Problem Set 5

1. Spontaneous Symmetry Breaking of SO(3)

In this exercise we continue the discussion on spontaneous symmetry breaking initiated in a previous Problem Set.

Consider the following Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i - V(\phi)\,,$$

where ϕ^i are three real scalar fields in the fundamental of SO(3), and summation over $i = 1, 2, 3$ is assumed. The potential is given by

$$V(\phi) = -\frac{\mu^2}{2}\phi^i\phi^i + \frac{\lambda}{4}(\phi^i\phi^i)^2\,,$$

with $\mu^2, \lambda > 0$.

a) Show that $\mathcal{L}$ is invariant under a global SO(3) symmetry, i.e. rotations in the three dimensional field-space.

b) Minimize the potential and determine the ground state of the system. What is the vacuum manifold, i.e. the manifold of all values of the vacuum expectation value (vev) ϕ_0 of the field that minimize the potential?

c) Does the vev break completely the SO(3) symmetry? If not, what is the unbroken group? How many Nambu-Goldstone bosons do you expect? Compute the quadratic Lagrangian for the perturbations on top of the vacuum to verify your expectation.

d) Let us now gauge the theory by promoting the global SO(3) to a local symmetry. Write down the corresponding Lagrangian.
Hint : You should generalize the partial derivative to a covariant one and add a kinetic term for the gauge fields.

e) Find the mass spectrum of the gauge fields by expanding the appropriate terms of the Lagrangian around the vacuum that you found before.

2. Explicit symmetry breaking and pseudo-Goldstone bosons

Consider the following Lagrangian capturing the dynamics of two real scalar fields

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1\ ,$$

where

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 + \frac{\mu^2}{2}\Big((\phi_1)^2 + (\phi_2)^2\Big) - \frac{\lambda}{4}\Big((\phi_1)^2 + (\phi_2)^2\Big)^2\ ,$$

and

$$\mathcal{L}_1 = \epsilon\, U(\phi_1)\ ,$$

where ϵ is a small parameter and $\underline{U \text{ depends non-trivially on the field } \phi_1 \text{ only}}$.

a) Take $\epsilon = 0$. What is the symmetry group of the Lagrangian? Find the ground state(s), the Noether current and the Nambu-Goldstone boson(s).

b) Take now $\epsilon \neq 0$. Find the lightest mode and its mass to the leading order in ϵ. This mode is called "pseudo-Goldstone mode."