Neutrino Physics Course

Lecture IV

5/15/2023

LMU

Spring 2023

From $\cancel{P}$ $\longrightarrow$ $V-A$ theory

$$\downarrow$$

Standard Model (SM)

$$W^- \longrightarrow e + \bar{\nu}_e$$

$$\Uparrow \qquad \Uparrow (e_L)$$

$$S_z = +1 \qquad \Uparrow (\bar{\nu}_e)_R \Leftrightarrow \nu_L$$

$$\Downarrow$$

$$Co^{60} \longrightarrow Ni^{60} + e + \bar{\nu}_e$$

$\Downarrow$ $\not{P}$ maximum

$$\mathcal{H}_{eff} = \frac{4 G_F}{4\sqrt{2}} J_\mu^W \bar{J}^\mu_W$$

$$J_\mu^W = \bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L$$

$\Updownarrow$

1957
Marshak, Sudarshan

$$\left(\frac{g}{\sqrt{2}} W_\mu^+ J_\mu^W + h.c \right)$$

$\boxed{\text{"V – A" theory}}$

$$L = \frac{1+\gamma_5}{2} \quad \text{(own)}$$

2009 Weinberg

"V-A was the key"

no coupling to ν_μ!

$$H_{eff} = + \; G_F' \left[\bar{\nu} (a+b\gamma_5) e + \cdots \right]^2$$

$$+ \; G_F'' \left[\bar{\nu} \gamma^\mu \gamma^\nu (a+b\gamma_5) e + \cdots \right]^2$$

1957 Schwinger

(Glashow)

gauge theory of ew (=

= em + weak) interactions

U(1) QED

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

$$\psi \to e^{i\alpha Q}\psi \qquad (Q\psi = q\psi)$$

if $\alpha = \alpha(x) \Rightarrow$ $\boxed{U \equiv e^{i\alpha Q}}$

$$D_\mu = \partial_\mu - ieQA_\mu$$

$$(D_\mu \psi) \to e^{i\alpha Q} (D_\mu \psi)$$

$$\therefore \boxed{A_\mu \to A_\mu + \frac{1}{e} \partial_\mu \alpha(x)}$$

$$Q A_\mu \to Q A_\mu + \frac{1}{e} \partial_\mu (Q \alpha(x))$$

$$\boxed{Q A_\mu \to U Q A_\mu U^{+} - \frac{1}{e} (\partial_\mu U) U^{+}}$$

$$\partial_\mu U = i e Q \partial_\mu (\alpha)$$

ew gauge theory $\Longleftrightarrow$

3 gauge bosons W^+, W^-, A

Q. What is the minimal gauge group?

1954 Mills, Yang

Shaw

Yang - Mills or non-Abelian gauge theory

- $\psi \longrightarrow U\psi$ $\qquad UU^{+} = U^{+}U = 1$

$U = e^{iH}$ $\qquad \det U = 1$

$$\Rightarrow \operatorname{tr} H = 0$$

$$\Rightarrow H = \theta_a T_a \qquad \xleftarrow{\;SU(N)\;} a = 1, \dots, N^2 - 1$$

$$[T_a, T_b] = i f_{abc} T_c$$

$$\left(SU(2) \; : \; f_{abc} = \varepsilon_{abc} \right)$$

- $\mathcal{L}_D = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi$

$$\psi \to e^{i \theta_a(x) T_a} \psi$$

$$\partial_\mu \to D_\mu = \partial_\mu - i g T_a A_\mu^a$$

$$T_a A_\mu^a \rightarrow \boxed{U T_a A_\mu^a U^+} - \frac{i}{g}(\partial_\mu U) U^+$$

$$- \frac{1}{g} i \, \partial_\mu \theta_a T_a$$

$\Rightarrow$ (gauge) (local) $\boxed{A_\mu^a \rightarrow A_\mu^a + \frac{1}{g} \partial_\mu \theta^a}$

the same as in

U(1) !

$\downarrow$ global

$$T_a A_\mu^a \to U T_a A_\mu^a U^+ \quad \text{(adjoint)}$$

$$= (1 + i\theta_b T_b)\, T_c A_\mu^c\, (1 - i\theta_b A_\mu^b)$$

$$= T_c A_\mu^c + i\, \theta_b\, [T_b, T_c]\, A_\mu^c$$

$$= T_a A_\mu^a + i\, \theta_b\, i f_{bca} T_a A_\mu^c$$

$$= T_a \left(A_\mu^a - f_{abc}\, \theta_b\, A_\mu^c \right)$$

$$A_\mu^a \to A_\mu^a - f_{abc}\, \theta_b\, A_\mu^c$$

$SU(2)$: $\quad f_{abc} = \varepsilon_{abc}$

$$\boxed{A_\mu^a \to A_\mu^a - \varepsilon_{abc}\,\theta_b\,A_\mu^c}$$

(vector)

vector of SO(3)

$$\boxed{\vec{V} \to \vec{V} - \vec{\varepsilon} \times \vec{V}}$$

Schwinger + Glashow $\boxed{\text{'57 – '60}}$

$\Downarrow$

SU(2) = minimal ew theory

Int

$$i\bar{\psi}\gamma^\mu D_\mu \psi \rightarrow i\bar{\psi}\gamma^\mu(-igT_a A_\mu^a)\psi$$

$$= g\bar{\psi}T_a A_\mu^a \psi$$

$$T_a = \frac{\sigma_a}{2}$$

$$= g\bar{\psi}\gamma^\mu T_3 \psi A_\mu^3 \quad + \qquad \text{leave}$$

$$+\frac{1}{2}g\bar{\psi}\begin{pmatrix} 0 & A_\mu^1 - iA_\mu^2 \\ A_\mu^1 + iA_\mu^2 & 0 \end{pmatrix}\gamma^\mu\psi$$

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$= \frac{1}{2} g \; \bar{u} \gamma^{\mu} \left(A_\mu^1 - i A_\mu^2 \right) d \; + h.c.$$

$$\begin{cases} W_\mu^+ = \dfrac{A_\mu^1 - i A_\mu^2}{\sqrt{2}} \\[2em] W_\mu^- = \dfrac{A_\mu^1 + i A_\mu^2}{\sqrt{2}} \end{cases}$$

$$\frac{g}{\sqrt{2}} W_\mu^+ J_W^\mu \; + h.c.$$

$$J_\mu^W = \bar{u} \gamma_\mu d$$

this could work!

$$+ \; g \, \bar{\psi} \gamma^\mu T_3 \psi A_\mu^3$$

photon

$$A_\mu^3 = A_\mu \Rightarrow \boxed{g = e}$$

but : $\boxed{Q_{em} = T_3}$

$$q_{em} = \pm 1/2$$

$\underline{SU(2)}$ $\underbrace{[T_a, T_b] = i\,\varepsilon_{abc}\,T_c}$

Lie Algebra

$\Downarrow$

$$\boxed{t_3 = n \cdot \frac{1}{2}}$$

$\underline{\text{"spin" } 1}$: $t_3 = (1, 0, -1)$

"spin" $3/2$: $t_3 = 3/2, 1/2, -1/2, -3/2$

- - - -

$\boxed{\text{charge is quantised}}$

$$T_a \qquad Y/2$$

$$SU(2) \longrightarrow SU(2) \times U(1)$$

minimal $\quad g \qquad g'$

extension

$$E = \cancel{ma^2}$$

$$E = \cancel{mb^2}$$

$$E = mc^2$$

$$Q_{em} = \mathrm{diag}\,(q_1, q_2, --)$$

$$\frac{Y}{2} = \mathrm{diag}\,(y_1, y_2, --)$$

$$T_3 = \sigma_3/2 = \text{diag}$$

$$\Downarrow$$

$$Q_{em} = T_3 + \frac{Y}{2}$$

$$\Updownarrow$$

$$Y = 2\left[Q - T_3\right]$$

know from nature

know from $SU(2)$

$$\boxed{\text{Building the SM}}$$

Glashow 1961

1. $G_{SM} = SU(2)_L \times U_Y(1)$

2. matter $= (q, \ell)$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R, d_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R, \cancel{\nu_R}$$

$$\psi \to U\psi = e^{i\theta_a T_a} \psi$$

$\Downarrow$ $\qquad\qquad\qquad\qquad$ $\Downarrow$

$$T_a = \sigma_a/2 \qquad\qquad T_a = 0$$

$\Downarrow$

doublet

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \longrightarrow e^{i\theta_a \sigma_{a/2}} \begin{pmatrix} u \\ d \end{pmatrix}$$

$$u_R \longrightarrow u_R = e^{i\theta_a \cdot 0} u_R$$

singlet

$$\partial_\mu \to D_\mu = \partial_\mu - ig \overbrace{T_a A_\mu^a}^{SU(2)} - ig' \overbrace{\tfrac{Y}{2} B}^{U(1)}$$

$$T_a = \frac{\sigma_a}{2} \longleftrightarrow \psi_L$$

$$T_a = 0 \longleftrightarrow \psi_R$$

$$Y = ?$$

u_R : $Y = 2\left[Q - T_3\right] = 2\,Q_u = 4/3$

d_R : $Y =$ —"— $= 2\,Q_d = -2/3$

e_R : $Y =$ - - - - $= -2$

- - - - - -

u_L : $Y = 2\left[\frac{2}{3} - \frac{1}{2}\right] = 1/3$

$$T_3 = \frac{\sigma_3}{2} = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

d_L : $Y = 2\left[-1/3 + \frac{1}{2}\right] = 1/3$

$$G_{SM} = SU(2)_L \times U(1)_Y$$

$$\left[T_a, \frac{Y}{2}\right] = 0$$

$\Downarrow$

$$\boxed{\begin{gathered} Y\,(\text{dublet}) = \text{fixed} \\ \Updownarrow \\ Y_u = Y_d \end{gathered}}$$

$\Rightarrow\ Y_{\nu_L} = Y_{e_L} = -1$ ✓

$SU(2) \times U(1)$

$(A^a_\mu\ ,\ B) = 4$ g.b.

$\downarrow$

$\boxed{W^+, W^-}\ \ A_3\ ,\ B$

$\Updownarrow$

correct weak int. | A, Z

new neutral gauge bosons

$\Downarrow$ neutral int.

$$D_\mu = -ig\, T_3 A^3_\mu - ig'(Q - T_3) B$$

$$\Downarrow$$

$$\mathcal{L}^{\text{neutral}}_{\text{int}} = \bar{\psi}\gamma^\mu \Big[\underbrace{(g A^3_\mu - g' B_\mu)}_{Z} T_3 + g' Q B_\mu \Big] \psi$$

if $\exists$ photon $(A) \longleftrightarrow Q$

$\Rightarrow \exists \; Z \longleftrightarrow \, ?$

$$\Downarrow$$

$$Z = \frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}} \perp A$$

$$\swarrow$$

$$A = \frac{g' A_3 + g B}{\sqrt{g^2 + g'^2}}$$

$$\tan \theta_W = g'/g$$

$$\Downarrow$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}, \quad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\Downarrow$$

$$A = \sin\theta_W A_3 + \cos\theta_W B \qquad (m_A = 0)$$

$$Z = \cos\theta_W A_3 - \sin\theta_W B \qquad (m_Z \simeq 100\,\mathrm{GeV})$$

$$\left(E \gg m_Z \iff m_Z \to 0 \right)$$

$$A_3 = \sin\theta_W A + \cos\theta_W B$$

$$B = \cos\theta_W A - \sin\theta_W Z$$

$$\Downarrow$$

$$\mathcal{L}_{int}^{neutral} = \bar{\psi}\left[\sqrt{g^2+g'^2}\, T_3 Z_\mu + g' Q \left(\cos\theta_W A_\mu - \sin\theta_W Z_\mu\right)\right]\gamma^\mu \psi$$

$$= \bar{\psi}\left[\left(\frac{g}{\cos\theta_W} T_3 - g' \sin\theta_W\, Q\right) Z^\mu + e Q A^\mu \right]\gamma_\mu \psi$$

$$\boxed{e = g' \cos\theta_W = g \sin\theta_W}$$

$$(e < g)$$

$$\mathcal{L}_{int}^{neutral} = e \bar{\psi} Q \gamma^\mu \psi A_\mu \quad (em)$$

$$+\frac{g}{\cos\theta_W}\bar{\psi}\, Q_Z \gamma^\mu \psi Z_\mu \quad \text{(weak)}$$

$\Downarrow$

$$\boxed{Q_Z = T_3 - Q\sin^2\theta_W}$$

$\downarrow$

LH fermions only

$$T_3 \psi_L = \pm \psi_L$$

$$T_3 \psi_R = 0$$

$$\theta_W \simeq 30^\circ$$

$$\sin^2\theta_W = 0.23$$