


Neutrino Physics Course

Lecture IV

5/15/2023

LMU

Spring 2023



From $\not{P} \rightarrow V - A$ theory



Standard Model (SM)

$$W^- \rightarrow e + \bar{\nu}_e$$

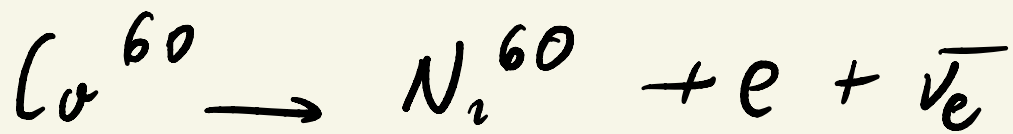


$$J_z = +1$$

$$\uparrow (e_L)$$

$$\uparrow (\bar{\nu}_e)_R \Leftrightarrow \downarrow$$

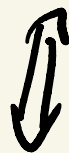




⇓ \neq maximum

$$\mathcal{H}_{\text{eff}} = \frac{4 G_F}{4\sqrt{2}} J_{\mu}^W \bar{J}_{\mu}^W$$

$$J_{\mu}^W = \bar{\nu}_l \gamma^{\mu} e_l + \bar{u}_l \gamma^{\mu} d_l$$



1957

Morishita, Sudshan

$$\left(\frac{g}{\sqrt{2}} W_{\mu}^{+} J_{\mu}^W + \text{h.c.} \right)$$

"V-A" theory

$$L = \frac{1 + \gamma_5}{2} \quad (\text{our})$$

2009 Weinberg

"V-A was the key"

no coupling to W_μ !

↓

~~$$L_{\text{lep}} = + G_F' [\bar{\nu} (a + b \gamma_5) e + \dots]^2$$~~

~~$$+ G_F'' [\bar{\nu} \gamma^\mu \gamma^\nu (a + b \gamma_5) e + \dots]^2$$~~

1957 Schwinger

(Glashow)

↙

gauge theory of ew (=
 = em + weak) interactions

U(1) QED

$$\mathcal{L}_0 = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi$$

$$\psi \rightarrow e^{i\alpha Q} \psi \quad (Q\psi = q\psi)$$

$$\text{if } \alpha = \alpha(x) \Rightarrow \boxed{U \equiv e^{i\alpha Q}}$$

$$D_\mu = \partial_\mu - ie Q A_\mu$$

$$(D_\mu \psi) \rightarrow e^{i\alpha Q} (D_\mu \psi)$$

$$\therefore \boxed{A_\mu \rightarrow A_\mu + \frac{1}{e} \partial_\mu \alpha(x)}$$

$$Q A_\mu \rightarrow Q A_\mu + \frac{1}{e} \partial_\mu (Q \alpha(x))$$

$$Q A_\mu \rightarrow U Q A_\mu U^\dagger - \frac{1}{e} (\partial_\mu U) U^\dagger$$

$$\partial_\mu U = i e Q \partial_\mu \alpha$$

EW gauge theory \Leftrightarrow

3 gauge bosons W^+ , W^- , A

Q. What is the minimal
gauge group?

1954 Mills, Yang
Show

Yang - Mills or non-Abelian
gauge theory

- $\psi \rightarrow U \psi$ $U U^\dagger = U^\dagger U = 1$
 $U = e^{i\theta}$ $\det U = 1$

$$\Rightarrow \bar{W} \psi = 0$$

$$\Rightarrow H = \theta_a T_a \quad a=1, \dots, N^2-1$$

$SU(N)$

$$[T_a, T_b] = i f_{abc} T_c$$

$$\left(SU(2) : f_{abc} = \epsilon_{abc} \right)$$

$$\bullet \mathcal{L}_0 = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi$$

$$\psi \rightarrow e^{i \theta_a(x) T_a} \psi$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ig T_a A_\mu^a$$

$$T_a A_\mu^a \rightarrow \left[U T_a A_\mu^a U^\dagger - \frac{i}{g} (\partial_\mu U) U^\dagger \right]$$

Φ

$$- \frac{1}{g} i \partial_\mu \theta_a T_a$$

$$\Rightarrow \begin{matrix} \text{(gauge)} \\ \text{(local)} \end{matrix} \left[A_\mu^a \rightarrow A_\mu^a + \frac{1}{g} \partial_\mu \theta^a \right]$$

the same as in

$U(1)$!

↓ Global

$$T_a A_\mu^a \rightarrow U T_a A_\mu^a U^\dagger \quad (\text{adjoint})$$

$$= (1 + i\theta_b T_b) T_c A_\mu^c (1 - i\theta_b T_b)$$

$$= T_c A_\mu^c + i\theta_b [T_b, T_c] A_\mu^c$$

$$= T_a A_\mu^a + i\theta_b i f_{bca} T_a A_\mu^c$$

$$= T_a (A_\mu^a - f_{abc} \theta_b A_\mu^c)$$

$$A_\mu^a \rightarrow A_\mu^a - f_{abc} \theta_b A_\mu^c$$

$$SU(2): \quad f_{abc} = \epsilon_{abc}$$

$$A_\mu^a \rightarrow A_\mu^a - \epsilon_{abc} \theta_b A_\mu^c$$

(vector)

vector of $SO(3)$

$$\vec{V} \rightarrow \vec{V} - \vec{\epsilon} \times \vec{V}$$

Schwartz + Peskin

'57-'60



$SU(2) =$ minimal en theory

Int

$$i \bar{\psi} \gamma^\mu D_\mu \psi \rightarrow i \bar{\psi} \gamma^\mu (-i g T_a A_\mu^a) \psi$$

$$= g \bar{\psi} T_a A_\mu^a \psi$$

$$T_a = \frac{\sigma_a}{2}$$

$$= g \bar{\psi} i T_3 \psi A_\mu^3 \rightarrow \text{leone} +$$

$$+ \frac{1}{2} g \bar{\psi} \begin{pmatrix} 0 & A_\mu^1 - i A_\mu^2 \\ A_\mu^1 + i A_\mu^2 & 0 \end{pmatrix} \gamma^\mu \psi$$

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$



$$= \frac{1}{2} g \bar{u} \gamma^\mu (A_\mu^1 - i A_\mu^2) d + \text{h.c.}$$

$$W_\mu^+ = \frac{A_\mu^1 - i A_\mu^2}{\sqrt{2}}$$

$$W_\mu^- = \frac{A_\mu^1 + i A_\mu^2}{\sqrt{2}}$$

$$\frac{g}{\sqrt{2}} W_\mu^+ J_\mu^+ + \text{h.c.}$$

$$J_\mu^+ = \bar{u} \gamma_\mu d$$

this could work!

$$+ g \bar{\psi} \gamma^\mu T_3 \psi A_\mu^3$$



photon

$$A_\mu^3 = A_\mu \Rightarrow \boxed{g = e}$$

but :

$$\boxed{Q_{em} = T_3}$$

$$Q_{em} = \pm 1/2$$

SU(2) $[T_a, T_b] = i \epsilon_{abc} T_c$

Lie Algebra



$$t_3 = n \cdot \frac{1}{2}$$

"Spin" 1 : $t_3 = (1, 0, -1)$

"Spin" $\frac{3}{2}$: $t_3 = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$

charge is quantized

$SU(2) \longrightarrow$
 T_a
 $Y/2$
 $SU(2) \times U(1)$
 \mathfrak{g}
 \mathfrak{g}'
 minimal
 extension

~~$E = ma^2$~~
 ~~$E = mv^2$~~
 $E = mc^2$

$Q_{em} = \text{diag} (\mathcal{Q}_1, \mathcal{Q}_2, \dots)$

$Y/2 = \text{diag} (Y_1, Y_2, \dots)$

$$T_3 = \sigma_{3/2} = \text{diag}$$



$$Q_{em} = T_3 + \frac{Y}{2}$$



$$Y = 2 [Q - T_3]$$



known

from nature



known

from SU(2)

Building the SM

Parsons 1961

$$\textcircled{1} \quad G_{SM} = SU(2)_L \times U(1)_Y$$

$$\textcircled{2} \quad \text{matter} = (2, \ell)$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$u_R, d_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$e_R, \cancel{\nu_R}$$

$$\Downarrow \quad \left[\psi \rightarrow U\psi = e^{i\theta_a T_a} \psi \right] \quad \Downarrow$$

$$T_a = \sigma_a / 2$$

$$T_a = 0$$



$$\begin{pmatrix} u \\ d \end{pmatrix}_L \xrightarrow{\text{doublet}} e^{i\theta_a \sigma_a} \begin{pmatrix} u \\ d \end{pmatrix}$$

$$u_R \xrightarrow{\text{singlet}} u_R = e^{i\theta_a \cdot 0} u_R$$

singlet

$$\not\partial_\mu \psi = D_\mu \psi = \not\partial_\mu \psi - ig \overset{SU(2)}{T_a} A_\mu^a - ig' \overset{U(1)}{\frac{Y}{2}} B$$

$$T_a = \frac{\sigma_a}{2} \leftrightarrow \psi_L$$

$$T_a = 0 \leftrightarrow \psi_R$$

$$Y = ?$$

$$u_R: Y = 2 [a - T_3] = 2 Q_u = \frac{4}{3}$$

$$d_R: Y = -11 = 2 Q_d = -\frac{2}{3}$$

$$e_R: Y = \dots = -2$$

$$u_L: Y = 2 \left[\frac{2}{3} - \frac{1}{2} \right] = \frac{1}{3}$$

$$T_3 = \frac{G_3}{Z} = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

$$d_L: Y = 2 \left[-\frac{1}{3} + \frac{1}{2} \right] = \frac{1}{3}$$

$$G_{SM} = SU(2)_L \times U(1)_Y$$

$$[T_a, \frac{Y}{2}] = 0$$



Y (doublet) = fixed



$$Y_u = Y_d$$

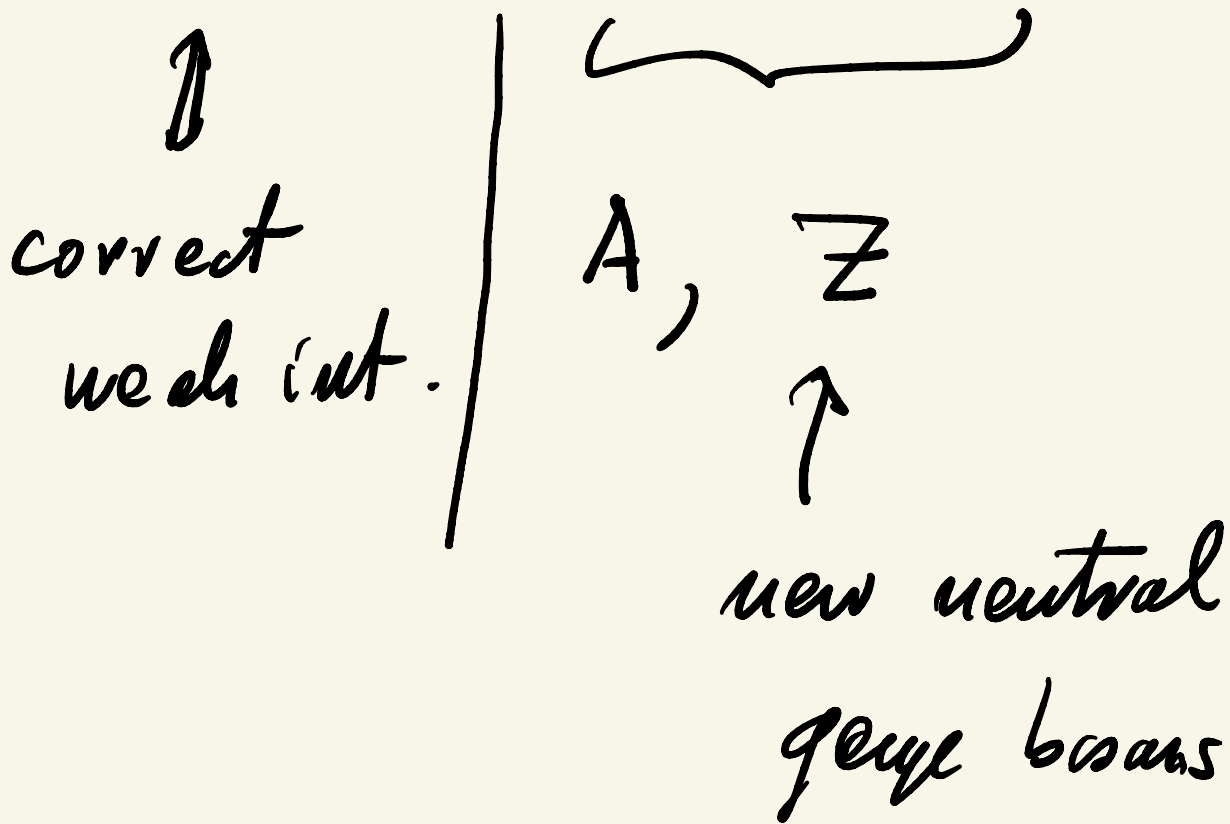
$$\Rightarrow Y_{\nu_L} = Y_{e_L} = -1 \quad \checkmark$$

$SU(2) \times U(1)$

$$(A_\mu^a, B) = 4 \text{ g.b.}$$



$$[W^+, W^-] \quad A_3, B$$



\Downarrow neutral int.

$$D_\mu = -ig T_3 A_\mu^3 - ig' (Q - T_3) B$$

\Downarrow

$$\mathcal{L}_{\text{int}}^{\text{neutral}} = \bar{\psi} \gamma^\mu \left[\underbrace{(g A_\mu^3 - g' B)_\mu}_{Z} T_3 + g' Q B_\mu \right] \psi$$

if Z photon $(A) \leftrightarrow Q$

$\Rightarrow Z \leftrightarrow ?$



$$Z = \frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}} \perp A$$



$$A = \frac{g' A_3 + g B}{\sqrt{g^2 + g'^2}}$$

$$\tan \theta_w = g'/g$$



$$\sin \theta_w = \frac{g'}{\sqrt{g'^2 + g^2}}, \quad \cos \theta_w = \frac{g}{\sqrt{g'^2 + g^2}}$$



$$A = \sin \theta_w A_3 + \cos \theta_w B \quad (u_A = 0)$$

$$Z = \cos \theta_w A_3 - \sin \theta_w B \quad (u_Z \approx 100 \text{ GeV})$$

$$(E \gg m_t \Leftrightarrow m_t \rightarrow 0)$$

$$A_3 = \sin \theta_w A + \cos \theta_w B$$

$$B = \cos \theta_w A - \sin \theta_w Z$$



$$\begin{aligned}
 \mathcal{L}_{\text{int}}^{\text{neutral}} &= \bar{\Psi} \left[\sqrt{g^2 + g'^2} \frac{T_3}{3} \gamma_\mu + g' Q (c_{\text{new}} A_\mu - s_{\text{new}} Z_\mu) \right] \gamma^\mu \Psi \\
 &= \bar{\Psi} \left[\left(\frac{g}{c_{\text{new}}} \frac{T_3}{3} - g' s_{\text{new}} Q \right) Z^\mu + e Q A^\mu \right] \gamma_\mu \Psi
 \end{aligned}$$

$$\boxed{e = g' c_{\text{new}} = g s_{\text{new}}}$$

$$(e < g)$$

$$\mathcal{L}_{\text{int}}^{\text{neutral}} = e \bar{\Psi} Q \gamma^\mu \Psi A_\mu \quad (\text{em})$$

$$+ \frac{g}{c_{\theta w}} \bar{\psi} Q_z \gamma^\mu \psi z_\mu \quad (\text{weak})$$



$$Q_z = T_3 - Q \sin^2 \theta_w$$



LH fermions only

$$T_3 \psi_L = \pm \psi_L$$

$$T_3 \psi_R = 0$$

$$\theta_w \approx 30^\circ$$

$$\sin^2 \theta_w = 0.23$$