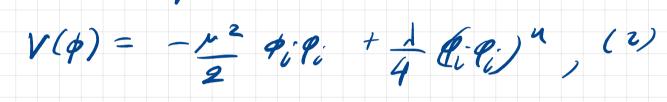
Problem 1

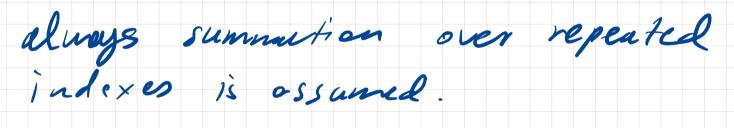


The starting point is:

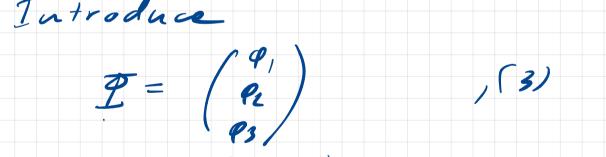
, (1) $\mathcal{L} = \frac{1}{2} \partial_{\mu} q_i \partial^{\mu} q_i - V(\phi)$



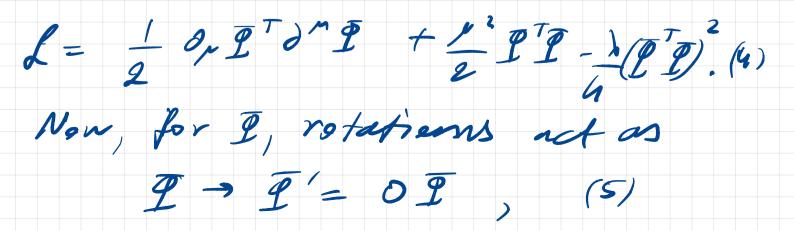


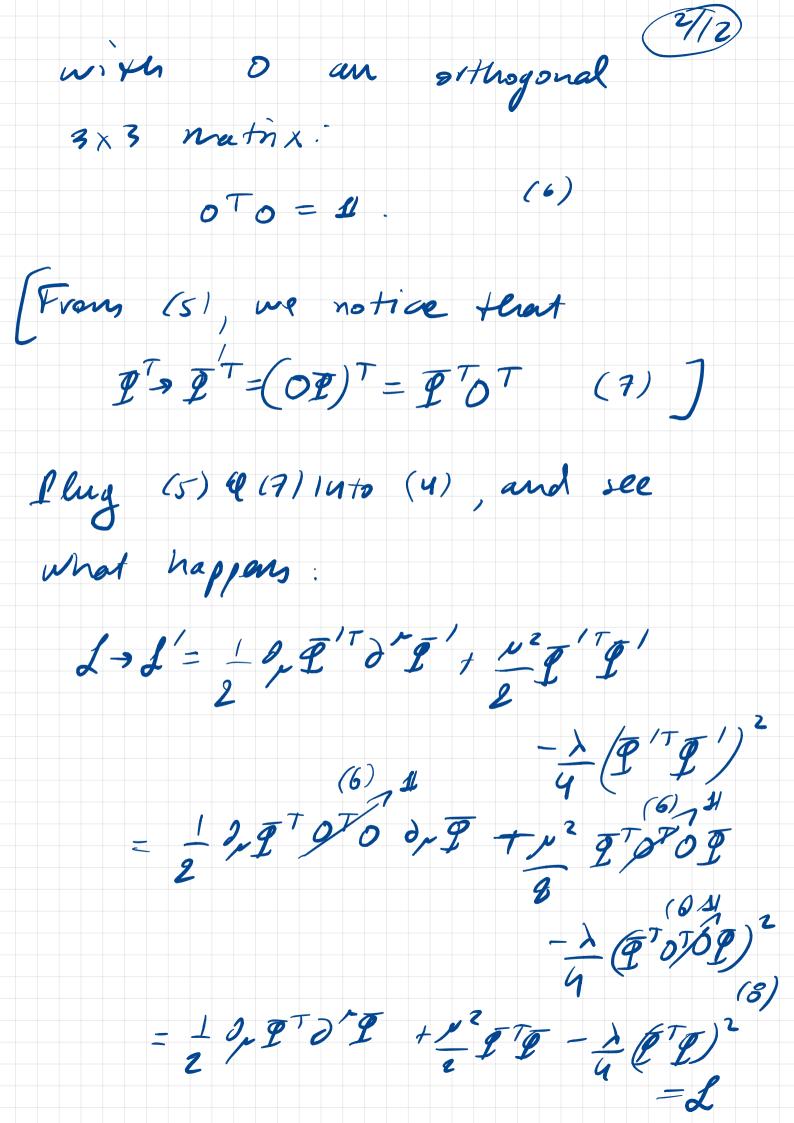












Indeed, ve have inværienne. [3/12]

under 30 rotations in the internal spiace of fields fi.

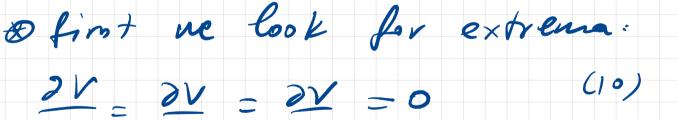
[Equindenty, you nay work with cach conjonant separately:

P; → Pí > Eiju .--)

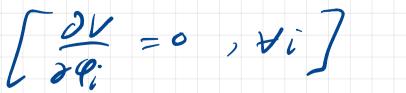


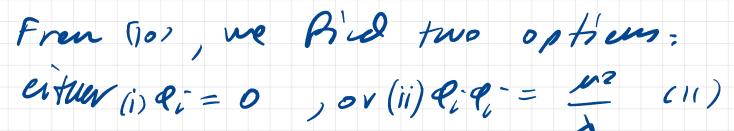
 $V(q) = -\frac{m^2}{2} e_i e_i + \frac{\lambda}{4} (e_i e_i)^2, (1)$

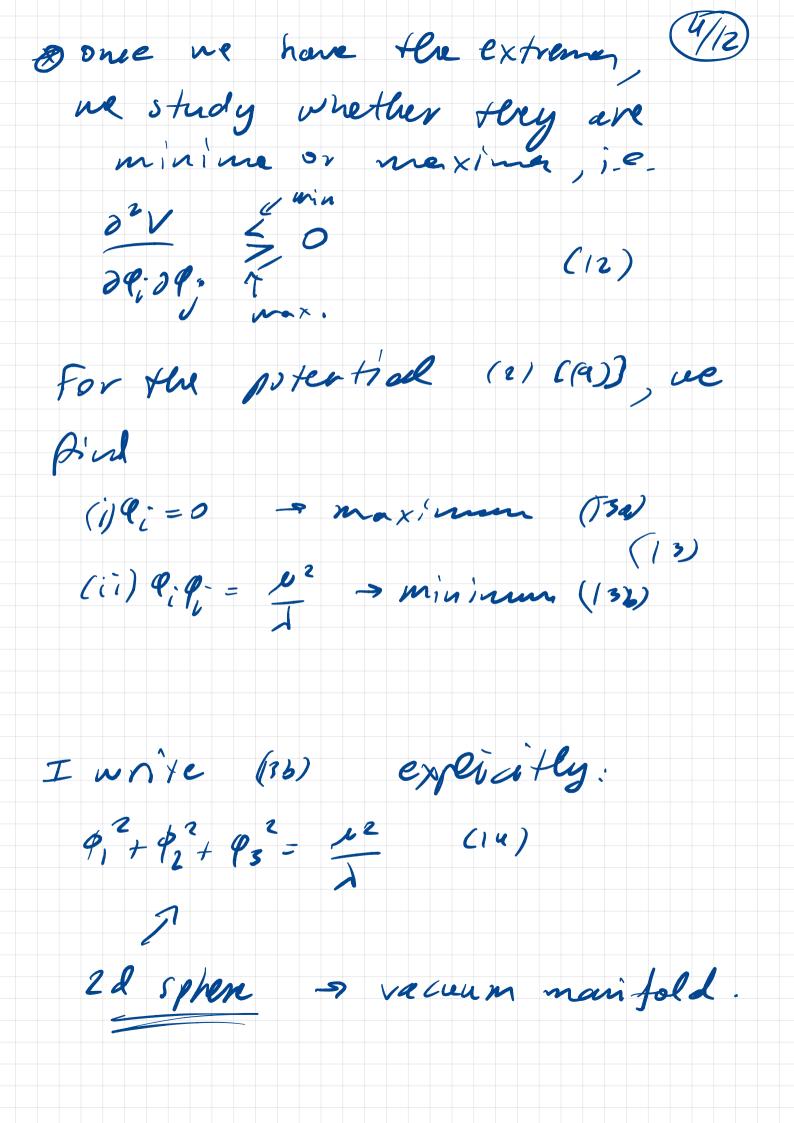


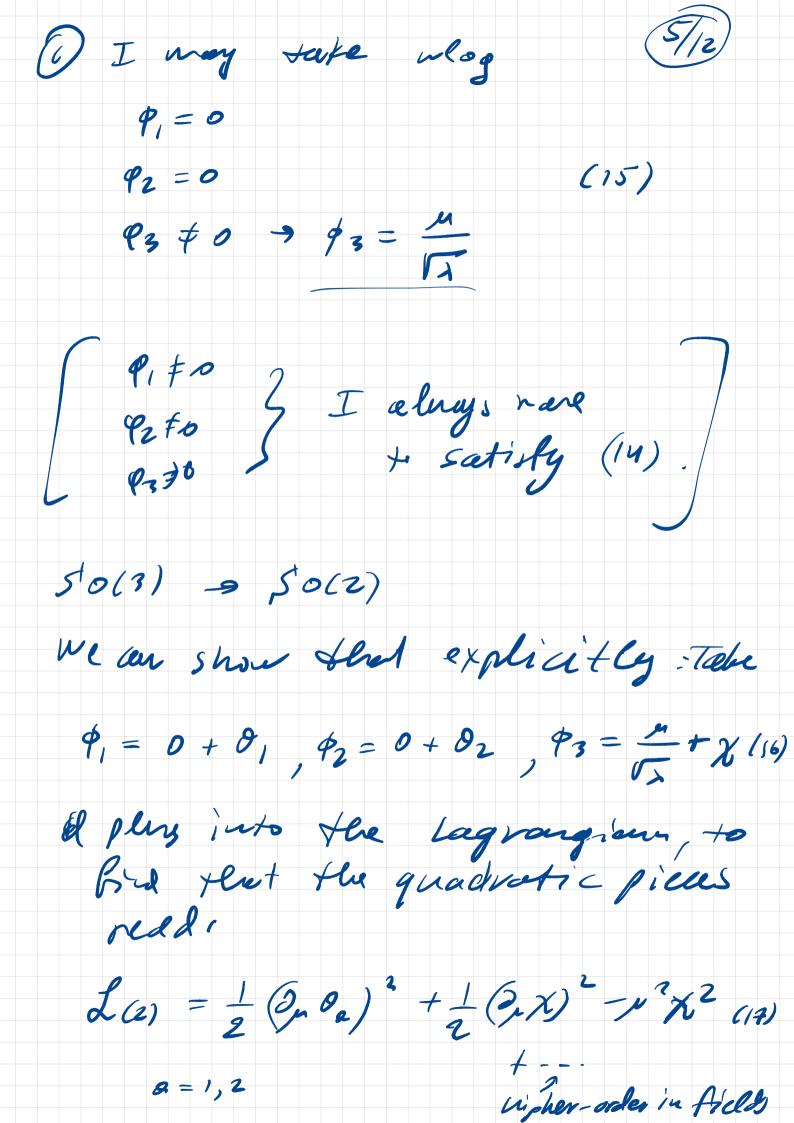


 $\frac{2V}{2q_1} = \frac{2V}{2q_2} = \frac{2V}{2q_3} = 0$









Notice two serings: (1) S'O(2) inverience because Da's equer in a symetry presering form (2) X is a nassive field with mass $m_{\chi} = 12^7 \mu$ 8, 82 which are nassless we have a situation in Mich So(3) -> SO(2) n(n-1) 2 # generators of so(n) = So(3) => 3 generators SO(2) => 1 generator # semenalin # gonnatives # so(3) - # so(2) = 2 -> # massless Bields Namme - Goldstone, bass

7/12) (a) Gauging of \$0(3): $\begin{pmatrix} \overline{\mathcal{P}} = \begin{pmatrix} \overline{\mathcal{P}}_1 \\ \overline{\mathcal{P}}_2 \\ \overline{\mathcal{P}}_2 \end{pmatrix} & \overline{\mathcal{P}} \to \overline{\mathcal{P}}' = \mathcal{O} \overline{\mathcal{P}} & (18) \\ \begin{pmatrix} \overline{\mathcal{P}}_3 \end{pmatrix} & & \mathcal{O} \to \mathcal{O} = \mathcal{O}(\mathcal{X}) \\ & \mathcal{O} \to \mathcal{O} = \mathcal{O}(\mathcal{X}) \end{pmatrix}$ Look at the kinetic part: 2, 2'T 2" -> 2, (T OT)2" (E) >] (2,07) 02, 9 + -what we have to do is modify the derivative eccordicyly, such fert it transform coursatly $\partial_{\mu} \rightarrow D_{\mu} = \partial_{\mu} + A_{\mu}$ (10) gange field (3×3 natrix] Alet transforms in a mananen Aleret ensures $(\mathcal{D}_{\mu}\overline{\mathcal{F}})' = \mathcal{O}\mathcal{D}_{\mu}\overline{\mathcal{F}}$ (20)

EThe messes of the gauge fields \$/12 come from the covariant derivative of I. On top of the vacuum, we find $m_{A_1} = m_{A_2} = g \frac{m}{\sqrt{\lambda}}, \quad m_{A_3} = 0$ (21)

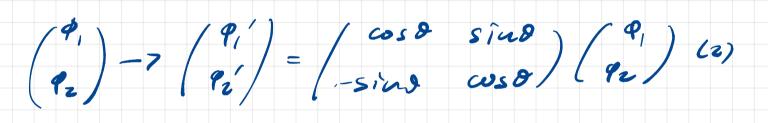
froblem 2

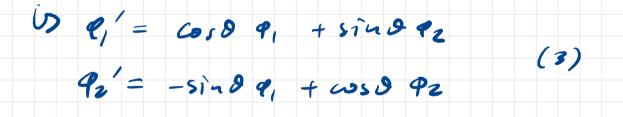


$(a) d_{0} = \frac{1}{2} \frac{g}{\mu} \frac{g}{a} \frac{g}{\mu} \frac{g}{a} + \frac{\mu^{2}}{2} \frac{g}{a} \frac{g}{a} - \frac{1}{4} \frac{g}{\mu} \frac{g}{a} \frac{g}{a} \frac{g}{\mu} \frac{g}{\mu}$

@ symmetry group = So(2) [U(1)]

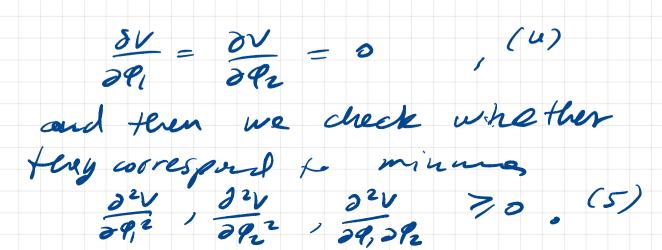
first let's understand sut happens in terms of Q1, 92:





Ø grone states(valua):

First we find extense



Fron (4) \$ (5) we find (0/12)

Eq. = 92 = 03 or Solver for the fiberty to have cither

 $\boldsymbol{\varphi}_{1} = \boldsymbol{O} \quad \boldsymbol{\mathcal{R}} \quad \boldsymbol{\varphi}_{2} = \frac{\boldsymbol{\mu}}{\boldsymbol{V}_{A}} \tag{2}$

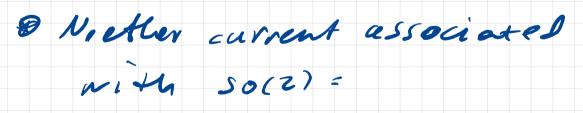
or na rena or materer compination ne lite.

Initially ve have SO(2) : 1 generator

"no thing "

I massless mode ; i the theory

A exactly the same as in Inollen !



 $i_{\mu} = \frac{\delta \mathcal{L}_{0}}{\delta \mathcal{I}_{\mu} + \frac{\delta \mathcal{L}_{0}}{\delta \mathcal{I}_{\mu} + \frac{\delta \mathcal{L}_{0}}{\delta \mathcal{I}_{\mu} + \frac{\delta \mathcal{I}_{0}}{\delta + \frac{\delta \mathcal{I}_{0}}{\delta + \frac{$ (8)

59, 592 are the infinites (med soco) rotations of the Giclds. Fren (3), or set (1/12) get $\delta \varphi_1 = \delta \varphi_2 , \quad \delta \varphi_2 = -\delta \varphi_2 \quad (9)$ So, fran (1), (3), (9), ne get • (107 $V_{\mu} = \phi_2 \partial_{\mu} \varphi_1 - \varphi_1 \partial_{\mu} \varphi_2$ Owe now take the following Lagranyies: $\begin{aligned} \mathcal{L} &= \frac{1}{2} \frac{\partial_{\mu}}{\partial_{\mu}} \frac{\partial^{\mu} q_{a}}{\partial_{\mu}} + \frac{\mu^{2}}{2} \frac{q_{a} q_{a}}{\partial_{\mu}} - \frac{\lambda}{4} \frac{q_{a} q_{a}}{\partial_{\mu}} + \varepsilon \mathcal{T}(\phi_{1}) \quad (11) \end{aligned}$ U(P,) = function at P, only (12) and E is a small paremeter. we look for minine af the potential. $\frac{\partial V}{\partial \phi_1} = \frac{\partial V}{\partial \phi_2} = 0$ 9 (13) ne find $\varphi_1(\mu^2 - \lambda \varphi_a \varphi_a) + \epsilon U' = 0 \quad \text{af } \varphi_2(\mu^2 - \lambda \varphi_a \varphi_a) = 0 \quad (14)$

from the above we notice the (12/12)

(i) $\phi_2 = 0 \rightarrow \phi_1 \simeq \frac{\mu}{14} + \epsilon \frac{V'}{2\mu^2}$, (15)

following optients:

(ii) $\phi_{c} \neq 0 \rightarrow \phi_{j} = 0, \ \sigma' = 0$. (16)

In either case (is , (i) , the spectrum of the theory contains a particle whose mass is proportional to E,

such feat sun se explicit breaking variales it becomes the genuine

NG beson of the broken generator.