Neutrino Physics Course

Lecture III

2/5/2023

LMU
Spring 2023

Majorana mass for neutrino?

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}, \quad \psi_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$u_{L,R} \to e^{i \vec{\sigma}/2 \, (\underset{\substack{\uparrow \\ \text{ROT}}}{\vec{\theta}} \pm i \underset{\substack{\uparrow \\ \text{BOOST}}}{\vec{\varphi}})} u_{L,R}$$

- $m_D \bar{\psi} \psi = m_D (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$

$$= m_D (u_L^\dagger u_R + u_R^\dagger u_L)$$

m_D = Dirac mass term

$\bullet\ m_M \left(u_L^T\, i\sigma_2\, u_L + h.c. \right) \frac{1}{2}$

Majorana 1937

see below

$$u_L \to e^{i\vec{\sigma}/2 \cdot \vec{\chi}}\, u_L \,, \qquad \vec{\chi} = \vec{\theta} + i\vec{\varphi}$$

$$\Rightarrow u_L^T i\sigma_2 u_L \to u_L^T e^{i\vec{\sigma}^T/2 \cdot \vec{\chi}} (i\sigma_2) e^{i\vec{\sigma}/2\, \vec{\chi}}\, u_L$$

$$= u_L^T i\sigma_2 \underbrace{e^{-i\vec{\sigma}/2 \cdot \vec{\chi}}\, e^{i\vec{\sigma}/2\, \vec{\chi}}}_{1}\, u_L$$

$$= u_L^T i\sigma_2 u_L \qquad \text{Q.E.D.}$$

$$\bullet\ u_L^T i\sigma_2 u_L = \boxed{\psi_L^T C\, \psi_L} =$$

$$= \left(u_L^T\ 0\right) i\gamma^2\gamma^0 \begin{pmatrix} u_L \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} u_L{}^T & 0 \end{pmatrix} \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} \begin{pmatrix} u_L \\ 0 \end{pmatrix}$$

$$\Downarrow$$

Q.E.D.

$$\psi_L{}^T C \psi_L = \text{Inv.} \iff \psi^c = C \bar{\psi}^T$$

is a spinor

<u>Proof:</u>

$$\psi = \psi_L \Rightarrow \psi^c = i\,\sigma_2 \gamma_0 \gamma_0\, \psi_L{}^*$$

$$= \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} u_L{}^* \\ 0 \end{pmatrix}$$

$$(\psi^c)_R = \begin{pmatrix} 0 \\ -i\sigma_2 u_L{}^* \end{pmatrix} \equiv C \bar{\psi}_L{}^T$$

$$\Downarrow$$

$$\overline{(\psi^c)_R}\ \psi_L = \text{inv.} \quad (*)$$

$$\underbrace{\overline{(\psi^c)_R} \uparrow \text{spinor} \quad \psi_L \uparrow \text{spinor}}_{\text{Lorentz}}$$

$$\underset{\text{inv.}}{\overline{\psi}\psi} \longrightarrow \overline{\psi_1}\,\psi_2 = \text{inv.}$$

$$(*)\ \overline{(\psi^c)_R}\ \psi_L = \overline{C\,\overline{\psi_L}^T}\ \psi_L =$$

$$= (C\gamma_0\psi_L^*)^+\gamma^0\,\psi_L$$

$$= \psi_L^T\,\gamma_0\,C^+\gamma^0\,\psi_L = \{C^+ = -C\}$$

$$= -\psi_L^T \gamma_0 C \gamma_0 \psi_L = \psi_L^T C \psi_L$$

$\boxed{\text{Q. E. D.}}$

$$m_M \underbrace{\psi_L^T C \psi_L}$$

breaks any global symmetry

$$\psi_L \to e^{i\alpha} \psi_L \quad :$$

$$\psi_L^T C \psi_L \to e^{2i\alpha} \psi_L^T C \psi_L$$

· electron $\quad \psi = e$

$$m_D \bar{e}_L e_R + \cancel{m_M^L e_L^T C e_L} + \cancel{m_M^R e_R^T C e_R}$$

$$m_M^L / m_D \leq 10^{-20}$$

$$m_M^R / m_D \leq 10^{-20}$$

- <u>neutrino</u> $\quad Q_\nu^{em} = 0$

$$\left(Q_\nu^{em} \leq 10^{-20} \right)$$

Majorana $\Longleftrightarrow$

<u>Lepton number (L)</u>

$$n \longrightarrow p + e + \bar{\nu}$$

$$\Rightarrow \quad L(\nu) = L(e) = 1$$

neutrino is a lepton

$$\nu \to e^{i\beta L}\nu = e^{i\beta}\nu$$

$$e \to e^{i\beta L} e = e^{i\beta} e$$

$$\boxed{\therefore \ \psi_L^T C \psi_L \Rightarrow \Delta L = 2}$$

Racah 1937, Furry 1938 } neutrino-less double beta decay

$$n + n \to e + e + p + p$$

$$\left(n + n \to p + p + e + e + \bar{\nu}_e + \bar{\nu}_e \right.$$

$$\left. \text{duble beta decay} \right)$$

<u>Dirac</u>

$$\Psi_D = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \quad \text{Independent}$$

$$\Leftrightarrow \quad \Psi = \underbrace{4}_{\text{Complex}} \text{ components}$$

<u>Mejorana</u>

$$\Psi_M = \Psi_L + C \, \bar{\Psi}_L^{\,T}$$

$$= \begin{pmatrix} u_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -i\sigma_2 \, u_L^* \end{pmatrix}$$

$$= \begin{pmatrix} u_L \\ -i\sigma_2 u_L^* \end{pmatrix}$$

$\Leftrightarrow \quad \psi_M = 2$ complex comp.

$$\bullet \; \overline{\psi_M}\,\psi_M = \left(\overline{\psi_L} + \overline{C\,\overline{\psi_L}^T}\right)\left(\psi_L + C\,\overline{\psi_L}^T\right)$$

$$= \cancel{\overline{\psi_L}\,\psi_L} + \overline{\psi_L}\,C\,\overline{\psi_L}^T +$$

$$+ \overline{C\,\overline{\psi_L}^T}\,\psi_L + \cancel{\overline{C\,\overline{\psi_L}^T}\,C\,\overline{\psi_R}^T} -$$

$$\downarrow$$

$$= \left(C\gamma_0\psi_L^*\right)^\dagger \gamma^0\,\psi_L + \overline{\psi_L}\,C\,\overline{\psi_L}^T$$

$$= \psi_L^T\,\gamma_0\,C^\dagger\gamma^0\psi_L + \psi_L^\dagger\gamma^0 C\left(\psi_L^\dagger\gamma^0\right)^T$$

$$= \psi_L^T(-C^\dagger)\psi_L + \psi_L^\dagger \gamma^0 C \gamma_0 \psi_L^*$$

$$= \underbrace{\psi_L^T C \psi_L}_{\text{Majorana}} + \underbrace{\psi_L^\dagger C^\dagger \psi_L^*}_{\text{h.c.}}$$

• $$i\bar{\psi}_M \gamma^\mu \partial_\mu \psi_M = i\left(\overline{\psi_L + C\bar{\psi}_L^T}\right)\gamma^\mu \partial_\mu \left(\psi_L + C\bar{\psi}_L^T\right)$$

$$= i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \overline{C\bar{\psi}_L^T}\gamma^\mu \partial_\mu C\bar{\psi}_L^T$$

$$\parallel$$

$$\text{--}\ \partial_\mu(\text{---})\ -$$

$$-\partial_\mu \overline{C\bar{\psi}_L^T}\gamma^\mu C\bar{\psi}_L^T$$

$$\parallel$$

$$= i\,\overline{\psi_L}\,\gamma^\mu\partial_\mu\psi_L + i\,\overline{\psi_L}\,\gamma^\mu\partial_\mu\psi_L$$

$$= 2i\,\overline{\psi_L}\,\gamma^\mu\partial_\mu\psi_L$$

$$\Downarrow$$

$$\mathcal{L}_M = i\,\overline{\psi_M}\,\gamma^\nu\partial_\nu\psi_M - \underbrace{\overline{\psi_M}\,\psi_M\, m_M}_{\text{mass term}} \qquad (1)$$

$$\left(E^2 = \bar{p}^2 + m_M^2\right)$$

$$= 2i\,\overline{\psi_L}\,\gamma^\mu\partial_\mu\psi_L - m_M\left(\psi_L^T C\,\psi_L + h.c.\right)$$

$$= 2\left[i\,\overline{\psi_L}\,\partial^\mu\partial_\mu\psi_L - \frac{m_M}{2}\left(\psi_L^T C\,\psi_L + h.c.\right)\right]$$

(2)

$\bar{\psi}_M \psi_M$;. $\psi_M \longrightarrow e^{i\alpha} \bar{\psi}_M$???

$$\psi_M = \psi_L + C\bar{\psi}_L^T = \psi_L + C\gamma_0 \psi_L^*$$

$$e^{i\alpha}\psi_L \qquad\qquad e^{-i\alpha}\ldots$$

50 % + 50 %

particle anti-particle

Rise and fall of parity

$$\mathcal{L}_{QED} = i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi$$

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu} \quad -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad (3)$$

$$D_\mu = \partial_\mu - ie\, Q^{em} A_\mu$$

photon

$$Q_{em} = \text{diag}\,(q_1, q_2, \ldots)$$

$$q_e = -1, \quad \underbrace{q_p = +1, \; q_n = 0}_{q_u = 2/3, \; q_d = -1/3}$$

$$|q_p|/|q_e| = 1 + \cancel{O(10^{-20})}$$

$$= 1$$

$$\boxed{q \propto n\, q_d}$$

charge quantization

Dirac if $\exists$ magnetic monopole

$$\Rightarrow \boxed{q_e \cdot q_m = 2\pi n}$$

---

eq. (31) $\psi \to e^{i\alpha(x) Q}\, \psi$

is inv.

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

$$\mathcal{L}_{int} = e\bar{\psi}\, Q\, \gamma^\mu \psi\, A_\mu$$

P: $A_i \xrightarrow{P} -A_i$, $A_0 \xrightarrow{P} A_0$

$$\psi \xrightarrow{P} \gamma^0 \psi$$

Proof: $\gamma^0 \gamma^i \gamma^0 = -\gamma^i$

$$\gamma^0 \gamma^0 \gamma^0 = \gamma^0 \qquad \text{Q.E.D.}$$

$$\begin{pmatrix} u_L \\ u_R \end{pmatrix} \xrightarrow{P} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

$$= \begin{pmatrix} u_R \\ u_L \end{pmatrix}$$

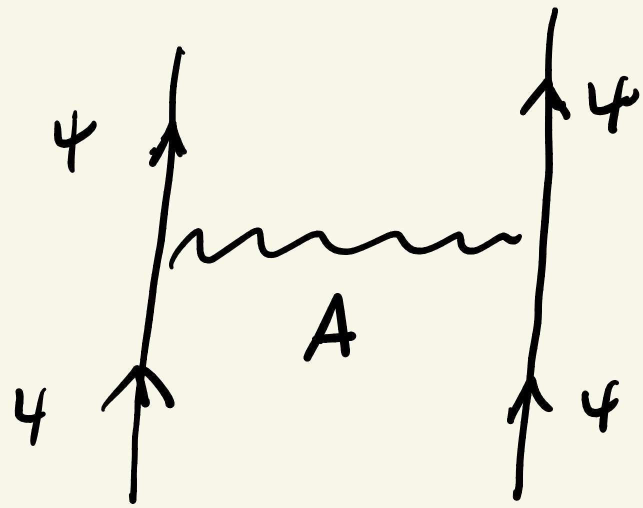
$$\boxed{u_L \overset{P}{\longleftrightarrow} u_R}$$

$$\mathcal{H}_{eff}^{em} \propto J_\mu^{em} J^\mu_{em}$$

$$J^\mu_{em} = \bar{\psi} \gamma^\mu Q_{em} \psi$$

$$\Leftrightarrow \quad \mathcal{L}_{fund} = e\, J_\mu^{em} A_\mu \quad \text{(photon)}$$

messenger



Fermi 1934

$$H_{eff}^{weak} \propto J^\mu_w \, J_\mu^{\bar{w}}$$

$$\mathcal{L}_{fund} = \frac{g}{\sqrt{2}} J^\mu_w \, W_\mu + h.c.$$

What is the form of J^μ_W?

$$J^\mu_W = \underbrace{\bar{u}\gamma^\mu d + \bar{e}\gamma^\mu \nu}_{\text{form?}}$$

$\bar{u}_L \gamma^\mu d_L$?

$\bar{u}_R \gamma^\mu d_R$?

$\bar{u}_L \gamma^\mu d_L + \bar{u}_R \gamma^\mu d_R$?

QED: $\bar{e}\gamma^\mu e = \bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R$

P

1956 Lee + Yang

Summer

1956 Wu et. al

Christmass

$$\Downarrow$$

$$\boxed{\cancel{P} \text{ maximal}}$$

Parity!

$$u_L \longleftrightarrow u_R$$

$$\psi_L \equiv L\psi \equiv \frac{1+\gamma_5}{2}\psi = \begin{pmatrix} u_L \\ 0 \end{pmatrix}$$

$$m_\psi = 0 \quad \Rightarrow \quad i\gamma^\mu \partial_\mu \psi = 0$$

$$\psi(x) = \tilde{\psi}(p)\, e^{ipx}$$

$$\Rightarrow \boxed{p_\mu \gamma^\mu \, \tilde{\psi}(p) = 0}$$

$$\Downarrow$$

$$\begin{pmatrix} 0 & E - \vec{p}\cdot\vec{\sigma} \\ E + \vec{p}\cdot\vec{\sigma} & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix} = 0$$

$$\Rightarrow \frac{1}{2} E\, u_{L,R} = \mp\, \vec{p}\cdot\vec{\sigma} \frac{1}{2} u_{L,R}$$

$$\Rightarrow \frac{1}{2} u_{L,R} = \mp \frac{\vec{p}\cdot\vec{\sigma}/2}{E} u_{L,R}$$

$$|\vec{p}| = E \qquad (m=0)$$

$$\vec{S} = \vec{\sigma}/2$$

$$\Rightarrow \boxed{\begin{array}{l} h\, u_{L/R} = \mp \frac{1}{2} u_{L/R} \\ h \equiv \vec{S} \cdot \hat{p} \end{array}}$$

$$Co^{60} \longrightarrow Ni^{60} + e + \bar{\nu}$$

$\Uparrow$ $\qquad\qquad$ $\Uparrow$

$J=5$ $\qquad\qquad$ $J=4$

$$W^- \longrightarrow e + \bar{\nu}_e \; (\nu^c)$$

$\Uparrow$

$S_z = +1$

$$M_W = 80 \text{ GeV} = 80\, m_p$$

$$m_e = 0 = m_\nu$$

$$\cancel{e_R \quad \Uparrow \quad (1/2)} \qquad\qquad e_L \quad \Uparrow \quad (1/2)$$

$$+ \qquad\qquad\qquad\qquad +$$

$$\cancel{(\overline{\nu_e})_L \quad \Uparrow \quad (1/2)} \qquad\qquad (\overline{\nu_e})_R \quad \Uparrow \quad (1/2)$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$1 \qquad\qquad\qquad\qquad 1$$

$$\Downarrow$$

$$\boxed{J^\mu_w = \overline{\nu_L}\, \gamma^\mu e_L + \overline{u_L}\, \gamma^\mu d_L}$$

$$\updownarrow \ P$$

$$\cancel{\overline{\nu_R}\, \gamma^\mu e_R + \overline{u_R}\, \gamma^\mu d_R}$$