

Neutrino Physics Course

Lecture VII

16 / 5 / 2023

LMU

Spring 2023

SM Higgs mechanism:

Weinberg theory

U(1) Higgs mechanism

$$\phi \rightarrow e^{i\alpha Q} \phi, \quad d = d(x)$$

$$(Q\phi = \phi)$$

$$\mathcal{L} = \frac{1}{2} |D_\mu \phi|^2 - V(\phi)$$

$$D_\mu = \partial_\mu - ig A_\mu Q$$

$$V = \frac{1}{4} (|\phi|^2 - v^2)^2$$



$$\mathcal{M}_0 = \{ \phi_0 \because v = v_{\text{min}} \} = \\ = \{ \phi_0^2 = v^2 \} = S_1$$

- $M_A^2 = g^2 v^2 |\phi_0|^2 \quad (\text{dof} = 3)$

- $\phi_{\text{un}} = (v+h) \quad (\text{dof} = 1)$

$$\phi = e^{i\theta/v} (v+h)$$

$$\rightarrow \underbrace{e^{-i\theta/v} \quad e^{i\theta/v}}_{\text{gauge}} (v+h)$$

gauge

Summary !

$$U(1) \xrightarrow{\phi_0 = \langle \phi \rangle} \mathbb{1}$$



Now

1967

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$$

T_a

$\frac{Y}{2}$

$$Q_{em} = T_3 + \frac{Y}{2}$$

- $M_W, M_Z \neq 0$
- $m_f (e, l) \neq 0$



fixes the Higgs multiplet

- down quark mass

$$m_d \bar{d} d = m_d (\bar{d}_L d_R + \bar{d}_R d_L)$$

matter of SM

$$L_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

u_R, d_R

\bar{Q}_L d_R \leftarrow not SO(2) inv.

$\mathcal{L}_Y = y_d \bar{Q}_L \Phi d_R + \text{h.c.}$

Higgs field
(Weinberg)

Quantum \neq

y_d

vacuum: $\Phi_0 \neq 0$

$\Rightarrow m_d \propto y_d \Phi_0$ (see below)

\Downarrow $SU(2)$ inv.

$\Phi = SU(2)$ doublet

$$(\Phi \rightarrow U \Phi)$$

$$\mathcal{L}_Y \rightarrow Y_d \bar{q}_L U^\dagger U \Phi d_R + \text{h.c.}$$

$$= \mathcal{L}_Y$$

$$Y(q_L) = 1/3, \quad Y(d_R) = -2/3$$

\Downarrow

$$\boxed{Y(\Phi) = +1}$$

$$\Phi : \bar{\Phi} \rightarrow U\bar{\Phi}, \quad Y(\Phi) = 1$$



$$D_\mu = \partial_\mu - ig \underbrace{T_a}_{SU(2)} A_\mu^a - ig' \underbrace{\frac{Y}{2}}_{U(1)} B_\mu$$

$$\therefore \text{for } \Phi : T_a = \sigma_a / 2$$

$$\text{for } : T_a = 0$$

$$\frac{Y}{2} = (Q_{em} - T_3)$$

$$\mathcal{L}_{SM} = \mathcal{L}_f + \mathcal{L}_\phi - \mathcal{L}_g + \mathcal{L}_{gb}$$

$$\mathcal{L}_\phi = \frac{1}{2} (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$$

$$V(\phi) = \frac{\lambda}{4} (\Phi^\dagger \bar{\Phi} - v^2)^2 + \dots$$

$$\cdot \Phi^\dagger \bar{\Phi} \rightarrow \Phi^\dagger U^\dagger U \bar{\Phi} = \Phi^\dagger \bar{\Phi}$$

$$\cdot \bar{\Phi}^T i \sigma_2 \bar{\Phi} \rightarrow \bar{\Phi}^T U^T i \sigma_2 U \bar{\Phi}$$

$$= \bar{\Phi}^T i \sigma_2 U^\dagger U \bar{\Phi} = \bar{\Phi}^T i \sigma_2 \bar{\Phi}$$

why not this term?

not $U_Y(1)$ inv.

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \phi_i \in \mathbb{C}$$

$$\begin{aligned} \Phi^T i \sigma_2 \Phi &= (\phi_1 \ \phi_2) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \\ &= \phi_1 \phi_2 - \phi_2 \phi_1 = 0 \end{aligned}$$

Simply, not there

... (true \checkmark):

$$(\Phi^\dagger \vec{\sigma} \Phi) (\Phi^\dagger \vec{\sigma} \Phi) = \text{inv.}$$

vectors

vectors

NOT independent

$$\propto (\Phi + \Phi)^2 \quad !$$

• $SU(2)$ algebra

$$[T_a, T_b] = i \epsilon_{abc} T_c$$

$$C = \{ T_i, [T_i, T_i] = 0 \} = T_3$$

Certain sub-algebra

$$\gamma(\text{rank}) = 1$$

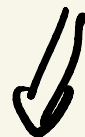
I expect me invariant

Explicit

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow U \Phi$$

choose $U \therefore \Phi \rightarrow \begin{pmatrix} 0 \\ \phi \end{pmatrix}$

** PROVE **



only one invariant \therefore

$$\bar{\Phi} + \Phi$$



$$V_{\text{general}} = \frac{\lambda}{4} (\bar{\Phi} + \Phi - v^2)^2$$

$$\mathcal{M}_0 = \{ \Phi_0 : V = \text{min} \}$$

$$= \{ \bar{\Phi}_0 + \Phi_0 = v^2 \}$$

$$= \{ |\phi_1^0|^2 + |\phi_2^0|^2 = v^2 \}$$

\nearrow

$$\underline{\Phi}_0 = \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}$$

$$\phi_2^0 = R_1^0 + i I_1^0$$

$$\Rightarrow M_0 = \left\{ (R_1^{02} + I_1^{02} + 1 - v^2) - v^2 \right\}$$

$$= S_3$$

\Downarrow simple choice on S_3

$$\boxed{\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \in R}$$

$$U(1) : \phi = e^{iQ\theta/v} (v+h)$$

$SU(2) \times U(1)$:

↗ analog

$$\boxed{\Phi = e^{i T_a \theta_a / v} \begin{pmatrix} 0 \\ v+h \end{pmatrix}}$$

$$T_a \theta_a = T_1 \theta_1 + T_2 \theta_2 + T_3 \theta_3$$

$$T_a = \sigma_a / 2$$

$$\Rightarrow \Phi = \left(1 + i \sigma_a / 2 \theta_a / v \right) \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\stackrel{?}{=} \begin{pmatrix} \theta_2 - i \theta_1 \\ v+h - i \theta_3 \end{pmatrix} \frac{1}{2} + \dots$$

(check)

$$\Phi \rightarrow U \Phi = e^{-i \sigma_a T_a / v} e^{i \sigma_a T_a / v}$$

$$\times \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\Phi_{un} = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

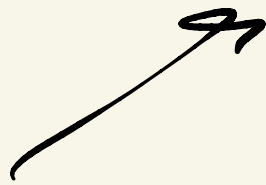
(Goldstone) - Higgs - Weinberg boson

Gauge bosons

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} |D_\mu \Phi|^2$$

$$\Phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$D_\mu \Phi = \begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} - \frac{i}{2} \begin{pmatrix} g A_3 + g' B & g(A_1 - i A_2) \\ g(A_1 + i A_2) & (-g A_3 + g' B) \end{pmatrix}$$



$$\times \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$-ig \frac{\sigma_a}{2} A_\mu^a - ig' \frac{1}{2} B_\mu$$



$$D_\mu \Phi = \dots - i \frac{1}{2} \begin{pmatrix} g(A_1 - i A_2) \\ (-g A_3 + g' B) \end{pmatrix} \quad (\text{with})$$

↖

$\partial_\mu \psi$



$$\frac{1}{2} (D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{1}{2} (\partial_\mu \psi)^2 \leftarrow \text{kin. energy}$$

$$+ \frac{1}{2} \frac{v^2}{4} \left[g^2 (A_1^2 + A_2^2) + \frac{(g A_3 - g' B)^2}{\left(1 + \frac{g}{g'}\right)^2} \right]$$

⇓

$$\left(W^\pm = \frac{A_1 - i A_2}{\sqrt{2}} \right)$$

$$M_w = \frac{g}{2} z \quad (1)$$

$$\tan \theta_w = g'/g$$

$$z = \frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}} = \cos \theta_w A_3 - \sin \theta_w B$$

$$\Rightarrow M_z = \frac{\sqrt{g^2 + g'^2}}{2} z \quad (2)$$

$$A = \frac{g' A_3 + g B}{\sqrt{g^2 + g'^2}} = \sin \theta_w A_3 + \cos \theta_w B$$

$$M_A = 0 \quad \text{predicted}$$

$$M_w = M_z \cos \theta_w \leftarrow \text{tran (1), (2)}$$

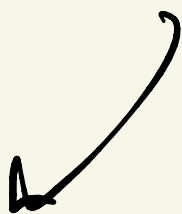


$S \ S \ B \ \dots$

$$\Phi_0 = \begin{pmatrix} 0 \\ \nu \end{pmatrix}, \quad T_i = \frac{p_i}{N}$$

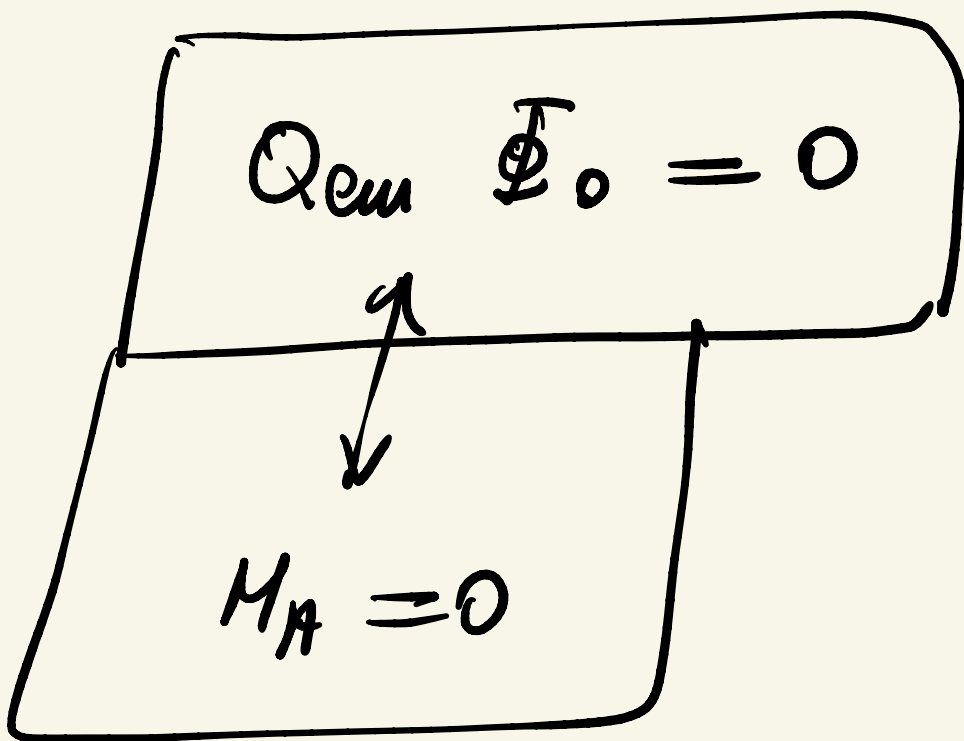
$$\Rightarrow T_1 \Phi_0 \neq 0 \quad T_3 \Phi_0 \neq 0$$

$$T_2 \Phi_0 \neq 0 \quad \gamma \Phi_0 \neq 0$$



$$\text{but } (T_3 + \frac{\gamma}{2}) \Phi_0 = \begin{pmatrix} +1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ a \end{pmatrix} = 0 \quad \times \begin{pmatrix} 0 \\ a \end{pmatrix}$$



conclude: $\left\{ \begin{array}{l} T_1 \Phi_0 \neq 0, \quad T_2 \Phi_0 \neq 0 \\ (T_3 - \frac{Y}{2}) \Phi_0 \neq 0 \end{array} \right.$

3 "broken" generators

SUN

56-60
 $\mu_p \approx 10$

Gravity matters!
matter gravitates!

Cosmology and

Vacuum energy

$$V = \frac{\lambda}{4} (\bar{\Phi} + \Phi - v^2)^2$$

$$\therefore V_{\text{min}} = 0$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

\downarrow \uparrow
 force energy-momentum

$$R_{\mu\nu} = f(g_{\mu\nu})$$

$$R = g_{\mu\nu} R^{\mu\nu} = f(g_{\mu\nu})$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} / M_{pl}^2$$

\uparrow

Minkowski

$$\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

$$G_N = \frac{1}{M_{pl}^2}, \quad M_{pl} \approx 10^{19} \text{ GeV}$$

$$\alpha_{gr} = \frac{E^2}{M_{pl}^2} \leq \left(\frac{10^4 \text{ GeV}}{10^{19} \text{ GeV}} \right)^2$$

today

$$T_{av} = \uparrow + \frac{T_{vac}}{M_{pl}^2}$$

matter

Cosmological constant

$$Q. \langle e \rangle = e_0 \neq 0 ?$$

$$\langle z_\mu \rangle \stackrel{?}{=} z_\mu^0 \neq 0 ?$$



A. NO \Rightarrow it breaks Lorentz!

$$V_{\text{min}} \approx (10^{-4} \text{ eV})^4 \approx (10^{-13} \text{ GeV})^4 \\ \approx 10^{-52} \text{ GeV}^4$$

Expected : $V_{\text{min}} \approx v^4$
 $\approx M_W^4$

$$\approx (10^2)^4 \text{ GeV}^4$$

$$\approx 10^8 \text{ GeV}^4$$

$$V_{\text{min}} \leq 10^{60} V_{\text{min}} (\text{expected})$$

Cosmological constant
problem

$$\mathcal{L}_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R, d_R$$



What if:

$$\mathcal{L}_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$\begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

\mathcal{P} conserving

Stay tuned!
