Ludwig-Maximilians-Universität München

QCD and Standard Model

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Guidelines :

- The exam consists of 5 problems.
- The duration of the exam is 2.5 hours.
- Please write your name or matriculation number on every sheet that you hand in.
- Your answers should be comprehensible and readable.

GOOD LUCK!

Exercise 1	12 P
Exercise 2	8 P
Exercise 3	20 P
Exercise 4	28 P
Exercise 5	32 P

Total	100 P

Problem 1 (12 points)

Let the Standard Model Higgs doublet take the following vacuum expectation value

$$H_0 = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \ .$$

a) Write down the unbroken generators, if there are any.

b) What is the unbroken group ?

c) How many gauge bosons acquire mass and how many remain massless ?

Problem 2 (8 points)

Consider the limit in which all the gauge and Yukawa couplings in the Standard Model are zero. What would be the symmetry of the Higgs sector in this case ?

Problem 3 (20 points)

a) Demonstrate that the hypercharge is free from gauge anomalies. Consider $[\mathrm{U}(1)]^3$, as well as the mixed anomalies including hypercharge with SU(2) and SU(3).

b) Consider a (gauge) U(1) theory with a massless gauge boson and 3 Dirac fermions with masses $m_1 = 4m_2 = \frac{5}{3}m_3 \neq 0$. What is the $[\mathrm{U}(1)]^3$ gauge anomaly in this case ?

Problem 4 (28 points)

Assume that the mass matrices for the up- and down- type quarks have the following forms (in the basis of weak interaction eigenstates)

$$M^{(u)} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \ , \qquad \text{and} \qquad M^{(d)} = m \begin{pmatrix} 1+a^2 & ab & 0 \\ ab & 1+b^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ ,$$

respectively. Here $m_i, [i = u, c, t]$ the mass of the respective quark flavor, m a parameter with dimensions of mass, and a, b real.

a) Find the CKM matrix. How many independent parameters does it have ? Parametrize them in terms of a and b.

b) Will there be a physical CP-violating phase ? Explain.

Problem 5 (32 points)

Let us now restrict ourselves to two generations of quarks. Take the mass matrix of the up-type quarks to be diagonal, and the one for the down-type quarks to be the following

$$M^{(d)} = m \begin{pmatrix} 0 & a \\ a & 2b \end{pmatrix} \ ,$$

with m a parameter with dimensions of mass and a, b real with $a \ll b$.

a) Find the 2×2 analog of the CKM matrix in terms of a and b.

b) Take $m_s/m_d \approx 20$ and compare the value of the mixing angle $\theta_{\rm mix}$ with its experimentally measured value $\theta_{\rm mix} \approx 13^o$.

c) Compute the following tree-level ratios of the W- and Z- boson decay rates to quarks as a function of $\theta_{\rm mix}$

$$\frac{\Gamma(W \to ud)}{\Gamma(W \to us)} \ , \quad \frac{\Gamma(Z \to u_L u_L)}{\Gamma(Z \to d_L d_L)} \ , \quad \frac{\Gamma(Z \to u_R u_R)}{\Gamma(Z \to d_L d_L)} \ .$$

Assumptions : Take the W- and Z- bosons at rest. Assume that the quark masses are negligible compared to their energies.