LMU Neutrino Course

Lecture 11

28/4/2023

LMU

Spring 2023

Basics of spinors

$m_\nu \le eV$ (KATRIN)

$$O_p \quad O_n \quad e \;\Big|\; \nu$$

u, d quarks : $p = uud$

$n = udd$

matter: u, d, e = universe

ν_e

$\Big\}$ generation (I)

c, s, μ ; ν_μ (II)

t, b, τ ; ν_τ (III)

$$\boxed{\nu_e, \nu_\mu, \nu_\tau = \text{around}}$$

Ptolomey?

$$e \longleftrightarrow \nu_e$$

$$\Updownarrow$$

$$n \longrightarrow p + e + \bar{\nu}_e$$

$$\mu \longleftrightarrow \nu_\mu$$

$$\mu \longrightarrow e + \bar{\nu}_e + \nu_\mu$$

Pontecorvo

'50's

$$\boxed{m_\nu \neq 0}$$

$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$

$$\Downarrow$$

physical : m_1, m_2

$$\nu_\mu = -\underline{\sin\theta}\,\nu_1 + \cos\theta\,\nu_2$$

SUN: produce ν_e!

$$P(\nu_e \to \nu_\mu) = ?$$

$$P(\nu_e \to \nu_\mu) = \left| \langle \nu_\mu \,|\, \nu_e(t) \rangle \right|^2$$

$$\nu_e(t) = \cos\theta\,\nu_1 e^{iE_1 t} +$$

$$+ \sin\theta\,\nu_2\, e^{iE_2 t}$$

$$E_1 = \sqrt{p_1 + m_1^2} \triangleq \sqrt{p^2 + m_1^2}$$

$$E_2 = \sqrt{p_2 + m_2^2} \simeq \sqrt{p^2 + m_2^2}$$

$$E_i \simeq p + \frac{1}{2}\frac{m_i^2}{p} + \cdots$$

$$\nu_e(t) = e^{iE_1 t}\left(\underline{\cos\theta\,\nu_1} + \sin\theta\,\nu_2\,e^{i\Delta E t}\right)$$

$$\therefore \quad \Delta E = E_2 - E_1 = \frac{1}{2}\frac{\Delta m^2}{p}$$

$$\Rightarrow \langle \nu_\mu | \nu_e(t) \rangle = e^{iE_1 t}\cos\theta\sin\theta\left(-1 + e^{i\Delta E t}\right)$$

$$\Downarrow$$

$$P(\nu_e \to \nu_\mu) = \cos^2\theta \, \sin^2\theta \, \times$$

$$\left(-1 + e^{i\Delta E t}\right)\left(-1 + e^{-i\Delta E t}\right)$$

$$= \cos^2\theta \, \sin^2\theta \left(\underbrace{1 + 1 - 2\cos \Delta E t}\right)$$

$$= 2\cos^2\theta \, \sin^2\theta \cdot 2\sin^2 \frac{\Delta E t}{2}$$

$$\Uparrow$$

$$\boxed{1 - \cos\alpha = 2\sin^2\frac{\alpha}{2}}$$

$$\Downarrow \qquad (v \simeq c = 1)$$

$$P(\nu_e \to \nu_\mu) = 4\sin^2\theta\cos^2\theta\sin^2\frac{\Delta m^2 L}{4p}$$

$$(p \simeq E)$$

$$\Downarrow$$

$$\boxed{P(\nu_e \to \nu_\mu) \cong \sin^2 2\theta_\odot \sin^2\frac{\Delta m_\odot^2 L}{4E}}$$

$$P(\nu_\mu \to \nu_\tau) \cong \sin^2 2\theta_{atm} \sin^2\frac{\Delta m_{atm}^2 L}{4E}$$

$$\Downarrow$$

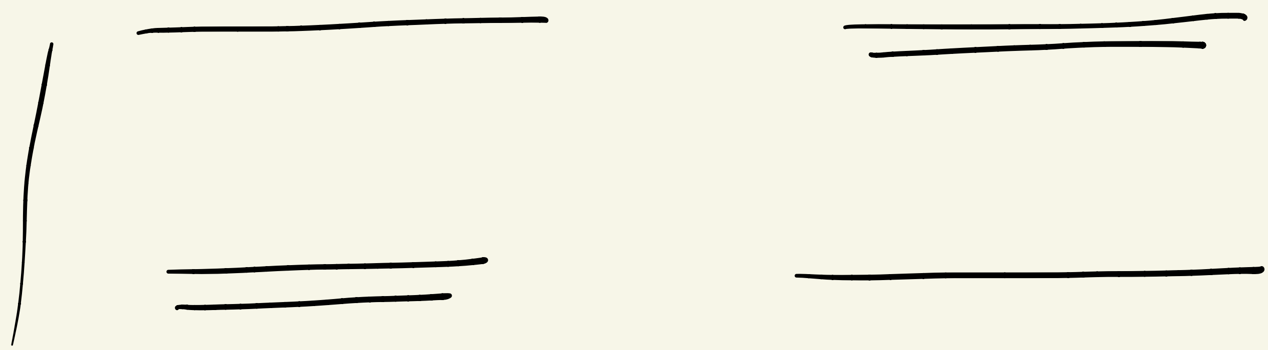
$$\Delta m_{atm}^2 \simeq 10^{-3}\,eV^2,\quad \theta_{atm} \simeq 45^0$$

$\Delta m^2_\odot \simeq 10^{-5}\, eV^2, \; \theta_\odot \simeq 30^\circ$

$$\boxed{m_\nu \gtrsim 1/30 \; eV}$$

$$\boxed{\begin{aligned} \theta^l_{12} &\simeq \theta_\odot \simeq 30^\circ \\ \theta^l_{23} &\simeq \theta_{atm} \simeq 45^\circ \\ \theta^l_{13} &\simeq 10^\circ \end{aligned}}$$

leptonic mixing



Normal hierarchy

Inverse hierarchy

or

degeneracy ?

$$\theta_{12}^{q} \simeq \theta_c \simeq 15^\circ$$

$$\theta_{23}^{q} \simeq 10^{-2}$$

$$\theta_{13}^{q} \simeq 10^{-3}$$

$$\Downarrow$$

$$\boxed{\theta^{q} \neq \theta^{\ell}}$$

Spinors

$$\boxed{\bar{\psi} = \psi^{+}\gamma^{0}}$$

$$\mathcal{L}_D = i\,\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\,\bar{\psi}\psi$$

- $\psi \to e^{i\alpha}\psi$, $\alpha = \text{cnst.}$

  $U(1)$ transf.

- $\boxed{\psi \longrightarrow \Lambda\psi}$ (def.)

$$\Lambda = \exp i\,\theta_{\mu\nu}\Sigma^{\mu\nu}$$

$\downarrow$

mixings $\quad \theta_{\mu\nu} = -\theta_{\nu\mu}$

$$\Sigma^{\mu\nu} = \frac{1}{4i}\left[\gamma^\mu, \gamma^\nu\right]$$

$$= -\Sigma^{\nu\mu}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$\left\{ \begin{array}{l} g_{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1) \\ \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & I \\ I & 2 \end{pmatrix} \end{array} \right\}$$

$\uparrow$

Pauli matrices

$$\Downarrow$$

$$\psi^c \to \Lambda \psi^c$$

$$\boxed{\begin{aligned} &\psi^c \equiv C \bar{\psi}^T = C \gamma_0 \psi^* \\ &C^T \gamma_\mu C = - \gamma_\mu^T \\ &C^T = C^\dagger = - C \end{aligned}}$$

$$\boxed{C = i \gamma_2 \gamma_0}$$

$$\cancel{\exists} \quad \exists\, \gamma_5 \quad : \quad \{\gamma_5, \gamma_\mu\} = 0$$

$$\Downarrow$$

$$[\gamma_5, \Sigma_{\mu\nu}] = 0 \qquad (*)$$

$$\gamma_5^2 = 1$$

$$\Rightarrow [L(R), \Sigma_{\mu\nu}] = 0$$

$$L(R) \equiv \frac{1 \pm \gamma_5}{2} \quad \text{(chirality)}$$

$$\boxed{\begin{gathered} \psi_L \equiv L\psi, \quad \psi_R \equiv R\psi \\ \psi = (L+R)\psi = \psi_L + \psi_R \end{gathered}}$$

$$\boxed{L^2 = L, \quad R^2 = R, \quad LR = 0}$$

$$\gamma_5 \overset{?}{=} -i\,\gamma_0\gamma_1\gamma_2\gamma_3 \overset{?}{=} +i\,\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\boxed{\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}}$$

$$\Rightarrow \psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \quad \therefore$$

2comp

$$\boxed{\begin{aligned} \psi_L &= L\,\psi = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \\ \psi_R &= R\,\psi = \begin{pmatrix} 0 \\ u_R \end{pmatrix} \end{aligned}}$$

~ 2comp

$$\psi^c = C\gamma_0\psi^* = i\gamma_2\gamma_0\gamma_0\psi^*$$

$$= i\gamma_2\psi^* = -i\begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}\psi^*$$

$$= \begin{pmatrix} -i\sigma_2 u_R^* \\ i\sigma_2 u_L^* \end{pmatrix}$$

$$\boxed{C: \text{ flips } L \leftrightarrow R}$$

$$(\psi^c)_L \equiv L\psi_c = \begin{pmatrix} -i\sigma_2 u_R^* \\ 0 \end{pmatrix}$$

anti-particle

• Lorentz $\psi \to \Lambda \psi$

$$\Lambda = \exp\left(i \, \Sigma^{\mu\nu} \theta_{\mu\nu} \right)$$

$$\Sigma^{ij} = \frac{1}{4i} [\gamma^i, \gamma^j]$$

$$= \frac{1}{4i} \begin{pmatrix} [\sigma_i, \sigma_j] & 0 \\ 0 & [\sigma_i, \sigma_j] \end{pmatrix}$$

$$= \frac{1}{4i} \, 2i \, \varepsilon_{ijk} \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}$$

$$\Sigma^{ij} = \frac{1}{2} \varepsilon^{ijk} \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}$$

$$\boxed{\begin{array}{c} u_{L(R)} \to e^{i\theta_i \frac{\sigma_i}{2}} u_{L(R)} \\ \theta_{ij} = \varepsilon_{ijk}\theta_k \end{array}}$$

$$\Uparrow$$

$$\boxed{SU(2) = \text{rotations}}$$

$$u \to \underbrace{U}_{\text{SU(2)}} u \leftarrow \text{spinor}$$

$$UU^+ = U^+U = 1 \qquad (1)$$

$$\det U = 1 \qquad (2)$$

$$(1) \Rightarrow U = e^{iH}, \quad H = H^+$$

$$(2) \Rightarrow \quad \mathrm{Tr}\, H = 0$$

$$\Rightarrow \quad H = \theta_i \frac{\sigma_i}{2} = \theta_i T_i$$

$$\boxed{[T_i, T_j] = i \varepsilon_{ijk} T_k}$$

- $$\Sigma^{0i} = \frac{1}{4i} [\gamma^0, \gamma^i] =$$

$$= \frac{1}{2i} \gamma^0 \gamma^i = \frac{1}{2i} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

$$= \frac{1}{2i} \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}$$

$$\boxed{\Sigma^{0i} = \frac{i}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & -\sigma_i \end{pmatrix}}$$

$$\Sigma^{0i}\,\theta_{0i} = \frac{i}{2}\begin{pmatrix} \sigma_i \chi_i & 0 \\ 0 & -\sigma_i \chi_i \end{pmatrix}$$

$$\chi_i = \theta_{0i}$$

$$\Downarrow$$

$$\boxed{u_{L,R} \to e^{\,i\vec{\sigma}/2\,(\underbrace{\vec{\theta}}_{\text{ROT}} \pm i \underbrace{\vec{\chi}}_{\text{BOOST}})}\,u_{L,R}}$$

· boost along z (v):

$\star\star\star$ $\boxed{\tanh \chi_3 = v}$ Prove!

$$\mathcal{L}_0 = i\,\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

- $$\bar{\psi}\gamma^\mu\partial_\mu\psi = \overline{\psi_L}\,\gamma^\mu\partial_\mu\psi_L + {}$$
$$+\ \overline{\psi_R}\,\gamma^\mu\partial_\mu\psi_R$$

- $$m\bar{\psi}\psi = m\ \psi^\dagger\gamma^0\ \psi$$
$$= m\,(\psi_L^\dagger + \psi_R^\dagger)\,\gamma^0\,(\psi_L + \psi_R)$$

$$= m\left(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L\right)$$

$$= m\left(u_L^\dagger u_R + u_R^\dagger u_L\right)$$

$$m_D \bar{\Psi}\psi = m\left(u_L^\dagger \; u_R^\dagger\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

<u>Dirac mass term</u>

$\boxed{\text{Majorana 1937}}$

$\Downarrow$

what about chiral fermions?

Suppose: $\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}$

Can I write a mass term?

$\boxed{\text{bi-linear}}$

$$\mathcal{L}_D = i\,\bar{\psi}\,\gamma^\mu \partial_\mu \psi - m_D\,\bar{\psi}\psi$$

$$\Rightarrow \quad i\,\gamma^\mu \partial_\mu \psi = m_D\,\psi$$

$$\psi = \tilde{\psi}(p)\, e^{ipx}$$

$$\Downarrow$$

$$p_\mu \gamma^\mu \tilde{\psi} = m_D \tilde{\psi} \quad \Big/ \, p_\nu \gamma^\nu$$

$$p_\mu \gamma^\mu p_\nu \gamma^\nu \tilde{\psi} = m_D^2 \tilde{\psi}$$

$$\parallel$$

$$p_\mu p_\nu \, \frac{1}{2}\{\gamma^\mu, \gamma^\nu\} \, \tilde{\psi} = m_D^2 \tilde{\psi}$$

$$\Downarrow$$

$$p_\mu p^\mu \, \tilde{\psi} = m_D^2 \tilde{\psi}$$

$$\Downarrow$$

$$\boxed{E^2 - \bar{p}^2 = m_D^2} \quad (\text{def.})$$

Q. Can I write mass term for ν_L?

$$\nu_L \quad \nu_L$$

spin $1/2$ $\quad u = \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}$

$$S = 0 \qquad |0\rangle = |\uparrow\downarrow - \downarrow\uparrow\rangle$$

$$\|$$

$$u^T \epsilon u \qquad \epsilon_{ij} = -\epsilon_{ji}$$

$$\|$$

$$i\sigma_2$$

$$\Downarrow$$

$$u_L^T i\sigma_2 u_L = \text{invariant}$$

$$\longrightarrow u_L^T U^T i\sigma_2 U u_L$$

$$U = e^{i\theta_i \sigma_i/2}$$

$$= u_L^T e^{i\theta_i \sigma_i^T/2} i\sigma_2 e^{i\theta_j \sigma_j/2} u_L$$

$$= u_L^T i\sigma_2 \underbrace{e^{-i\theta_i \frac{\sigma_i}{2}} e^{i\theta_j \sigma_j/2}}_{1} u_L$$

$$= u_L^T i\sigma_2 u_L$$

$$\boxed{m_M \; u_L^T \, i \, \sigma_2 \, u_L}$$

possible for neutral particles