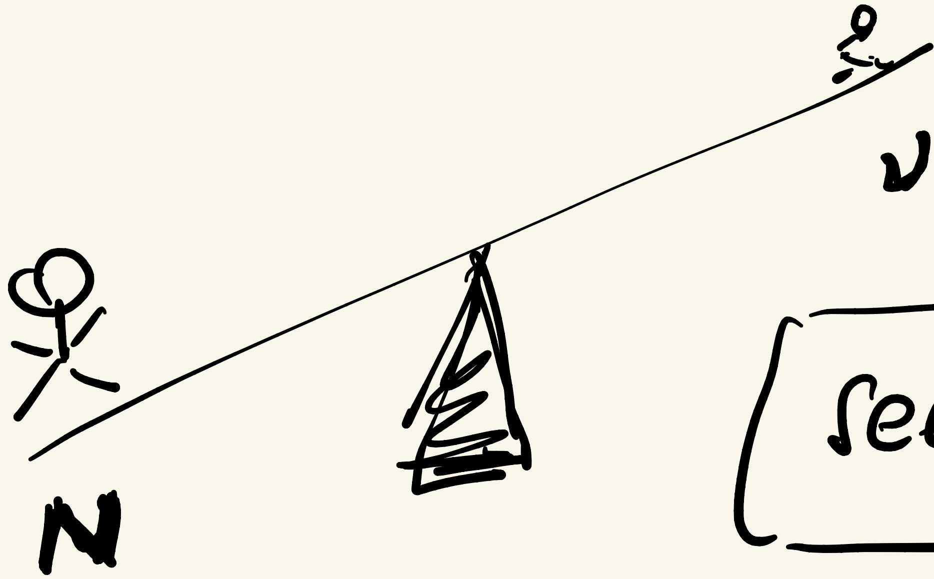
$$\Downarrow$$

$$m_L \, m_N = - m_D^2$$

$$\boxed{m_L \simeq - \frac{m_D^2}{m_N}}$$

m_ν

see-saw



see - saw

$$\begin{pmatrix} \nu \\ N \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix} = \begin{pmatrix} \nu' \\ N' \end{pmatrix}$$

$$\Rightarrow \boxed{\theta \simeq m_D / m_N} \quad \text{(physics)}$$

$\boxed{a, b, c \in \mathbb{R}}$

$$M_S = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2} \tan 2\theta = \frac{b}{c-a}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\Downarrow$ seesaw

$$\boxed{m_\nu \simeq - \frac{m_D^2}{m_N}}$$

(got rid of ' notation)

SM reminder

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad ; \quad e_R$$

$$\Downarrow$$

$$\boxed{m_\nu = 0}$$

$\boxed{\text{LRSM}}$

$$m_N \propto M_R$$

$$m_\nu \propto \frac{m_D{}^2}{M_R}$$

SM limit : $M_R \to \infty$

$\Rightarrow$ lightness of neutrino

$\Updownarrow$

(new) maximality of $\not P$ in weak int.

· limit on M_R ???

$$m_D = y_D \langle \phi_{SM} \rangle \lesssim M_W$$

$$(\Leftrightarrow \quad y_D \lesssim O(1))$$

$\Downarrow$

$$m_\nu \le \frac{M_W^2}{M_R}$$

$$\Rightarrow \quad M_R \le \frac{M_W^2}{m_\nu} \simeq \frac{(100\,\mathrm{GeV})^2}{10^{-1}\,\mathrm{eV}}$$

$$\simeq \frac{10^4}{10^{-10}}\,\mathrm{GeV}$$

$$\boxed{M_R \le 10^{14}\,\mathrm{GeV}}$$

$$G_{LR} = \underbrace{SU(2)_L \times SU(2)_R}_{\substack{\Delta_L,\ \Delta_R \\ \text{triplets}}} \times U(1)_{B-L}$$

$$(B-L)(\Delta_L\ (\Delta_R)) = 2$$

$$\Downarrow$$

$\langle \Delta_R \rangle = v_R$ breaks $B-L$
by 2 units

$$\Uparrow$$

$$l_R \langle \Delta_R \rangle l_R \Rightarrow \nu_R \, \underbrace{\nu_R \ \langle \Delta_R \rangle}_{\boxed{\Delta L = 2}}$$

$\beta: \quad n \to p + e + \bar{\nu}_e \quad (\Delta L = 0)$

$2\beta: \quad n + n \to p + p + e + e$
$\qquad + \bar{\nu}_e + \bar{\nu}_e \quad (\Delta L = 0)$

$0\nu 2\beta: \quad n + n \to p + p + e + e$
$\qquad (\Delta L = 2)$

Majorana fermion

$f_L \; (\psi_L) \; \therefore$

$$\mathcal{L}_M(f_L) = i\,\overline{f_L}\,\gamma^\mu \partial_\mu f_L - \frac{1}{2} m_M \left(\overline{f_L}^T C f_L + h.c.\right) \quad (1)$$

analogy with Dirac

$$\mathcal{L}_D(f_L, f_R) = i\,\overline{f_L}\,\gamma^\mu \partial_\mu f_L + i\,\overline{f_R}\,\gamma^\mu \partial_\mu f_R - m_D\left(\overline{f_L} f_R + \overline{f_R} f_L\right)$$

$$f_D = f_L + f_R$$

$$\Downarrow$$

$$\mathcal{L}_D(f_D) = i\,\overline{f_D}\,\gamma^\mu \partial_\mu f_D -$$

$$- m_D \bar{f}_D f_D$$

$$\boxed{p^2 = m_D^2}$$

$$S_F(p) = \frac{1}{\not{p} - m_D}$$

$$\not{p} = p_\mu \gamma^\mu$$

$$f_M = f_L + (\underbrace{C \bar{f}_L^T}_{(f^c)_R} \alpha f_L^*)$$

$$f_L = \begin{pmatrix} \nu_L \\ 0 \end{pmatrix} \Rightarrow \boxed{f_M = \begin{pmatrix} f_L \\ -i\sigma_2 f_L^* \end{pmatrix}}$$

$$(f^c)_R$$

$\Downarrow$ Prove:

- $\bar{f}_M f_M = f_L^T C f_L + h.c.$

- $\bar{f}_M \gamma^\mu \partial_\mu f_M = 2 \bar{f}_L \gamma^\mu \partial_\mu f_L$

$\Downarrow$

From (1):

$$\mathcal{L}_M (f_L) = \frac{1}{2} i \bar{f}_M \gamma^\mu \partial_\mu f_M \; -$$

$$-\frac{1}{2}\, m_M\, \bar{\psi}_M \psi_M$$

$$= \frac{1}{2}\left[i \bar{\psi}_M \gamma^\mu \partial_\mu \psi_M - m_M \bar{\psi}_M \psi_M \right]$$

Dirac like !!

$$\Downarrow$$

$$S_F(\psi_M) = \frac{i}{\not{p} - m_M}$$

with :

(i) 1/2 degrees of Dirac

(ii) $\Delta L = 2 \iff \psi_M = \psi + (\alpha)\psi^*$

N generations seesaw

$$\mathcal{L}_{mass}(\nu_L, N_L) =$$

$$= \bar{\nu}_R \, M_D \, \nu_L + h.c$$

$$+ \frac{1}{2} \nu_R^T M_{\nu_R} C \nu_R + h.c.$$

$$= N_L^T C \, M_D \, \nu_L + h.c.$$

$$+ \frac{1}{2} N_L^T C \, M_N \, N_L + h.c$$

$$\nu_L = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \vdots \end{pmatrix}_L , \quad N_L = \begin{pmatrix} N_1 \\ N_2 \\ \vdots \end{pmatrix}_L$$

$$= \frac{1}{2}\left[\underline{N_L^T C M_D \nu_L} + \underline{\nu_L^T C M_D^T N_L}\right]$$

$$+ \frac{1}{2} N_L^T \underline{M}_N C N_L + h.c.$$

$$\Downarrow$$

$$\underline{M}_{\nu N} = \begin{array}{c c} & \begin{array}{cc} \nu_L \qquad & \qquad N_L \end{array} \\ \begin{array}{c} \nu_L \\ \\ N_L \end{array} & \begin{pmatrix} 0 & M_D^T \\ & \\ M_D & \underline{M}_N \end{pmatrix} \end{array}$$

$$\boxed{\begin{pmatrix} \nu \\ N \end{pmatrix}_L \longrightarrow U(\theta) \begin{pmatrix} \nu \\ N \end{pmatrix}_L}$$

$$= \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$U(\theta) = \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix} \quad !.$$

$$U^+ U = 1 + O(\theta^2)$$

$$U^+ = \begin{pmatrix} 1 & -\theta^+ \\ \theta & 1 \end{pmatrix}$$

$$\Downarrow$$

$$M_{\nu N} \rightarrow \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix}$$

with :

$$\Theta = \frac{1}{M_N} M_D$$

$$\boxed{M_\nu = - M_D^T \frac{1}{M_N} M_D}$$

T. Majorana mass matrix is symmetric.

Proof:

$$N^T C M_N N =$$

$$= (N_1\ N_2\ \cdots)^T C\ M_N \begin{pmatrix} N_1 \\ \vdots \end{pmatrix}$$

$$= - N^T C^T M_N^T N$$

$$= - N^T (-C) M_N^T N$$

$$= N^T C\ M_N^T N \Rightarrow$$

$$\boxed{M_N = M_N^T}$$

$$\uparrow$$

Q.E.D.

$$N_i^T C\ M_{ij}\ N_j =$$

$$= - N_j^T C^T M_{ij} N_i$$

$$= - N_j^T C^T (M_{ji})^T N_i \quad \ldots$$

$$\Downarrow$$

M_ν = symmetric

$$\Downarrow$$

$$\boxed{M_\nu = - M_D^T \frac{1}{M_N} M_D}$$

[General see saw formula]

N couplings

$$\mathcal{L}_{ww} = \frac{g}{\sqrt{2}}\, \bar{\nu}_L \gamma^\mu e_L W_\mu^+ + h.c.$$

$$\nu \to \nu + \Theta N$$

$$\frac{g\,\Theta}{\sqrt{2}}\, \bar{N}_L \gamma^\mu e_L W_\mu^+ + h.c.$$

$$N \to e + W^+$$

$$\Theta = \frac{m_D}{m_W}$$

$$\begin{pmatrix} \nu \\ N \end{pmatrix} \longrightarrow U(\theta) \begin{pmatrix} \nu \\ N \end{pmatrix}$$

$$\theta = \frac{1}{M_N} M_D$$

<u>generation</u>

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R, \; d_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R, \; \nu_R \; ?$$

- $m_f = y_f \, v_{SM}$

  $y_f \lesssim O(1)$

$$\Rightarrow \quad m_f \lesssim v_{SM} \quad (600 \text{ GeV})$$