# Neutrino Physics Course

## Lecture VII

16/5/2023

LMU

Spring 2023

# SM Higgs mechanism:

## Weinberg theory

### U(1) Higgs mechanism

$$\phi \to e^{i\alpha Q}\,\phi\,, \quad \alpha = \alpha(x)$$

$$(Q\phi = \phi)$$

$$\mathcal{L} = \frac{1}{2}|D_\mu \phi|^2 - V(\phi)$$

$$D_\mu = \partial_\mu - ig A_\mu Q$$

$$V = \frac{1}{4}\left(|\phi|^2 - v^2\right)^2$$

$$\Downarrow$$

$$\mathcal{M}_0 = \{ \phi_0 : V = V_{min} \} =$$

$$= \{ \phi_0^2 = v^2 \} = S_1$$

- $M_A^2 = g^2 v^2 |\phi_0|^2$ (dof = 3)
- $\phi_{un} = (v + h)$ (dof = 1)

$$\Uparrow$$

$$\phi = e^{iG/v} (v + h)$$

$$\to \underbrace{e^{-iG/v}}_{\text{gauge}} e^{iG/v} (v + h)$$

Summary :

$$U(1) \xrightarrow[\phi_0 = \langle \phi \rangle]{} \mathbb{1}$$

Now

1967

$$\boxed{\underset{(g)}{SU(2)_L} \times \underset{Y\ (g')}{U(1)} \longrightarrow U(1)_{em}}$$

$T_a$ , $\frac{Y}{2}$ $Q_{em} = T_3 + \frac{Y}{2}$

- $M_W, M_Z \neq 0$

- $m_f \; (q, l) \neq 0$

fixes the Higgs multiplet

- down quark mass

$$m_d \bar{d} d = m_d (\bar{d}_L d_R + \bar{d}_R d_L)$$

<u>matter of SM</u>

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R, \; d_R$$

$$\downarrow$$

$\bar{q}_L \quad d_R \leftarrow$ <u>not</u> SO(2) inv.

$$\downarrow$$

Higgs field

(Weinberg)

$$\mathcal{L}_Y = y_d \bar{q}_L \, \Phi \, d_R + h.c.$$

Yukawa

quantum #

$$\Downarrow$$

vacuum: $\Phi_0 \neq 0$

$\Rightarrow \quad m_d \propto y_d \Phi_0$ (see below)

$\Downarrow$ SU(2) inv.

$\Phi =$ SU(2) doublet

$(\Phi \to U\Phi)$

$$\mathcal{L}_Y \to y_d\, \bar{q}_L U^+ U \Phi\, d_R + h.c.$$

$$= \mathcal{L}_Y$$

$$Y(q_L) = 1/3, \quad Y(d_R) = -2/3$$

$\Downarrow$

$$\boxed{Y(\Phi) = +1}$$

$$\boxed{\Phi : \quad \Phi \to U\Phi, \quad Y(\Phi) = 1}$$

$$\Downarrow$$

$$D_\mu = \partial_\mu - ig \overbrace{T_a A_\mu^a}^{SU(2)} - ig' \overbrace{\frac{Y}{2} B_\mu}^{U(1)}$$

$\therefore \quad f_L, \Phi : \quad T_a = \sigma_a/2$

$\quad\ \ f_R \quad\ : \quad T_a = 0$

$$\boxed{\frac{Y}{2} = (Q_{em} - T_3)}$$

$$\mathcal{L}_{SM} = \mathcal{L}_f + \mathcal{L}_\phi - \mathcal{L}_Y + \mathcal{L}_{gb}$$

$$\mathcal{L}_\phi = \frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$$

$$\boxed{V(\phi) = \frac{\lambda}{4}\left(\Phi^\dagger \Phi - v^2\right)^2} + \cancel{\cdots}$$

- $\Phi^\dagger \Phi \to \Phi^\dagger U^\dagger U \Phi = \Phi^\dagger \Phi$

- $\Phi^T i\sigma_2 \Phi \to \Phi^T U^T i\sigma_2 U \Phi$

$$= \Phi^T i\sigma_2 U^\dagger U \Phi = \Phi^T i\sigma_2 \Phi$$

why not this term?

not $U_Y(1)$ inv.

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_1 \end{pmatrix}, \quad \phi_i \in \mathbb{C}$$

$$\Phi^T i\sigma_2 \Phi = \begin{pmatrix} \phi_1 & \phi_2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$= \phi_1 \phi_2 - \phi_2 \phi_1 = 0$$

simply, not there

.... (term V) :

$$\underbrace{(\Phi^+ \vec{\sigma}\, \Phi)}_{\text{vector}} \underbrace{(\Phi^+ \vec{\sigma}\, \phi)}_{\text{vector}} = \text{inv.}$$

**NOT** independent

$\propto (\Phi^+ \Phi)^2$ !

· SU(2) algebra

$$[T_a, T_b] = i\, \varepsilon_{abc} T_c$$

$$C = \{ T_i,\ [T_i, T_j] = 0 \} = T_3$$

Cartan sub-algebra

$$\boxed{\gamma(\text{vecth}) = 1}$$

I expect <u>me</u> invariant

<u>Explicit</u>

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to U\,\Phi$$

$$\boxed{\text{choose } U \therefore \ \Phi \to \begin{pmatrix} 0 \\ \phi \end{pmatrix} \qquad ** \text{PROVE} **}$$

$$\boxed{\text{only one invariant: } \Phi^{+}\Phi}$$

$$\Downarrow$$

$$V_{general} = \frac{\lambda}{4}\left(\Phi^{+}\Phi - v^{2}\right)^{2}$$

$$\mathcal{M}_0 = \{ \Phi_0 : V = \min \}$$

$$= \{ \Phi_0^{+}\Phi_0 = v^2 \}$$

$$= \{ |\phi_1^0|^2 + |\phi_2^0|^2 = v^2 \}$$

$$\Uparrow$$

$$\Phi_0 = \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}$$

$$\phi_1^0 = R_1^0 + i\, I_1^0$$

$$\Rightarrow \quad \mathcal{M}_0 = \left\{ \left( R_1^{0\,2} + I_1^{0\,2} + 1 \to 2 \right) - v^2 \right\}$$

$$= S_3$$

$$\Downarrow \quad \text{single choice on } S_3$$

$$\boxed{\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \in R}$$

$U(1)$ : $\phi = e^{iQ\, G/v}(v+h)$

$\updownarrow$ analogy

$SU(2)\times U(1)$:

$$\boxed{\Phi = e^{i\, T_a G_a / v} \begin{pmatrix} 0 \\ v+h \end{pmatrix}}$$

$$T_a G_a = T_1 G_1 + T_2 G_2 + T_3 G_3$$

$$T_a = \sigma_a / 2$$

$$\Rightarrow \Phi = \left(1 + i\, \sigma_a/2\, G_a / v\right) \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\overset{?}{=} \begin{pmatrix} G_2 - i\, G_1 \\ v+h - i\, G_3 \end{pmatrix} 1/2 + \cdots$$

(check)

$$\Phi \to U\Phi = e^{-i\,G_a T_a / v}\, e^{i\,G_a T_a / v} \times \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\boxed{\Phi_{un} = \begin{pmatrix} 0 \\ v+h \end{pmatrix}}$$

(Goldstone) - Higgs - Weinberg boson

# Gauge bosons

$$\mathcal{L}_{kin} = \frac{1}{2} |D_\mu \Phi|^2$$

$$\Phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$D_\mu \Phi = \begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} - \frac{i}{2} \begin{pmatrix} g A_3 + g' B & g(A_1 - iA_2) \\ g(A_1 + iA_2) & (-g A_3 + g' B) \end{pmatrix} \times \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$-ig \frac{\sigma_a}{2} A_\mu{}^a - ig' \frac{1}{2} B_\mu$$

$$\Downarrow$$

$$D_\mu \Phi = \underset{\partial_\mu h}{\cdots} - \frac{i}{2} \begin{pmatrix} g(A_1 - i A_2) \\ (-g A_3 + g' B) \end{pmatrix} (v+h)$$

$$\Downarrow$$

$$\frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{1}{2} (\partial_\mu h)^2 \leftarrow \text{kin. energy}$$

$$+ \frac{1}{2} \frac{v^2}{4} \left[ g^2 (A_1^2 + A_2^2) + (g A_3 - g' B)^2 \right] \underline{\phantom{xx}} (1 + \frac{h}{v})^2$$

$$\Downarrow$$

$$\left( W^\pm = \frac{A_1 \mp i A_2}{\sqrt{2}} \right)$$

$$\boxed{M_W = \frac{g}{2} v} \quad (1)$$

$$\tan\theta_W = g'/g$$

$$\Downarrow$$

$$Z = \frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}} = \cos\theta_W A_3 - \sin\theta_W B$$

$$\Rightarrow \boxed{M_Z = \frac{\sqrt{g^2+g'^2}}{2} v} \quad (2)$$

$$A = \frac{g' A_3 + g B}{\sqrt{g^2+g'^2}} = \sin\theta_W A_3 + \cos\theta_W B$$

$$\boxed{M_A = 0}$$

predicted

$$\boxed{M_W = M_Z \cos\theta_W} \Leftarrow \text{tree (1), (2)}$$

$$\Updownarrow$$

SSB ...

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad T_i = \frac{\sigma_i}{2}$$

$$\Rightarrow \quad T_1 \Phi_0 \neq 0 \qquad T_3 \Phi_0 \neq 0$$

$$T_2 \Phi_0 \neq 0 \qquad Y \Phi_0 \neq 0$$

$$\text{but } \left(T_3 + \frac{Y}{2}\right)\Phi_0 = \begin{pmatrix} +1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\times \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = 0$$

$$\boxed{Q_{em}\, \Phi_0 = 0}$$

$$\Updownarrow$$

$$\boxed{M_A = 0}$$

conclude: $\left\{ \begin{array}{l} T_1\, \Phi_0 \neq 0, \quad T_2\, \Phi_0 \neq 0 \\ \left(T_3 - \frac{Y}{2}\right) \Phi_0 \neq 0 \\ 3 \text{ "broken" generators} \end{array} \right.$

SUN

$$N_p \simeq 10^{56-60}$$

Gravity matters!

matter gravitates!

## Cosmology and Vacuum energy

$$V = \frac{\lambda}{4}\left(\Phi^\dagger \Phi - v^2\right)^2$$

$$\therefore \quad V_{min} = 0$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

force

energy-momentum

$$R_{\mu\nu} = f(g_{\mu\nu})$$

$$R = g_{\mu\nu} R^{\mu\nu} = f(g_{\mu\nu})$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{pl}$$

Minkowski

$$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$$G_N = \frac{1}{M_{pl}^2}, \quad M_{pl} \simeq 10^{19}\,\mathrm{GeV}$$

$$\alpha_{gr} = \frac{E^2}{M_{pl}^2} \leq \left(\frac{10^4\,\mathrm{GeV}}{10^{19}\,\mathrm{GeV}}\right)^2$$

today

$$\boxed{T_{\mu\nu} = \quad + \quad T_{vac}/M_{pl}^2}$$

matter

cosmological constant

Q. $\langle e \rangle = e_0 \neq 0$ ?

$\langle Z_\mu \rangle \simeq Z_\mu^0 \neq 0$ ?

$$\Downarrow$$

A. NO $\Rightarrow$ it breaks Lorentz!

$$V_{min} \simeq (10^{-4}\,eV)^4 \simeq (10^{-13}\,GeV)^4$$

$$\simeq 10^{-52}\,GeV^4$$

expected : $V_{min} \simeq v^4$

$$\simeq M_W^4$$

$$\simeq (10^2)^4 \text{ GeV}^4$$

$$\simeq 10^8 \text{ GeV}^4$$

$$\boxed{V_{min} \leq 10^{60}\, V_{min}\,(\text{expected})}$$

Cosmological constant problem

$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$ $u_R, \; d_R$

What if: $q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$

$\boxed{P \text{ conserving}}$

Stay tuned!